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Sufficient and Necessary Conditions for Information to have Social Value in Pure Exchange

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ABSTRACT

This paper extends, corrects, and unifies earlier statements concerning the social value of public information as well as the no-trading conditions in pure exchange. Sufficient and necessary conditions are provided for both the single-period and two-period cases in a postsignal trading model. The social value of information is shown to be closely linked to the allocational efficiency of the market, the degree of homogeneity of prior beliefs, and of information structures, the time-additivity of preferences, and the efficiency of endowments. We conclude that the case in favor of public information is much stronger than previously suggested.

IN 1971, HIRSHLEIFER, IN A seminal paper [3], elegantly contrasted the value of an information system at the private level (i.e. when access to the information is limited to a few) with the value of that same system at the social level, i.e. when all market participants have access to the information.[1] His main conclusion was that, under pure exchange, information does not have social value: either no one is better off with information or if some individual should happen to gain in expected utility it could only be at the expense of someone else. This conclusion has subsequently been confirmed by a number of other authors (e.g. Fama and Laffer [1]), Marshall [5], Ng [6], and Wilson [10]). There appear to be only three examples in which public information does have social value: in two of them investors have heterogeneous prior beliefs and homogeneous posterior beliefs (Marshall [5, p. 387], Ng [7], while in the third the market is either incomplete or has time-dependent preferences and homogeneous beliefs (Jaffe [4]); all except the example of Marshall also assume that the endowments represent equilibrium holdings if no information is released.

Despite its very special assumptions (identical preferences of the type that imply portfolio separation, identical prior probability beliefs, identical information structures, identical endowments, and a complete market), Hirshleifer's original conclusion has taken on the prominence of a general theorem. This is remarkable

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[1] Weather forecasts, crop forecasts, macro-economic forecasts, and corporate financial reports are examples of signals generated by information systems accessible to the public.

only because the particular assumptions which are responsible for his result to our knowledge have never been identified, the cited examples notwithstanding. In other words, the apparent "contradiction" between the Hirshleifer result and the Marshall-Jaffe-Ng papers has never been satisfactorily resolved.

The main purpose of this paper is to set forth a simple set of sufficient conditions for public information to have social value as well as necessary conditions for this to be the case. These conditions provide a sharp distinction between the single-period and the two-period cases. The sufficiency conditions derive their significance from the fact that they depend, in the single-period setting, on only three things: whether the market is fully efficient allocationally, whether information *structures* are essentially homogeneous, and on certain aspects of investor endowments. Thus, these conditions are independent of prior (and posterior) beliefs, of preferences, and of most aspects of investor endowments, i.e. of the major heterogeneities that may exist among investors (except to the extent that the latter impinge on the allocational efficiency on the market—see Section I). In the two-period case, however, it is also pivotal whether prior beliefs are essentially homogeneous and whether preferences are time-additive. Furthermore, it will be shown that the above conditions are very closely related to conditions which are necessary for information to have social value. Our results therefore represent substantial generalizations and extensions of extant papers, all of which will be seen to represent rather special cases—this applies both to the papers which have shown that information may have social value as well as to those which have shown that it does not. Our main inference is that the cases in which information *has* social value should not be viewed as being in any sense implausible. Thus, previous studies, which on the whole have thrown their weight behind the conclusion that public information is socially worthless in pure exchange, may be thought of as having given rise to a "biased" perspective. This is primarily due to the explicit or implicit use of a strong set of assumptions which, despite appearances to the contrary, are also closely related. Of course, the preceding does not alter the fact that there are critical differences between the private and social value of information, and from this perspective it is difficult to overstate the significance of the original papers in this area.

