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Tests of Two Models for Valuing Call Options on Stocks with Dividends

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ABSTRACT

Roll has recently formulated an option pricing model which allows dividend payments on the underlying stock. This paper compares the performance of the exact Roll model with a modified, but inexact, Black-Scholes model. The results indicate that the Roll model prices are significantly closer to actual market prices.

THE ORIGINAL BLACK-SCHOLES [1] call option pricing model does not take account of dividend payments on the underlying stock and does not allow for the possibility of early exercise which may be optimal when the stock pays dividends. Black [2] has suggested that the original B-S model can be modified to take account of dividends and Sharpe [10, p. 383] concludes that the modified B-S approach, "while not exact, is probably sufficient for many listed options".

Results of empirical tests of the B-S model are mixed. Black [2] reports that the B-S model underprices (overprices) deep out-of-the-money (in-the-money) call options when standard deviations on the underlying stocks are estimated using historical stock price data. Merton [8] states that practitioners observe B-S model prices to be less than market prices for deep in-the-money as well as deep out-of-the-money options. More recently, Macbeth and Merville [5, 6] tested the B-S model on the options of six firms and found biases which were the exact opposite of those reported by Black. These results are widely believed to be related at least partly to the dividend problem which may persist in the modified, but inexact, B-S model.

Roll [9] has recently solved the problem of valuing an American call written on an underlying stock with known dividends by noting that the call can be valued as a portfolio of three European options.[1] Since this portfolio duplicates the relevant cash flows of an American call option on an underlying dividend-paying stock, it is an exact formulation and should lead to results superior to those based on the modified B-S model.

The major goal of this paper is to test the modified Black-Scholes option pricing approach against the Roll option pricing model using the testing procedure suggested by Macbeth and Merville. Application of the modified B-S approach to a larger group of firms than that employed by Macbeth and Merville causes the original biases reported by Black to reappear. However, the results indicate that

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[1] The Roll model does not require the assumption of a continuous dividend generating process as assumed by Merton [7].

the Roll model, using implied standard deviations, significantly reduces and sometimes completely eliminates these biases.

I. The Data

Daily call option and underlying stock price data were gathered from the *Wall Street Journal* during the month of October 1979 for CBOE options maturing in December 1979 and January and February 1980. Dividend information was taken from Standard and Poor's Stock Record. Interest rates on United States Treasury Bills of the same maturity as the options were gathered from the *Wall Street Journal* and updated daily. An average of the bid and asked discount rate was calculated and converted to an equivalent bond yield.

Since Roll's model assumes that there is only one dividend during the life of the option, options on stocks which paid more than one dividend during the life of the option were eliminated from the sample. The resulting sample consisted of data for one month on 181 options of 63 firms, in contrast to the Macbeth and Merville [5, 6] sample which consisted of all options on 6 firms over a period of one year.

II. Methodology

A. The Modified Black-Scholes Model

As stated earlier, it is assumed in the original B-S model that the underlying security pays no dividends. To account for dividend payments and the possibility of early exercise, the B-S model is modified as follows:

(1) The option value is calculated from the B-S formula after reducing the stock price by the present value of the actual dividend payment on the underlying stock; and
(2) The option value is recalculated, assuming that the option is exercised just before the ex-dividend date.

The larger of the two calculated option values is then used as an estimate of the option's value. This is only an approximation, however, to the exact Roll solution which follows.

B. The Roll Model

Roll [9] has recently used the Geske [3] compound option formula to develop an option valuation model which values call options on stocks with known dividends. Implementation of this model requires the determination of the stock price at which an option holder would be indifferent between holding or exercising the option at the ex-dividend date. The Roll model is:[2]

$$\begin{aligned} C = {} & S \cdot N[q(S, T, X)] - N[q(S, T, X) \\ & - \sigma\sqrt{T}]e^{-rT}X + S \cdot N[q(S, t, S_t^* + \alpha D)] \end{aligned}$$

[2] More recently, Geske [4] has developed a simpler version of Roll's model. Roll's original model forms the basis of this research.

$$
\begin{aligned}
&- (S_t^* + \alpha D)N[q(S, t, S_t^* + \alpha D) \\
&- \sigma\sqrt{t}]e^{-rt} - S \cdot N[q(S, t, S_t^*), q(S, T, X)] \\
&+ X \cdot N[q(S, t, S_t^*) - \sigma\sqrt{t}, q(S, T, X) - \sigma\sqrt{T}]e^{-rT} + (S_t^* + \alpha D - X) \cdot \\
&\quad N[q(S, t, S_t^*) - \sigma\sqrt{t}]e^{-rt} \qquad (1)
\end{aligned}
$$

where

C = the market value of an American call option

$q(s, \tau, x) = [\ln(s/x) + (r + \sigma^2/2)\tau]/\sigma\sqrt{\tau}$

$N(q)$ = the univariate standard normal probability distribution function

$N(q, p)$ = the bivariate standard normal p.d.f. with correlation coefficient $\sqrt{(t/T)}$

