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A Dynamic Theory of the Banking Firm

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ABSTRACT

Dynamic analysis has greatly increased our understanding of the microfoundations of the general firm. Unfortunately, however, little attention has been focussed on the dynamic nature of the banking firm. Instead, most theoretical work has derived the optimal behavior for the bank in a single period context. This approach, while yielding insight into the function of the bank as a broker between borrowers and lenders, has proven incapable of capturing many essential elements of the banking firm. The role of capital, for example, is understated if the risk of bankruptcy is not considered. Perhaps more importantly, the role of banks in transforming risks in an uncertain environment cannot be captured if the problem of handling these risks over time is not considered. This paper incorporates these dual roles of brokerage and risk transformation in a cohesive model of the banking firm.

NUMEROUS APPROACHES HAVE BEEN taken to model and explain the banking firm. Portfolio selection models, production theory models, models of resource allocation, all have been suggested as the proper paradigm for the banking firm. Yet, as Baltensperger notes in a recent survey paper [2], these approaches "have not yet been forged together to form a coherent, unified, and generally accepted theory of bank behavior."

Much of the difficulty in modelling the banking firm stems from the complexity of the bank's operations. As an intermediary, the bank brings together borrowers and lenders of funds. Because the bank is subject to both default and market risks on the funds it lends, and to withdrawal risk on the funds it borrows, the bank must contend with risk in both its assets and its liabilities. In addition to its intermediary role, however, the bank is also a firm, presumably run to benefit its stockholders. Because management and ownership are separate, the stockholders must somehow force the management to operate the bank for their benefit. Finally, the banking firm is a regulated enterprise and thus is subject to the rules and supervision of the bank regulatory agencies.

A theory of the banking firm should therefore incorporate the roles of the bank as a financial intermediary, a firm, and a regulated enterprise. There remains, however, the problem of deciding what it is that the bank is actually maximizing. Klein [9] and numerous other authors have assumed that the bank is operated to maximize the return on equity. In a one-period model this forces the bank to maximize short-term profits. However, in a multiperiod model the optimal actions for any period clearly depend on expectations of future opportunities and the bank's current position. A further complication is that although stockholders may prefer the maximization of return on equity, they are dependent upon the

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management to actually achieve this. Unless management and ownership are unitary, "agency" problems must be considered.

The effect of agency on firm operations recently has received much attention. Although Fama [5] argues that a free market for managers' services will force profit-maximizing behavior, Edwards's [4] study of the operating behavior of commercial banks in 44 metropolitan areas indicates that agency problems are found in the banking industry.[1] This suggests that an analysis of the banking firm should allow for agency problems to affect bank operations.

In this paper decisions are viewed from the perspective of the bank manager and it is assumed that the manager maximizes his utility subject to the constraints imposed upon him by the regulators and the stockholders.[2] The model utilizes stochastic dynamic programming to capture the multiperiodic nature of the manager's problem. This approach is consistent with a variety of short-run strategies; profit maximization, growth maximization, et al., are all possible objectives for the manager. Clearly, the tighter the control exercised by the stockholders, the closer the bank's operating results will be to those desired by the stockholders.

In the next section a dynamic model is derived which allows for the endogenous determination of the composition of the bank's asset portfolio; the composition of the bank's deposit liabilities and borrowing; the equilibrium scale of the firm; and the equilibrium level of capital retained in the firm. In subsequent sections, the interrelatedness of bank assets and liabilities is examined as well as the impact of shareholder demands. The paper concludes by looking at the impact of statutory capital requirements on the operations of the banking firm.

I. A Model of the Banking Firm

Because we are not concerned with new firm entry or development, the bank is assumed to be already operating with a given level of equity.[3] As a depository intermediary, the bank accepts deposits and makes loans. Deposit inflows and loan demand are random, and the bank also has access to an established short-term market for borrowing and lending.

The bank's portfolio is composed of two assets, L^1 and L^2, which have different levels of risk and return. The manager finances his portfolio with deposits, money market borrowing, and equity. The bank faces a competitive deposit market in which deposits are a function of the resources allocated by the firm and a random

[1] Edwards's empirical results are consistent with Shavell's finding that, if the shareholders are risk neutral and the manager is risk averse, under a Pareto Optimal incentive contract the manager will not operate the firm to maximize profits. If a free market for manager's services exists, its effect may be to limit the inefficiencies of the bank's operations but not necessarily to the profit maximizing level.

[2] The model can also be adapted to view decisionmaking from the point of view of the owner. In that case, the owner-manager receives utility from the long run stream of dividends and hence growth in the value of the equity in the firm. However, since it is not possible to charter a bank with only one owner, agency problems will still arise because of the minority shareholders. It seems more reasonable, then, to view the bank as being run by a manager.

[3] Bank capital requirements are not fully binding until the 20th year of operation. To avoid the differences of new firms and seasoned firms we look only at fully established firms.

element. The bank pays an exogenously given interest rate to its depositors and competes for deposits by providing services to the depositors. The volume of services determines the cost of deposits and, thus, the expected "price" for each new deposit. While the bank manager may influence the size of deposits he must also be concerned with withdrawal risk. Equity growth is assumed to occur only through retained earnings.[4] The manager is permitted, however, to raise funds by borrowing in the money market if profitable investment opportunities occur.