I. Preliminaries

We consider a pure exchange economy with a single commodity which lasts for one or two periods under the standard assumptions.[2] That is, at the end of period 1 the economy will be in some state s, where $s = 1, \cdots, n$. There are I consumer-investors indexed by i, whose probability beliefs over the states are given by the vectors $\pi_i = (\pi_{i1}, \cdots, \pi_{in})$, where $\pi_{is} \geq 0$ for all i and s and there is agreement on the subset of states for which $\pi_{is} > 0$. Preferences are represented by the functions $U_{is}(c_i, w_{is})$, where c_i is the consumption level in period 1 and w_{is} is the consumption level in period 2 if the economy is in state s at the beginning of that period. U_{is} is assumed to be monotone increasing in each argument and

[2] The single-period model obtains whenever the variables c_i (and quantities $\bar{c}_i$) in the sequel are suppressed.

strictly concave, i.e. consumer-investors prefer more to less and are risk-averse. At the beginning of period 1 (time 1), consumer-investors may allocate their resources among current assumption and a portfolio chosen from J securities indexed by j. Security j pays $a_{js} \geq 0$ per share at the end of period 1, where $a_{js} > 0$ for some s for each j, and the total number of outstanding shares is Z_j. Let z_{ij} denote the number of shares of security j purchased by investor i at time 0; his portfolio $z_i = (z_{i1}, \cdots, z_{iJ})$ then yields the payoff

$$w_{is} = \sum_j z_{ij} a_{js}$$

available for consumption in period 2, if state s occurs at the end of period 1. Aggregate wealth in state s is given by

$$W_s \equiv \sum_j Z_j a_{js} > 0, \text{ all } s$$

Investor endowments are denoted $(\bar{c}_i, \bar{z}_i)$ and markets, as is usual, are assumed to be competitive and perfect.[3] The rank of matrix $A = [a_{js}]$ is never less than 2; if it is full (equals n), the financial market will be called complete; if not, it will be called incomplete. The matrix A is referred to as the market structure.

Prior to trading, investors may obtain information bearing on the state that will occur at time l via a signal y from an information system $Y = (y_l, \cdots, y_m)$. This causes consumer-investor i to update his prior beliefs π_i^o to the posterior beliefs π_i^y via Bayes's rule. That is, if $p_i(y|s)$ denotes investor i's perceivec probability that signal y will be emitted if s is about to occur, Bayes's Theorem gives

$$\pi_{is}^y = \frac{p_i(y \mid s)\pi_{is}^o}{\sum_s p_i(y \mid s)\pi_{is}^o} = \frac{p_i(y \mid s)\pi_{is}^o}{p_i(y)}$$

When $\pi_i^y \neq \pi_i^o$, the information system Y is said to be non-null for investor i.

The $n \times m$ matrix of conditional probability numbers $p_i(y \mid s)$ for investor i will be called his *information structure*. Whenever

$$p_i(y \mid s) = p_1(y \mid s) \text{ for all } i \geq 2, y, \text{ and } s \tag{1a}$$

the information structures will be said to be *homogeneous*; if (1a) does not hold, they will be called nonhomogeneous. Finally, when there exist numbers $k_i(y)$ such that

$$p_i(y \mid s) = k_i(y) p_1(y \mid s) \text{ for all } i \geq 2, \text{ all } s \text{ and } y \tag{1b}$$

the information structures will be said to be *essentially homogeneous*.[4] This is because, as a practical matter, (1b) is only slightly more general than (1a).

[3] That is consumer-investors perceive prices as beyond their influence, there are no transaction costs or taxes, securities are perfectly divisible, and the proceeds from short sales can be invested.

[4] For example, the two information structures

	y_1	y_2	y_3	y_1	y_2	y_3
s_1	.4	.4	.2	.20	.70	.10
s_2	.3	.4	.3	.15	.70	.15
s_3	.2	.4	.4	.10	.70	.20

are essentially homogeneous but not homogeneous.

Under our assumptions, each consumer-investor i maximizes

$$\sum_s \pi_{is} U_{is}(c_i, \sum_j z_{ij} a_{js}) \tag{2}$$

with respect to the decision vector (c_i, z_i), subject to his budget constraint

$$c_i P_0 + \sum_j z_{ij} P_j = \bar{c}_i P_0 + \sum_j \bar{z}_{ij} P_j$$

as a price-taker, where P_0 is the price of a unit of period 1 consumption and P_j is the price (at $t = 0$) of security j. Assuming interior solutions (with respect to the non-negativity constraint on consumption, $(c_i, w_i) \geqq 0$), the equilibrium conditions may be written