S_t^* = the solution to $S_t^* N[q(S_t^*, T - t, X)] - Xe^{-r(T-t)} \cdot N[q(S_t^*, T - t, X) - \sigma\sqrt{(T - t)}] - S_t^* - \alpha D + X = 0$

S = the current stock price, net of escrowed dividend

T = time until expiration

X = exercise price

r = the riskless rate of interest

σ^2 = the variance rate of return on S

D = the dividend

t = the time until the ex-dividend instant

α = the known decline in the stock price at the ex-dividend instant as a proportion of the dividend.

Here S_t^* is that value of the ex-dividend stock price at time t which would cause the option holder to be indifferent between exercising or holding the option at the ex-dividend date. If S_t exceeds S_t^* the option holder would exercise the option at the ex-dividend date rather than hold the option to maturity.

C. Calculation of Implied Standard Deviations

Observed market prices were used to determine daily implied Black-Scholes and Roll instantaneous standard deviations.[3] Options with implied annual standard deviations below 1% or about 120% were ignored. On any given day there will be several implied standard deviations for each firm: one for each option. Based on statements by Black [2], Macbeth and Merville [5, 6] assumed that the B-S model performs best for at-the-money options. Following them, an estimated standard deviation was determined for an at-the-money option by regressing the

[3] As discussed by Macbeth and Merville [5, 6], the correct implied standard deviation to use in the modified B-S approach is the smaller of the two calculated standard deviations.

Table I

Differences between Market Prices and Model Prices[a]

$$\text{Average Value of } D = \frac{C_{\text{market}} - C_{\text{model}}}{C_{\text{model}}}$$

	Black-Scholes	Roll	Difference	Number of Observations
All Options	5.38%[b]	3.26%[b]	−2.12%[b]	3232
	(.59)	(.54)	(.80)	
$M \leq 1.0$ (out-of-the-money options)	12.03%[b]	6.74[b]	−5.29%[b]	1536
	(1.21)	(1.12)	(1.65)	
$M > 1.0$ (in-the-money options)	−.64%[b]	.11%	.75%[b]	1696
	(.14)	(.15)	(.20)	
$C_{\text{market}} \geq \$1.00$	.17%	.31%	.14%	2387
	(.19)	(.18)	(.26)	
$C_{\text{market}} < \$1.00$	20.12%[b]	11.60%[b]	−8.52%[b]	845
	(2.11)	(1.97)	(2.89)	
$C_{\text{market}} < \$.50$	30.63%[b]	18.20%[b]	−12.43%[b]	449
	(3.54)	(3.35)	(4.87)	
$C_{\text{market}} < \$.25$	35.90%[b]	23.00%[b]	−12.90%	171
	(6.66)	(6.30)	(9.16)	

[a] Numbers in parentheses are standard errors of the mean.

[b] Significantly different from zero at 5% level.

implied standard deviations for an individual firm against the percentage difference between the stock price, S, and the discounted exercise price. This procedure was repeated daily for both the B-S and Roll models.[4] The implied at-the-money Black-Scholes and Roll instantaneous standard deviations were then used to calculate modified Black-Scholes and Roll option values.

III. Empirical Results

To facilitate a comparison of the results, deviations of market prices from model prices, D, were plotted against the standardized stock price, M, where

$$D = \frac{C_{\text{market}} - C_{\text{model}}}{C_{\text{model}}} \times 100\% \tag{2}$$

and

$$M = \frac{S}{Xe^{-rT}}$$

Results for the full set of observations are presented in Figure 1, Parts A and B, and Table I.[5] As can be seen in Table I, the Roll model results in significantly

[4] Options with model values less than 0.0625 and simultaneous market values equal to 0.0625 were ignored, since this is the lowest acceptable trade for CBOE options.

[5] Outliers with either B-S or Roll percent deviations exceeding 300% are not included in the results presented in Table I. These extremely large percentage deviations occur when model prices are very close to zero but market prices are above .0625. They are probably due to the non-simultaneity of the stock and option quotes.