The manager of the bank seeks to maximize his utility subject to the constraints imposed by the regulators and the stockholders. The manager receives utility from the compensation, X_t, he extracts from the bank in every period. The manager's problem is

$$\text{MAX } E\left[\sum_{t=0}^{\infty} \alpha^t U(X_t)\right] \tag{1}$$

where α is a discount rate applied to future consumption, $0 \leq \alpha < 1$.[5] U is a utility function assumed strictly increasing ($U' > 0$), strictly concave ($U'' < 0$), continuous, bounded above, and twice continuously differentiable. The manager may receive utility from factors exogenous to his work but these factors are assumed to have a zero covariance with his benefits from the firm.[6] Utility is normalized to focus on his firm related utility so that $U(0) = 0$.

Because the manager wishes to maximize his utility not only in the present period but also in all future periods, the condition of the bank is of crucial importance. The state of the bank at the beginning of period t is given by Y_t where Y_t equals the accumulated net worth. The manager's actions in period t and the stochastic environment will influence the state of the bank in the next period, Y_{t+1}. Hence, the manager's actions today are important both for their effect on his current utility and for their effect on the future state of the bank. This future state, in turn, affects his future return from the bank, X_{t+1}.

The manager's actions are constrained by the demands of the bank's regulators. These regulatory constraints include both reserve requirements and capital requirements. The bank is required to hold reserves, equal to a given percentage, κ, of deposits. Additionally, the bank must meet a certain minimum net worth requirement, B, and failure to do so results in bankruptcy.

The manager's actions are also constrained by the demands of the stockholders. One mechanism the stockholders can utilize to manifest these demands is an

[4] Hence, equity is not a fixed component but rather changes over time. This treatment contrasts with previous models, for example Klein [9] and Hart and Jaffee [7], where equity is assumed stationary. I do not explicity consider the case where owners issue new equity. Given that banks rarely issue new equity, however, abstracting from new issues does not constitute an unreasonable assumption. See also Footnote 11.

[5] The model employs an infinite horizon as the appropriate time frame. The alternative is to view the problem as having a finite horizon but there are several drawbacks to that. First, placing a strict horizon on the number of years implies that the manager has absolute certainty as to when he will leave the firm. Second, if the manager is rational he will take all the assets in the last period and drive the firm into bankruptcy. This problem can be mitigated by assuming a positive scrap value for the firm but any scrap value will be clearly arbitrary. Since, for large T, the finite horizon plans converge to the infinite horizon plans, the policies analyzed here approximate those of the finite horizon case.

[6] The manager's labor-leisure tradeoff has been omitted for simplicity. It can easily be examined by defining the utility function as $U(X_t, H_t)$ and allowing H_t, leisure, to be a choice variable.

incentive contract which is a risk-sharing agreement dictating the division of returns between the stockholders and the manager. The optimal design of such contracts has been considered by many researchers (see Holmström [8]; Shavell [12]; Harris and Raviv [6]). In our model, the return to the stockholders is modelled as a function, $I(\cdot)$, of the bank's portfolio condition, equity, and profit flow. Because bankruptcy is a real possibility, I assume that if management increases the bank's riskiness, the stockholders demand a higher return to compensate them for the additional risk. However, I do not require any specific form for the incentive contract; hence, the model permits, but does not assume, a Pareto optimal contract. The return to the stockholders is assumed paid out as a cash dividend.[7]

The stockholders may also limit the manager's behavior by imposing absolute, or nonstate-contingent, bounds on his actions. One such bound could be a ceiling on the total compensation the manager can extract in any one period (e.g., $X_t \leq \bar{X}$). This constraint limits the manager's ability to "take the money and run" and, hence, curtails self-dealing behavior by the manager.[8]

At the beginning of period t the manager chooses his consumption, X_t, his loan portfolio, L_t^1 and L_t^2, and his level of deposit services, P_t. The manager may not choose a level of X_t that immediately drives the equity below B; that is, $0 \leq X_t \leq Y_t - B$, for $Y_t > B$. Deposit services P_t cost $C(P_t)$ and generate deposits which will be in the bank for period t. These deposits are subject to a random element, $\tilde{q}_t \epsilon [\underline{q}, \bar{q}]$ and are given by $D(P_t, \tilde{q}_t)$. The loans to which the manager commits the bank, L_t^1 and L_t^2, are loaned out for the period t and pay off at the end of period t at the rates $\tilde{\imath}_t^1$ and $\tilde{\imath}_t^2$ where $\tilde{\imath}_t^j \epsilon [\underline{i}^j, \bar{\imath}^j]$. The interest rate randomness results from both default risk and market risk.[9]

The model is structured so that the random variables in period t are realized after the manager takes his actions. The set of possible realizations of the random variables is denoted by $\Omega = [\underline{i}^1, \bar{\imath}^1] \times [\underline{i}^2, \bar{\imath}^2] \times [\underline{q}, \bar{q}]$, and an element of Ω is denoted $\tilde{w}$. The randomness is described by a continuous joint density function $\Gamma(\tilde{\imath}^1, \tilde{\imath}^2, \tilde{q})$.[10]

[7] The incentive contract allows the stockholders to control implicitly the manager's return. Because the manager must meet these shareholder demands, the optimal incentive contract encourages the manager to maximize profits while rewarding him with sufficient compensation. Clearly, if the stockholders' demands are set too high, the manager will either quit or have little incentive to pursue profit maximizing policies; if their demands are too low, the manager will be able to extract excessive compensation. The optimal form of the incentive contract thus depends upon the manager's opportunities and the return stockholders could earn on alternative investments of the same risk.