$$\sum_s \pi_{is} \frac{\partial U_{is}(c_i, w_{is})}{\partial c_i} = \lambda_i \qquad \text{all } i \tag{3}$$

$$\sum_s \pi_{is} \frac{\partial U_{is}(c_i, w_{is}) a_{js}}{\partial w_{is}} = \lambda_i P_j \qquad \text{all } i, j \tag{4}$$

$$c_i + z_i P = \bar{c}_i + \bar{z}_i P \qquad \text{all } i \tag{5}$$

$$\sum_i c_i = \sum_i \bar{c}_i, \sum_i z_{ij} = Z_j, \qquad \text{all } j \tag{6}$$

where the λ_i are the Lagrange multipliers; (6) represents the market clearing equations; and $P_0 \equiv 1$ has been chosen as numeraire. In the one period case, (3) drops out, $P_1 \equiv 1$ and $c_i = \bar{c}_1 \equiv 0$ in (5) and (6).

We note that an allocation (c^*, z^*) which constitutes a solution to system (3)–(6) (along with a price vector P and a vector λ) is Pareto-efficient with respect to market structure A; no other allocation (c, z) obtainable within market structure A can make some consumer-investors better off without making others worse off. However, there may exist allocations *outside* the market (e.g. accomplished by the invention of new securities) which yield allocations that Pareto-dominate the market allocation (c^*, z^*). When this is not the case, (c^*, z^*) will be said to be fully Pareto-efficient or *fully allocationally efficient.*

To be more precise, let

$$P_{is} \equiv \frac{\pi_{is}}{\lambda_i} \frac{\partial U_{is}(c_i, w_{is})}{\partial w_{is}} \equiv \pi_{is} u_{is} / \lambda_i$$

By reference to (4), it can be seen that P_{is} denotes the investor i's shadow price of wealth in state s; roughly, it represents the marginal value of receiving an additional unit of wealth in state s. It is well known that (3)–(6) plus

$$P_{is} = P_{1s} \qquad \text{all } i \geq 2, \text{ all } s \tag{7}$$

is a necessary and sufficient condition for the market allocation (c^*, z^*) to be fully allocationally efficient. This follows because (7) ensures that the marginal rates of substitution of wealth (consumption) *between any two states* are the same for all investors i. Condition (7) plays a particularly important role in what follows, a role which has previously not been well appreciated.

It is sufficient (but not necessary) for (7) to obtain that the security market be complete, i.e. that the rank of A be n. Clearly (7) also occurs when *all* consumer-investors are *completely* identical. A third set of well-known sufficient conditions which imply (7) is the following: at least two securities in the market, one of which is risk-free; homogeneous beliefs; and additive utility, with the second period's utility function being a member of the HARA-class, the exponent being the same across all consumer-investors (except in the negative exponential case).[5] It is noteworthy that most studies dealing with the absence of social value of information have employed at least one of the above sufficient conditions, thus *implicitly* obtaining (7) (e.g. Hirshleifer [3], Ng [6, 7], and Wilson [10]).

The following is a direct consequence of Bayes's Theorem.

Lemma 1: Suppose $\pi_{i1}^y > 0$ all y for at least one i ($i = 1$). Then

$$(\pi_{is}^y/\pi_{i1}^y)/(\pi_{is}^o/\pi_{i1}^o) = (\pi_{1s}^y/\pi_{11}^y)/(\pi_{1s}^o/\pi_{11}^o), \text{ all } i \geq 2,\ s \geq 2, \text{ and } y$$

if and only if the information structures are essentially homogeneous, i.e. (1b) holds.

Paraphrased, the lemma states that whenever investors have essentially homogeneous information structures, and only then, does the revised (posterior) probability ratio between any two states change by the same proportion for all investors no matter how heterogeneous their priors may be.

With this background, we are ready to take up the sufficiency question.