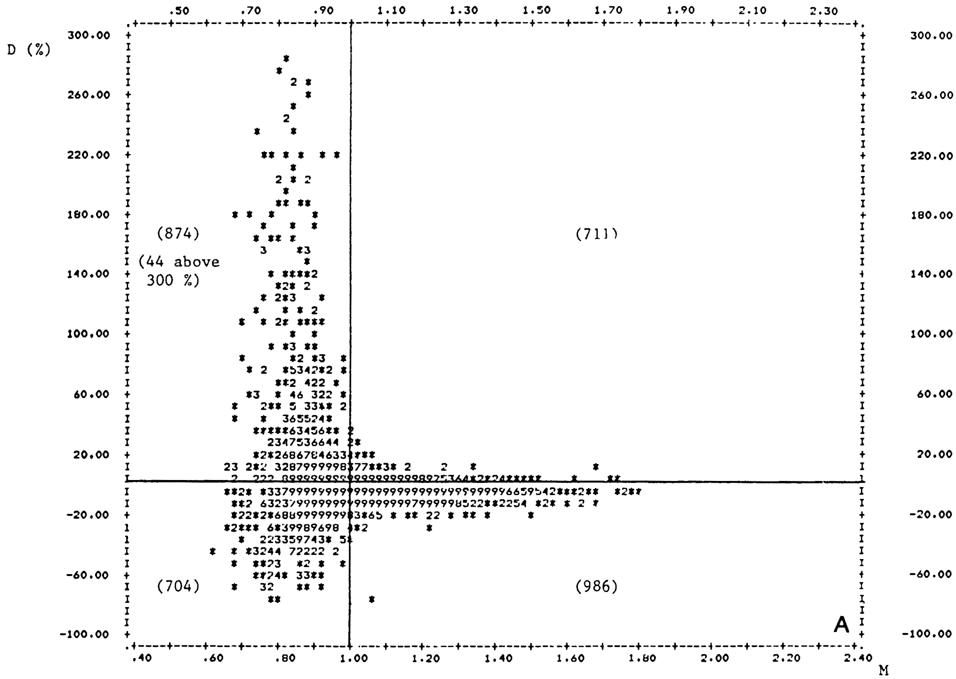

Figure 1A. Black-Scholes Model: Difference between Market Prices and Model Prices (D) for Different Standardized Stock Prices (M).

Numbers in parentheses indicate number of observations in the associated quadrant.

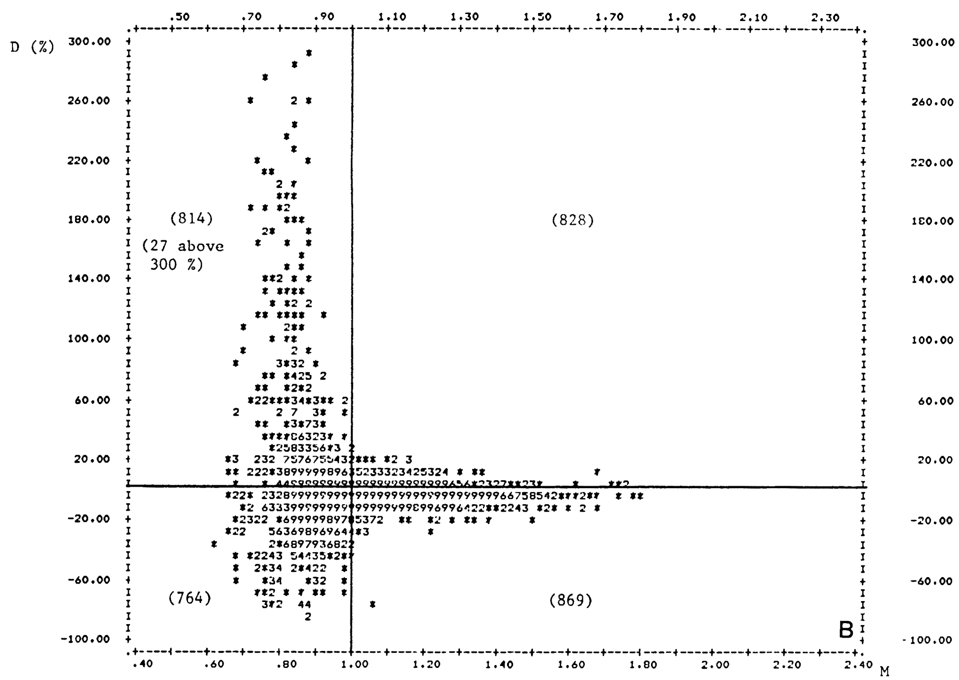

Figure 1B. Roll Model: Difference between Market Prices and Model Prices (D) for Different Standardized Stock Prices (M).

Numbers in parentheses indicate number of observations in the associated quadrant.