[8] The shareholders can also limit the manager's actions through the net worth constraint. The required equity level could include both a regulatory minimum and an additional amount specified by the shareholders. This specification would permit the shareholders to determine the minimum capital level of the firm, below which the current manager would be fired.

[9] I am assuming that a secondary market exists and that the manager evaluates all loans at their market value. Hence, at the end of the period his return is equal to the interest payment plus a capital gain (loss) if the asset maturity is greater than one period. The manager may then decide to renew such loans or sell them in the secondary market.

[10] We could assume that w_t follows a Markov process specified by a transition density function $\Gamma(\tilde{w}_{t+1}, w_t)$. That is, given $w_t \in \Omega$, the probability that $\tilde{w}_{t+1}$ is in any Borel measurable subset B of Ω is $\int_B \Gamma(\tilde{w}_{t+1}, w_t)\, d\, \tilde{w}_{t+1}$. However, this modification complicates the notation (in particular Ψ and V will both have w_t as a separate argument) but does not lead to interesting economic differences in the conclusions.

Because the manager makes his loan commitments for period t before the deposit uncertainty, $\tilde{q}_t$, is resolved, he may face a shortage (or glut) of deposits. If the manager's loan commitments are greater than his deposit inflow (net of reserves) and equity, he must borrow the residual defined as $T_t = L_t^1 + L_t^2 - (1 - \kappa)D_t - (Y_t - X_t)$.[11] The cost of these borrowings is given by the function $\rho(T)$. Conversely, if his loan commitments are less than his resources, T_t is loaned out at the risk-free rate r_f.

At the end of period t, the manager knows that period's profit (excluding his return) defined as $\pi_t = \tilde{\imath}_t^1 L_t^1 + \tilde{\imath}_t^2 L_t^2 - rD(P_t, \tilde{q}_t) - C(P_t) - \rho(T_t)$. At this time he must pay the stockholders the amount $I(Y_t, \pi_t, L_t^1, L_t^2)$. The state of the bank at the beginning of period $t + 1$ is thus given by

$$\begin{aligned} Y_{t+1} = Y_t - X_t + \tilde{\imath}_t^1 L_t^1 + \tilde{\imath}_t^2 L_t^2 - rD(P_t, \tilde{q}_t) - C(P_t) - \rho(T_t) \\ - I(Y_t, \pi_t, L_t^1, L_t^2) \end{aligned} \tag{2}$$

where the variables are defined in the text and

$D(P_t, \tilde{q}_t)$ = a deposit function assumed bounded, convex, increasing and twice continuously differentiable in P, for all q, and $D(0, \tilde{q}) = 0$;

$\rho(T)$ = a money-market borrowing (or lending) cost function assumed convex, increasing, and twice continuously differentiable;

$C(P_t)$ = a cost function assumed bounded, convex, increasing, and twice continuously differentiable;

$I(Y_t, \pi_t, L_t^1, L_t^2)$ = an incentive contract determining the stockholders dividend where I is assumed to be a separable function, increasing in Y, π, L_t^1, and L_t^2, twice continuously differentiable, and $I(Y_t, 0, 0, 0) = r_f Y_t$

Bankruptcy occurs when the value of the equity, Y_t, falls below the exogenously given net worth constraint, B. If $Y_t < B$, the manager loses his job at the bank and receives no remuneration from the bank in period t or in any period $\tau > t$.[12] If $Y_t \geq B$, the manager has satisfied the demands of both the shareholders and the regulators and, hence, may continue to operate the bank.

We can now define the constraints which the manager faces as a function of the state of the bank. The set of feasible actions, X_t, L_t^1, L_t^2, P_t, at the beginning of period t, given Y_t, is:

$$\Psi(Y_t) = \left[\begin{array}{r} \{X_t, L_t^1, L_t^2, P_t : 0 \leq X_t \leq Y_t - B,\ P_t \geq 0,\ L_t^1 \geq 0 \\ L_t^2 \geq 0,\ X_t \leq \bar{X}\} \quad \text{if } Y_t > B \\ \{X_t, L_t^1, L_t^2, P_t : X_t = 0,\ L_t^1 = 0,\ L_t^2 = 0,\ P_t = 0\} \\ \text{if } Y_t \leq B \end{array} \right] \tag{3}$$

[11] Note that the manager extracts his compensation at the beginning of the period. Hence, the amount of funds retained in the firm that can be used to finance loans is $Y_t - X_t$.

[12] In practice, regulators rarely allow banks to be declared insolvent. Instead, the bank will be forced to issue new equity and to replace the management. From the point of view of the manager, therefore, the bank is no longer operable regardless of what the regulators decide.