II. Sufficiency

Denote the maximum of (2) in the no-information case, i.e. the consumer-investor's equilibrium expected utility when (all) investors make decisions on the basis of their prior beliefs π_i^o, by V_i^o, and let (c_i^o, z_i^o) be the resulting equilibrium allocation to consumer-investor i. When the information system Y is in use, there will be a different equilibrium for each signal y. Let V_i^y and (c_i^y, z_i^y) represent, respectively, consumer-investor i's conditional expected utility and final allocation when equilibrium is based on revised beliefs π_i^y (signal y). The probability of receiving signal y is $p_i(y) = \sum_s p_i(y \mid s)\pi_{is}^o$; thus, the relevant expected utility of consumer-investor i in the information case, which we label $V_i(Y)$ is

$$V_i(Y) = \sum_y p_i(y) V_i^y = \sum_s \sum_y \pi_{is}^o p_i(y \mid s) U_{is}(c_i^y, \textstyle\sum_j z_{ij}^y a_{js})$$

Following standard practice, information will consequently be said to have social value if

$$\begin{aligned} V_i(Y) &\geq V_i^o, \quad \text{all } i \\ V_i(Y) &> V_i^o, \quad \text{some } i \end{aligned} \tag{8}$$

but not otherwise.

Consider now the case in which endowments are equilibrium allocations with-

[5] The HARA-class (hyperbolic absolute risk aversion) consists of the following utility functions of wealth with the properties $u'(w) > 0$, $u''(w) < 0$ (the first over at least a finite range of positive wealth):

$$u(w) = \begin{cases} \dfrac{1}{\gamma}(w + a)^\gamma & \gamma < 1 \text{ (decreasing absolute risk aversion)} \\ -\exp(\gamma w) & \gamma < 0 \text{ (constant absolute risk aversion)} \\ -(a - w)^\gamma & \gamma > 1, \text{ a large (increasing absolute risk aversion).} \end{cases}$$

out information:

$$(c_i^o, z_i^o) = (\bar{c}_i, \bar{z}_i), \qquad \text{all } i \tag{9}$$

This immediately gives

$$V_i(Y) \geq V_i^o, \qquad \text{all } i \tag{10}$$

since $\pi_{is}^0 = \sum_y p_i(y)\pi_{is}^y$ and rational consumer-investors will not choose an allocation worse than their endowment. Thus, whether the information system has value in this case depends *solely* on whether nontrivial trading takes place in the information case.[6] The conditions which rule out trading are given by:

Lemma 2: If endowments are an equilibrium allocation without information (i.e. (9) holds), sufficient conditions for the receipt of information to result in no trading, i.e.

$$(c_i^y, z_i^y) = (\bar{c}_i, \bar{z}_i) \qquad \text{all } i \text{ and } y \tag{11}$$

are that information structures are essentially homogeneous, i.e.

$$p_i(y \mid s) = k_i(y)p_1(y \mid s) \qquad \text{all } i \geq 2, \text{ all } s \text{ and } y \tag{1b}$$

full allocational efficiency is achieved, i.e.

$$P_{is}^o = P_{1s}^o, \qquad \text{all } i \geq 2, \text{ all } s \tag{12}$$

prior beliefs are essentially homogeneous, i.e.

$$p_i(y) = k_i(y)p_1(y), \qquad \text{all } i \geq 2, \text{ all } y \tag{13b}$$

and utilities are time-additive, i.e.

$$U_{is}(c_i, w_{is}) = f_i(c_i) + g_{is}(w_{is}), \qquad \text{all } s \text{ and } i \tag{14}$$

where the last two conditions do not apply in the single-period case. For arbitrary preferences and non-null information structures, the above conditions are also necessary for no trading to occur.

Proof: Consider sufficiency and the two-period case first[7, 8]; it will be shown that, given (9), (1b), (12), (13b), and (14), (11) satisfies the individual equilibrium

[6] Nontrivial trading is trading that not only changes the security allocation (c, z) but also the final consumption allocation (c, w); trivial security trading leaves (c, w) unaltered and may occur if the rank of A is less than J.

[7] Starr [8] has (implicitly) shown that, given (9), (12) and (14), (completely) homogeneous beliefs are sufficient to imply that there will be no trading. However, as will be shown, these conditions can be relaxed. Verrecchia [9] has also attempted to establish sufficiency for special cases of (1b) and (12) in the single-period case. However, Verrecchia's sufficiency conditions are false, as are his necessity conditions.