Table II

Dollar Differences between Market Prices and Model Prices[a]

Average Value of $Y = C_{\text{market}} - C_{\text{model}}$

	Black-Scholes	Roll	Difference	Number of Observations
All Options	−.0331[b]	−.0157[b]	.0174[b]	3275
	(.0053)	(.0058)	(.0078)	
$M \leq 1.0$ (out-of-the-money options)	.0097[b]	−.0127[b]	−.0224[b]	1579
	(.0047)	(.0049)	(.0068)	
$M > 1.0$ (in-the-money options)	−.0729[b]	−.0184	.0545[b]	1696
	(.0092)	(.0101)	(.0137)	
$C_{\text{market}} \geq \$1.00$	−.0493[b]	−.0148	.0345[b]	2387
	(.0070)	(.0076)	(.0103)	
$C_{\text{market}} < \$1.00$	.0104	−.0180[b]	−.0284[b]	888
	(.0058)	(.0058)	(.0082)	
$C_{\text{market}} < \$.50$	.0178[b]	−.0104	−.0302[b]	489
	(.0056)	(.0061)	(.0083)	
$C_{\text{market}} < \$.25$	.0179[b]	−.0043	−.0222[b]	199
	(.0064)	(.0077)	(.0100)	

[a] Numbers in parentheses are standard errors of the mean.
[b] Significantly different from zero at 5% level.

lower average deviations. The average deviation is 5.4% for the B-S model, but only 3.3% for the Roll model.

Figure 1A reveals a preponderance of points in the northwest quadrant indicating that the B-S model tends to underprice out-of-the-money options. This is consistent with Black's [2] original finding but conflicts with the results of Macbeth and Merville [5, 6]. The Roll model reduces this out-of-the-money bias. This is apparent from Figure 1B which contains fewer points in the northwest quadrant and more in the southwest quadrant than Figure 1A. As shown in Table I, the average value of D for out-of-the-money options is 12.0% for the B-S model, but only 6.7% for the Roll model.

The preponderance of points in the southwest quadrant of Figure 1A indicates that the B-S model tends to overprice in-the-money options. Again, this is consistent with Black's original results, but is inconsistent with the findings of Macbeth and Merville. The Roll model again tends to eliminate the bias as may be seen in Figure 1B and Table I. There is a shift of points from the southeast quadrant to the northeast quadrant.

Neither bias is as pronounced as those reported by Black or Macbeth and Merville. That is, there is a fairly large number of observations in all four quadrants of the figures. The average deviation for in-the-money options is very close to zero for both models. All of these results are true not only for the period as a whole, but also for each individual day within the period.

The reason these findings contradict the results of Macbeth and Merville may be due to the larger number of underlying stocks used in this study.[6] In addition,

[6] It is also possible that the reversal is due to the different sample periods, although these results did hold for each individual day within the period.

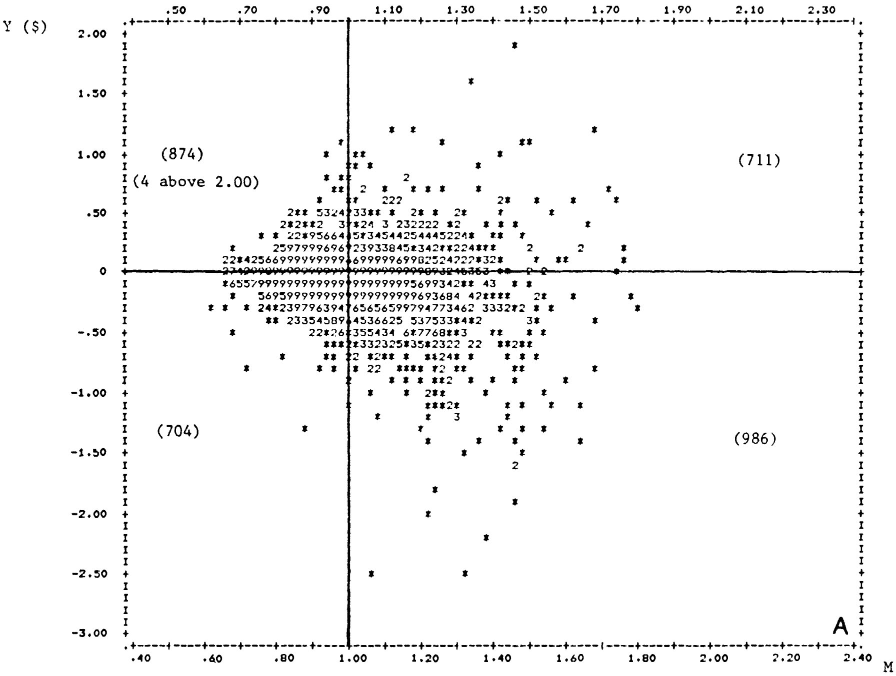

Figure 2A. Black-Scholes Model: Dollar Differences between Market Prices and Model Prices (Y) for Different Standardized Stock Prices (M).