We also assume that there is a finite bound A on compensation, total resource allocation, and total loan activity.[13] If the bank's equity goes below the bankruptcy point, the choice variables are set so that the manager cannot return the bank to a profitable state. Thus, if $Y_t < B$ it can easily be shown that $Y_{t+1} < B$ and $X_t = 0 \; \forall \; \tau > t$.[14]

The set-up and dynamics of the model have now been defined. However, to solve this model it is necessary to show that the manager's indirect utility received from the state of the bank can be expressed by a value function, V. The value function represents the maximum expected future utility that the manager can obtain given the current state of the bank. Additionally, we must show that optimizing policies exist. The manager's decision over his choice variables can then be represented by optimal policies which he evaluates every period based upon the current state. The proofs of the existence and properties of the value function and the optimal policies are available from the author.

Since an optimal value function exists, the manager's multiperiod maximization problem can now be written as a simple two-period decision problem

$$\text{MAX } U(X_t) + \int V(\tilde{Y}_{t+1})\Gamma(\tilde{w})\, d\tilde{w}$$

$$\text{s.t. } (X_t, L_t^1, L_t^2, P_t) \in \Psi(Y_t) \tag{4}$$

where the discount factor α is subsumed in V. We can write Y_{t+1} as the only argument of the value function since the state and random variables affect future opportunities only through equity (see the definition of the constraint Ψ). The value function written here incorporates the effect of current actions on future expected utility given that future actions are chosen optimally. Hence, the problem can be represented by the simple two-period problem given by (4).

The traditional problem with dynamic programming has been that although solutions are shown to exist, they cannot usually be characterized by differentiable first-order conditions. However, using a technique developed by Blume, Easley, and O'Hara (see [3]), the manager's problem can be treated as a classical optimization problem in which differentiable first-order conditions determine the maximizing plan at each date.[15]

[13] The purpose of the finite bound A is only to insure that the set of actions is compact, and A can be so large as to be very unlikely to be binding (i.e., A could be the total money supply in the economy).

[14] To see this, recall that $I(Y_t, 0, 0, 0) = r_f Y_t$. Then if $Y_t < B$, $Y_{t+1} = Y_t - \rho(-Y_t) - I(Y_t, 0, 0, 0) \leq Y_t$ as $I(Y_t, 0, 0, 0) = r_f Y_t$ and $-\rho(-Y_t) = r_f Y_t$.

[15] For the analysis of Sections II and III, it is useful to note under what conditions the optimal policy is a continuous function. Since the value function is not concave over the entire state space (a kink occurs at $Y = B$), the optimal policies may not always be single-valued. However, if there exists a $\bar{Y} > B$ such that an optimal policy for $Y_t \geq \bar{Y}$ results in $Y_{t+1} \geq \bar{Y}$, the policies will be continuous. Further, under this assumption both the optimal policy and the value function will be sufficiently differentiable to use comparative static analysis to characterize the optimal policy. The assumption that $Y_t \geq \bar{Y}$ is maintained throughout the paper.

II. The Optimal Strategy for the Bank

The manager's problem is

$$\begin{aligned}&\text{MAX } U(X_t) + E[V(\tilde{Y}_{t+1})]\\&\text{s.t. } X_t, L_t^1, L_t^2, P_t \in \Psi(Y_t)\end{aligned} \tag{5}$$

Since U and V are concave and differentiable, the optimal strategy, if it is interior, is given by solving

$$\begin{aligned}\text{MAX } \theta = {} & U(X_t) + E[V[Y_t - X_t + \tilde{\imath}_t^1 L_t^1 + \tilde{\imath}_t^2 L_t^2\\& - rD(P_t, \tilde{q}_t) - C(P_t) - \rho(T_t) - I(Y_t, \pi_t, L_t^1, L_t^2)]]\end{aligned} \tag{6}$$

$$\frac{\partial\theta}{\partial X_t} = U' - E[V'(\cdot)[1 + \rho'(\cdot)(1 - I_\pi)]] = 0 \tag{7}$$

$$\frac{\partial\theta}{\partial L_t^1} = E[V'(\cdot)(\tilde{\imath}_t^1 - \rho'(\cdot) - \{I_{L_1}(\cdot) + I_\pi(\cdot)(\tilde{\imath}_t^1 - \rho'(\cdot))\})] = 0 \tag{8}$$

$$\frac{\partial\theta}{\partial L_t^2} = E[V'(\cdot)(\tilde{\imath}_t^2 - \rho'(\cdot) - \{I_{L_2}(\cdot) + I_\pi(\cdot)(\tilde{\imath}_t^2 - \rho'(\cdot))\})] = 0 \tag{9}$$

$$\frac{\partial\theta}{\partial P_t} = E[V'(\cdot)(\rho'(\cdot)(1 - \kappa)D'(\cdot) - rD'(\cdot) - C'(\cdot))\{1 - I_\pi(\cdot)\})] = 0 \tag{10}$$

where primes denote derivatives of single argument functions and subscripts denote derivatives of multiple argument functions.