[8] Given (1b), (13b) is always satisfied when it is also assumed that prior beliefs are homogeneous but also in certain other cases. For example, (13b) holds when $\pi_1^o = (3/7, 1/7, 3/7)$ and $\pi_2^o = (1/7, 5/7, 1/7)$ in the presence of the (homogeneous) information structure

	y_1	y_2
s_1	5/6	1/6
s_2	1/2	1/2
s_3	1/6	5/6

conditions (3) and (4) in an economy with information system Y. Of course, the budget constraints (5) and the market-clearing conditions (6) are always satisfied then.

Eliminating prices in the individual equilibrium conditions (4) for a given signal y, one obtains

$$\sum_s \pi_{is}^y(u_{is}^y/\lambda_i^y)a_{js} = \sum_s \pi_{1s}^y(u_{1s}^y/\lambda_1^y)a_{js}, \qquad \text{all } y, j, i \geq 2$$

where λ_i^y is defined by (3).[9] This expression is equivalent to

$$\sum_s [(\pi_{is}^y/\pi_{is}^o)(\pi_{is}^o u_{is}^y/\lambda_i^y) - (\pi_{1s}^y/\pi_{1s}^o)(\pi_{1s}^o u_{1s}^y/\lambda_1^y)]a_{js} = 0, \text{ all } y, j, i \geq 2 \qquad (15)$$

which in turn will be satisfied for $(c_i^y, z_i^y) = (\bar{c}_i, \bar{z}_i)$ if

$$(\pi_{is}^y/\pi_{is}^o) = (\pi_{1s}^y/\pi_{1s}^o) \qquad \text{all } y, s, i \geq 2 \qquad (16)$$

This follows because (9) and (11) imply that $u_{is}^y = u_{is}^o$ and (9), (14), and (11) imply that $\lambda_i^y = \lambda_i^o$ which, combined with (12), gives

$$\pi_{is}^o u_{is}^y/\lambda_i^y = \pi_{is}^o u_{is}^o/\lambda_i^o \equiv P_{is}^o = P_{1s}^o \equiv \pi_{1s}^o u_{1s}^y/\lambda_1^y \qquad \text{all } y, s, i \geq 2. \qquad (17)$$

Since (16) follows directly from (1b) and (13b), (15) is indeed satisfied; the no-trading solution is an equilibrium for each message y.

The sufficiency proof for the one-period case is most straightforward if one puts the market structure A equal to the identity-matrix; this entails no loss of generality in view of the full allocational efficiency assumption (12). Equations (3) and (4) now reduce to

$$\frac{(\pi_{is}^y/\pi_{is}^o)(\pi_{is}^o u_{is}^y)}{(\pi_{i1}^y/\pi_{i1}^o)(\pi_{i1}^o u_{i1}^y)} = \frac{(\pi_{1s}^y/\pi_{1s}^o)(\pi_{1s}^o u_{1s}^y)}{(\pi_{11}^y/\pi_{11}^o)(\pi_{11}^o u_{11}^y)}, \qquad \text{all } y, s, i \geq 2 \qquad (18)$$

In view of assumption (1b) it follows from Lemma 1 that

$$\frac{(\pi_{is}^y/\pi_{is}^o)}{(\pi_{i1}^y/\pi_{i1}^o)} = \frac{(\pi_{1s}^y/\pi_{1s}^o)}{(\pi_{11}^y/\pi_{11}^o)}$$

Thus, (11) is indeed a solution since (9) and (11) imply that $u_{is}^y = u_{is}^o$ for all i, s, and y and (12) implies that $P_{is}^o = P_{1s}^o$ for all $i \geq 2$ and all s.