Numbers in parentheses indicate number of observations in the associated quadrant.

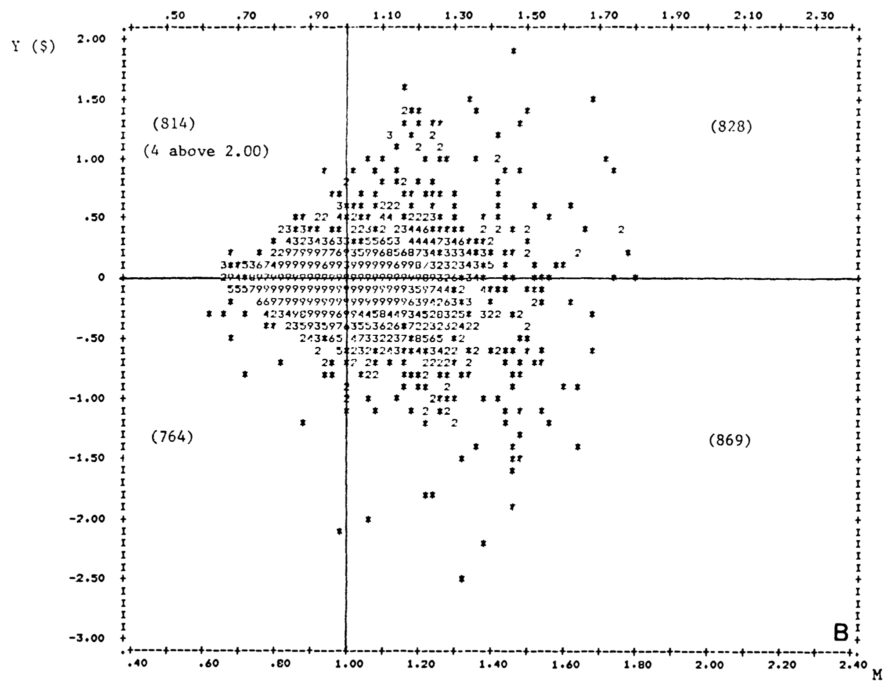

Figure 2B. Roll Model: Dollar Differences between Market Prices and Model Prices (Y) for Different Standardized Stock Prices (M).

Numbers in parentheses indicate number of observations in the associated quadrant.

both dividend modifications to the original Black-Scholes model suggested by Black were used in this research. Macbeth and Merville assumed that the early exercise effect was insignificant and therefore they used only the first dividend modification suggested by Black. Twenty-seven of the 182 options used in this study show a strong possibility of early exercise (the current stock price exceeds S^*). All of these options were in-the-money, and a majority of them appear in the southeast quadrant of Figure 1A. This would offset the bias found by Macbeth and Merville (more observations in the northeast quadrant).

Since the Roll model performs much better (relative to the B-S model) for out-of-the-money options, one might suspect that the Roll model would perform better (relative to the B-S model) for lower priced options (which are out-of-the-money). As can be seen in Table I, this appears to be true. Average deviations for options with market prices below $1.00 are significantly smaller for the Roll model.

Relative deviations from market prices appear large for out-of-the-money options, because the prices are typically small for out-of-the-money options. Therefore, dollar deviations, Y, are also calculated where

$$Y = C_{\text{market}} - C_{\text{model}} \tag{3}$$

The results are presented in Table II and Figure 2, Parts A and B. Table II reveals that average dollar deviations for both models are typically very small and usually significantly lower for the Roll model. The number of observations in each quadrant in Figure 2 is the same as before and the same comments apply. That is, the Roll model alleviates both the slight in-the-money and out-of-the-money B-S bias.

IV. Summary and Conclusions

This paper has demonstrated empirically that the Roll [9] option valuation model for American call options on dividend paying stocks leads to significantly smaller deviations from market prices than the Black-Scholes model modified to include dividends.

Furthermore, when options on a large number of firms are used, it is seen that the Black-Scholes biases directly contradict the findings recently presented by Macbeth and Merville [5, 6]. The modified B-S model, on average, underprices (overprices) out-of-the-money (in-the-money) options. That is, the biases originally reported by Black [2] reappear. When implied standard deviations calculated using the technique developed by Macbeth and Merville are used, the Roll model is seen to alleviate both the in-the-money and the out-of-the-money biases.

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