The interpretation of (7) is straightforward. The manager selects the amount of funds he extracts from the firm, X_t, to equate his marginal utility of present consumption to the discounted marginal utility of his future consumption. This condition also implicitly describes the retained earnings decision for the firm. Leaving funds in the firm adds to excess equity held above the required levels. Hence, the decision to augment equity reflects the manager's perception of future riskiness, his rate of time preference, and the expected investment opportunities of the bank.

Note that these conditions hold when $X_t < \bar{X}$, the shareholder specified maximum compensation level. If the $\bar{X}$ constraint is binding, the manager selects the corner solution, $X_t = \bar{X}$. The first-order condition for X_t would then become $U' - E[V'(\cdot)(1 + \rho'(\cdot)(1 - I_\pi))] < 0$. Conditions (8), (9), and (10) would remain the same but all first-order conditions would now be evaluated at $X_t = \bar{X}$.

The loan decision conditions (8) and (9) illustrate the relationship between the return on loans, and the marginal cost of funds for the bank. The $\rho'(\cdot)$ term reflects the marginal cost of borrowing. If all loans are financed by internal funds, $T = 0$ and $\rho'(T) = r_f$. Alternatively, if loans exceed deposits plus equity, borrowings increase and $\rho'(T)$ grows increasingly larger than r_f.[16] Additionally, the stockholders' demands will influence the return through the loan decisions

[16] The model also captures the bank's excess reserves decision. If the bank wishes to hold excess reserves, it lends them out as Fed Funds at interest rate r. Excess reserves thus generate income for the bank, similar to their role in modern banking.

effect on income, $I_\pi(\cdot)$, and through the portfolio adjustment, $I_L(\cdot)$. The term $(\tilde{\imath}_t^j - \rho'(\cdot) - \{I'(\cdot)\})$, therefore, reflects the expected spread on making type j loans. Notice that the bank generally will not make only one type of loan. Although the marginal cost of borrowing, $\rho'(\cdot)$, is the same for all loan types, the shareholders' demands need not be. Hence, if $I_{L^1} \neq I_{L^2}$, the bank's portfolio will consist of both asset types. This result will hold regardless of the manager's risk preferences. Finally, the $V'(\cdot)$ term incorporates the effect of each loan type's risk-return tradeoff on the behavior of the manager. The portfolio allocation decision therefore depends on the manager's risk preference, the marginal cost of funds, and the impact that the loan decision has on the required return to the stockholders.[17]

The optimal scale of the firm is determined by Equation (10). The amount of resources to expend, P_t, depends upon the marginal cost of new deposits, $(1 - I_\pi)(rD'(\cdot) + C'(\cdot))$, and the marginal benefit, $(1 - I_\pi)\rho'(1 - \kappa)D'(\cdot)$. The marginal benefit term is a function of the cost of money-market borrowing. The higher this cost, the cheaper it is to finance loans by increased deposits. The marginal cost of deposits, however, is also increasing. The scale of the firm is therefore determinant; above some point, the cost of acquiring funds through either deposits or borrowings will be greater than the return from investing them.

Finally, the composition of the bank's liabilities depends on the cost of raising funds and the investment opportunities of the firm. As long as the bank's investment return is greater than the marginal cost of funds, the bank will increase its liabilities. Whether the loan expansion is financed by deposits or borrowing depends upon their relative costs. Given the bank's cost functions, however, an optimal liability structure is determined.

The model thus provides an endogenous determination of the composition of the bank's asset portfolio; the composition of the bank's deposit liabilities and borrowings; the equilibrium scale of the firm; and the equilibrium level of capital retained in the firm. The model also indicates that the optimal structure of the bank's assets is not independent of the optimal structure of the bank's liabilities.[18] The bank manager must jointly determine the total structure of the bank.

Further, the amount of excess equity retained in the firm depends on the perceived riskiness of the bank's future operations. The structure of the bank's assets, liabilities, and capital is thus interrelated. In the next section, this interrelationship is further explored by examining some comparative static results.

[17] Michael Klein [9] notes that the interest rate, $\tilde{\imath}$, earned on an investment may decrease as more of the investment is undertaken. This occurs because loan losses can be expected to rise as the bank extends credit to less trustworthy customers. This effect can be incorporated into the model by making

$$\tilde{\imath}_{t+1}^j = \tilde{\imath}_{t+1}^j(L_t^j, \tilde{w})$$

The effect in conditions (8) and (9) is to reduce the size of the spread which in turn will further limit the scale of the firm.

[18] The independence of the bank's asset and liability structure was a characteristic of Klein's model. Since then, numerous authors have demonstrated that this is inaccurate. See Baltensperger [2] for more discussion.

III. The Dynamics of Bank Decisionmaking

Having established the optimal policies for the bank, we now consider how these policies are affected by changes in the environment faced by the bank. We examine the effects of changes in the return on loans, the cost of deposits, and the demands of stockholders.

From the first-order conditions, we know that the manager's risk preferences will influence how he operates the bank. As noted by Arrow [1], the assumptions of Decreasing Absolute Risk Aversion (DARA) and Increasing Relative Risk Aversion (IRRA) are consistent with observable economic behavior. DARA implies that as wealth increases, an individual is more likely to take on added risk. IRRA states that if both wealth and the size of the risky endeavor are increased in the same proportion, the willingness to take on the added risk will decrease. We assume that the manager's risk preferences are characterized by DARA and IRRA.

A. The Effect of Changes in Loan Rates

We first consider the effect of a change in the bank's return on loans. Define $\tilde{\iota}_t = \bar{\iota}_t + \tilde{\varepsilon}_t$. The $\bar{\iota}$ term can either be thought of as the stated interest rate minus expected loan losses or as simply the level of interest rates. We now consider the effect of a change in $\bar{\iota}$ on the manager's consumption decision, the level of equity retained in the firm, and the size of the bank's loan portfolio.

The bank's loan portfolio, L, is defined as L^1 plus L^2. The bank's investment decision is thus a choice between a risky portfolio of loans, L_t, and lending at the risk free rate $\rho'(T)$ where, for all T, $\rho'(T) = r_f$. To keep the analysis tractable, we assume the bank's resource allocation decision P_t is set at the optimal level P^* so that deposits are a random variable with a given mean and variance and that the incentive contract is linear in its arguments.

The manager thus chooses X_t and L_t to maximize his utility. His consumption decision in turn dictates the amount of earnings retained in the firm. To examine the effect of changes in $\bar{\iota}$ on the decision variables, we look at the total derivatives. Let $\phi = (\tilde{\iota} - \rho' - I_L - I_\pi(\tilde{\iota} - \rho'))$. Then

$$\frac{\partial L_t}{\partial \bar{\iota}} = \frac{(1 - I_\pi)\{U''(-LE[V''\phi] - E[V']) - (1 + \rho'(1 - I_\pi))^2 E[V'']E[V']\}}{|H|} \tag{11}$$

where H is the determinant of the Hessian for the maximization problem and $|H| > 0$ for a maximum. From concavity $U'' < 0$. As Arrow [1] shows, the sign of $E[V''\phi]$ depends upon the manager's risk preferences. If the manager has Decreasing Absolute Risk Aversion, $E[V''\phi] > 0$. Conversely, if the manager has Increasing Absolute Risk Aversion, $E[V''\phi] < 0$. This latter characteristic, however, is not observed in the "real world." Hence, under reasonable conditions

$$\frac{\partial L_t}{\partial \bar{\iota}} > 0$$

Thus, if the return on loans increases, the manager increases the bank's holding of loans.

Now consider $\partial X_t/\partial \bar{\imath}$.

$$\frac{\partial X_t}{\partial \bar{\imath}} = \frac{L(1-I_\pi)(1+\rho'(1-I_\pi))\{E[V'']E[V''\phi^2] - E[V''\phi]E[V''\phi]\} - (1-I_\pi)E[V']E[V''\phi]}{|H|} \tag{12}$$

Consider first the terms in the brackets ($\{\cdot\}$). Recall that from the definition of covariance

$$E[V'']E[V''\phi^2] + \mathrm{COV}(V'', \phi^2 V'') = E[(V''\phi)^2]$$

Clearly

$$E[(V''\phi)^2] > E[V''\phi]E[V''\phi] > 0$$

Hence, if $\mathrm{COV}(V'', \phi^2 V'') < 0$ the bracketed term is strictly positive. This covariance will be negative if the manager possesses Decreasing Absolute Risk Aversion and Increasing Relative Risk Aversion.[19]

The remaining term in the numerator, however, is strictly negative, given DARA. The manager is thus subject to two influences on his consumption. The positive bracketed term corresponds to the "income effect." If the mean interest rate on loans increases, for example, the bank is now more profitable and the manager can increase his present consumption. Conversely, the negative intertemporal "substitution effect" indicates that because the loan rates are permanently higher, the bank will have greater profitability in the future. These dual effects can be illustrated more clearly by expressing the derivative as the sum of the change in X induced by a one-time change in $\bar{\imath}$ and the change in X induced by a change in $\bar{\imath}$ in all future periods. Then

$$\frac{\partial X_t}{\partial \bar{\imath}} = \frac{\partial X_t}{\partial \bar{\imath}_t}\bigg|\, \bar{\imath}_\tau \text{ constant} + \frac{\partial X_t}{\partial \bar{\imath}}\bigg|\, \bar{\imath}_t \text{ constant} \qquad \forall\, \tau > t \tag{13}$$

The first term corresponds to the bracketed term in (12) while the second term equals the remaining term in the numerator. The manager's consumption decision thus depends on the relative magnitudes of these income and substitution effects.

The effect of changes in $\bar{\imath}$ on the level of equity retained in the firm can be determined from (12) and (13). Examining how the expected net worth changes we find

$$\frac{\partial E[\tilde{Y}_{t+1}]}{\partial \bar{\imath}} = L(1-I_\pi) - (1 + r_f(1-I_\pi))\frac{\partial X}{\partial \bar{\imath}} + ((\bar{\imath} - r_f)(1-I_\pi) - I_L)\frac{\partial L}{\partial \bar{\imath}} \tag{14}$$

Clearly, if the intertemporal substitution effect dominates the income effect (i.e. $\partial X \mid \partial \bar{\imath} < 0$), the sign of (14) is positive. However, even if the manager increases his

[19] For the covariance to be negative we need

$$\frac{\partial V''}{\partial \phi} = LV''' > 0 \quad \text{and} \quad \frac{\partial \phi V''}{\pi \phi^2} = 2\phi^2 V_{yy} + \phi L V_{yy} < 0$$

The first partial is positive by DARA (see Arrow [1]). The second partial is negative by IRRA (see Rothschild and Stiglitz [11]).

own remuneration, the level of retained earnings may still increase because of the additional expected income generated by the increased loan portfolio. Finally, (14) illustrates the crucial role of the stockholders' demands. If the stockholders demand too large a share of the return, the manager may find it optimal to greatly reduce the retained earnings and take the funds for his own consumption.

If the return on loans changes, the model indicates that the bank's optimal policies change as well. If the mean interest rate on loans increases, for example, the manager will increase the bank's loan portfolio and, under reasonable conditions, increase the bank's excess equity. Additionally, if the income effect dominates the substitution effect, the manager will increase his own remuneration.

B. The Effect of a Change in Shareholder Demands

The previous case examined the effect of an increase in the return on assets. We now consider the effect of shareholders demanding an increase in the return on equity. For simplicity, define $I = m + \beta L_t + \theta Y_t + \gamma \pi_t$. Examining how the system reacts to a change in m, the base return to the shareholders, we find

$$\frac{\partial L}{\partial m} = \frac{U'' E[V''\phi]}{|H|} \tag{15}$$

which is less than zero given Decreasing Absolute Risk Aversion. Thus, if the shareholders increase their demands, the manager decreases the risky loan portfolio of the bank.

Alternatively, the effect on the manager's consumption decision is

$$\frac{\partial X}{\partial m} = \frac{(1 - \rho'(1 - \gamma))\{-E[V'']E[V''\phi^2] + E[V''\phi]E[V''\phi]\}}{|H|} \tag{16}$$

From our analysis of (12) with Decreasing Absolute Risk Aversion and Increasing Relative Risk Aversion, it follows that the sign of $\partial X/\partial m$ is negative. The manager, therefore, decreases the remuneration he extracts from the bank. He also responds to the change in shareholder demands by decreasing the risky loan portfolio of the bank.

C. The Effect of Changes in the Deposit Rate

We have examined how changes in the return on loans and the return on equity affect the banking firm. We now consider the effects of a change in the cost of liabilities by analyzing a change in r, the interest rate paid to depositors. Unfortunately, if deposit levels depend to some extent on a random variable, these effects are difficult to sign. If we view deposits as a nonrandom variable, however, we can examine $\partial X/\partial r$ and $\partial L/\partial r$. Then

$$\frac{\partial X}{\partial r} = \frac{(1 + I_\pi)(1 + \rho'(1 - I_\pi))D(\cdot)\{[E[V''\phi]E[V''\phi] - E[V'']E[V''\phi^2]\}}{|H|} < 0 \tag{17}$$

given DARA and IRRA. Further,

$$\frac{\partial L}{\partial r} = \frac{(1 + I_\pi)U''D(\cdot)E[V''\phi]}{|H|} < 0 \tag{18}$$

given DARA.

Thus, if the deposit rate r increases, the manager responds by decreasing the risky loan portfolio and decreasing his present consumption.[20]

IV. Regulation and Bank Decision Making

The cases presented in the previous section have illustrated the interrelatedness of the bank's optimal decisions. It is also interesting, however, to examine how regulation affects the operations of the bank. In particular, changes in the net worth requirement, B, may yield some insights into the problems of capital adequacy. As this section will show, changes in B will not likely result in neutral shifts in the banks operating policies and hence by raising or lowering B, the regulators may have to deal with perverse reactions by the banks.[21]

The net worth requirement, or bankruptcy level, enters the model through the constraint correspondence. If firms are faced with a higher B, or higher required level of equity, the initial effect is to limit the actions which are feasible to the manager. Figure 1 depicts the qualitative effect on the manager's value function when the net worth requirement changes from B_1 to B_2. Because the manager's activities are now curtailed, the effect of changing the capital requirement is to shift the manager's value function to a lower position, representing a lower future value of the firm to him. If we examine how this change affects the optimal policies of the manager, we find that the results are not clear cut.

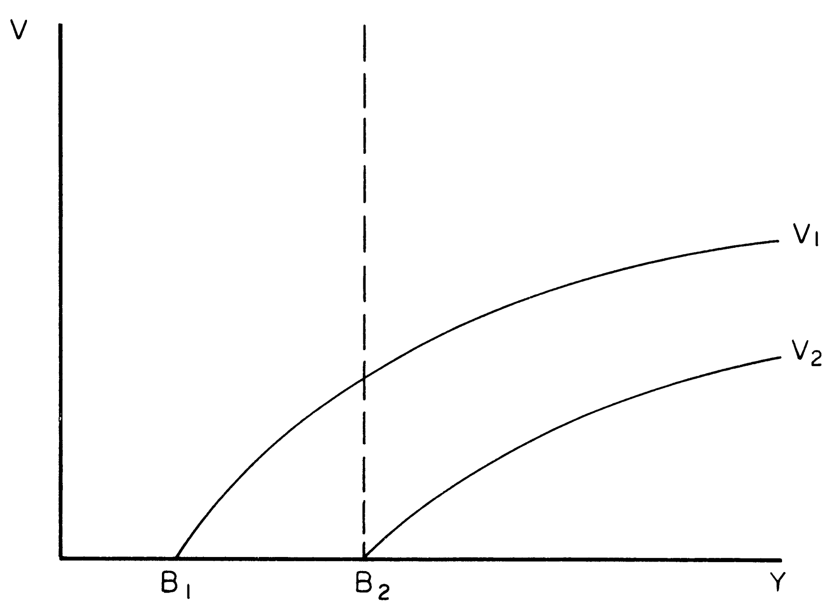

Figure 1. Net Worth Requirements and the Value Function

[20] The effect of changing reserve requirements can also be considered. If κ increases, the manager responds by decreasing his consumption and decreasing the riskiness of the firm's portfolio.

[21] The capital requirement modelled here is not a function of bank risk. If B is a function of the portfolio's riskiness we conjecture that the effects elaborated here will still hold, but will be smaller in size.

Consider first how the manager's X decision changes given a change in B. Introducing this change in B into the model analyzed in Section III yields.[22]

$$\frac{\partial X}{\partial B} = \frac{(1 + \rho'(1 - I_\pi))\{E[V_{yB}]E[V''\phi^2] - E[V''\phi]E[V_{yB}\phi]\}}{|H|} \tag{19}$$

The denominator is positive. The sign of the numerator depends upon the sign of $E(V_{yB})$ which could be either positive or negative depending upon how the manager perceives the higher B to affect the firm's prospect of bankruptcy (see Figure 1). If the bank's value Y is above B_2, the sign of $E(V_{yB}) > 0$. Conversely, if Y is below B_2, the sign of $E(V_{yB}) < 0$. Because the value function is dependent upon future income, the sign of $E(V_{yB})$ changes depending upon whether this expected future value is above the new net worth constraint.

To sign the numerator, however, we must also determine the sign of $E[V_{yB}\phi]$. Unfortunately, there is no way to determine whether this is positive or negative. The sign of $E[V_{yB}\phi]$ depends upon the covariance of V_{yB} and ϕ. Note that if this covariance is small, managers' of firms whose expected future value is above B_2 (i.e., $E[V_{yB}] > 0$) will extract less X from the firm. Alternatively, if the future value is below B_2, it becomes optimal for the manager to take as much X as possible and drive the firm towards bankruptcy.[23]

The risk exposure of the bank is also subject to this same indeterminancy.

$$\frac{\partial L}{\partial B} = \frac{-U''E[V_{yB}\phi] - (1 + \rho'(1 - I_\pi))^2\{E[V_{yB}]E[V_{yB}\phi] - E[V'']E[V_{yB}\phi]\}}{|H|} \tag{20}$$

Again, the sign depends upon $E[V_{yB}\phi]$ and thus the sign of $\partial L/\partial B$ is impossible to determine. Hence, unlike in the previous changes we considered, the assumptions of Decreasing Absolute Risk Aversion and Increasing Relative Risk Aversion are not sufficient to guarantee a determinant sign.

Equations (19) and (20) suggest that changing the net worth requirement imposes conflicting effects upon the firm and, hence, the manager's optimal policies are not clear cut. If the manager perceives that the future operations of the bank cannot sustain the new required net worth level, it may be optimal for the manager to "take the money and run." Another possibility, however, is that the manager invests in a riskier portfolio in the hopes that the higher mean return on such investments can provide a higher profit level for the bank. This, in turn, creates the paradoxical situation that increasing the required net worth to presumably make the bank safer results in an increased risk exposure for the bank. This same type of indeterminancy has also been demonstrated using a portfolio theory approach by Koehn and Santomero [10].

It would be extremely useful to know the magnitude of changes in riskiness induced by different capital requirements. Unfortunately, in the absence of specific assumptions on the manager's utility function, these magnitudes are

[22] We are assuming that V is differentiable with respect to B.

[23] The shareholder imposed maximum compensation bound, $\bar{X}$, may limit the manager's ability to do this in one period. Hence, the manager will extract $X_t = \bar{X}$ and, because Y_t is now permanently lower, will eventually violate the net worth constraint. The manager's selection of maximum present consumption occurs because the bank's future income prospects are not high enough for the manager to justify investing funds in the bank.

impossible to determine. What the model does show, however, is that changes in regulation will not necessarily produce predictable results in bank behavior.

V. Conclusion

The banking firm is a complex organization. As a financial intermediary, it performs both a brokerage and a risk transformation function. As a business, it must yield a return to its owners. As a regulated enterprise, it must operate within the bounds specified by the bank supervisory agencies. This paper has developed a theory of the banking firm that incorporates all of these roles into a dynamic framework. The behavior of the bank was shown to depend on the uncertainties the bank faces over time as well as on the demands of shareholders and regulators. The model also illustrates that the optimal decisions for the banking firm can not be considered in isolation of each other. Rather, it is the interaction of these decisions over time that results in the process of financial intermediation.

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