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since $p_i(y_1) = p_i(y_2) = .5$, all i. Note that what happens in the example is that condition (1a) (i.e. homogeneous information structures) combined with homogeneous signal beliefs

$$p_i(y) = p_1(y) \qquad \text{all } i \geq 2 \text{ and } y \qquad (13a)$$

have the same effect on relative prior-posterior beliefs as do conditions (1b) and (13b). More generally, (1b) and (13b) imply that

$$\frac{p_i(y \mid s)}{p_i(y)} = \frac{p_1(y \mid s)}{p_1(y)} \qquad \text{all } i \geq 2 \text{ and } s, y$$

and these relations hold if and only if

$$\frac{\pi_{is}^y}{\pi_{is}^o} = \frac{\pi_{1s}^y}{\pi_{1s}^o} \qquad \text{all } i \geq 2 \text{ and } s, y$$

It might be added that the above analysis also indicates why we have chosen to label (13b) essentially homogeneous *prior* beliefs rather than essentially homogeneous *signal* beliefs.

[9] In other words, in equilibrium there is for each signal equality across investors of marginal rates of substitution between securities and current consumption.

Turning to necessity, we note that this result should be viewed as rather straightforward because (11) must hold for *all* utilities within the broad boundaries posed by (12) and (14); (12) alone in the single-period case. In other words, it is not ruled out that (11) may just happen to hold for certain singular preference structures. It therefore follows that both (16) and (17) must hold if (15) is to be satisfied across different preferences. An immediate consequence is that the market must attain full allocational efficiency (12). Likewise, if time-additivity is not satisfied, then λ_i^y can be made to depend on y in an arbitrary fashion so that (14) is indeed necessary. Conditions (1b) and (13b) are necessary because (16) is equivalent to (1b) and (13b). This completes the necessity part for the two-period case. With respect to the proof for the one-period case,[10] we need only observe that (1b) is necessary because of (18) and Lemma 1.

It should be noted that if a presignal round of trading were added to the model, such trading would result in (9) being automatically satisfied, given full allocational efficiency, time-additive utility, and prior beliefs consistent with (1b) and (13b).

Observe that whenever endowments are equilibrium allocations without information, nontrivial trading implies not only that nobody is worse off with information (i.e. (10) holds) but also that

$$V_i(Y) > V_i^o$$

for all who trade. Consequently, Lemma 2 implies that we have obtained a particularly simple set of sufficient conditions for public information to have social value:

Theorem 1: For non-null public information to have social value under pure exchange, it is sufficient (for all but singular preference structures) that

endowments constitute equilibrium allocations without information (i)

and that *either*

the market is not fully allocationally efficient (iia)

or that

information structures are not essentially homogeneous (iib)

or that

prior beliefs are not essentially homogeneous (iiia)

or that

the (two-period) utility function is nonadditive (iiib)

where (iiia) and (iiib) do not apply in the single-period case. That is, for (8) to be valid, it is sufficient that (9) holds and that *one* of conditions (12), (1b), (13b), or (14) is violated.

In the single-period case, it is noteworthy that the above conditions are (essentially) independent of preferences and prior (as well as posterior) beliefs and of investor homogeneity or heterogeneity in general. What counts is the

[10] This corrects the "theorem" in Verrecchia [9, p. 280].

allocational efficiency of the financial market and the way the *information itself* is perceived by investors. However, when we move to two periods, certain properties of investor preferences and beliefs, in particular (13b) and (14), also enter the picture in explicit fashion.

Suppose now that the information system Y is costly in terms of real resources. Let $V_{ic}(Y)$ be consumer investor i's expected utility in equilibrium with costly information (where c is paid prior to the receipt of the message y, and thus before trading, say). Then for sufficiently small c, if we generalize into a setting in which all investors trade, (10) becomes

$$V_{ic}(Y) > V_i^o, \qquad \text{all } i \tag{19}$$

so that even *costly* information may have social value under pure exchange.[11] That is, it may pay to exchange real resources for public information even in pure exchange.

III. Necessity

We shall now show that either nonhomogeneous information structures or nonhomogeneous signal beliefs or (iia) or (iiib) is also *necessary* for (public) information to have social value under pure exchange; (9), on the other hand, does not have to hold. We begin with the following negative result.

Lemma 3: In a single-period setting, public information cannot have social value under pure exchange when the financial market achieves full allocational efficiency and information structures are homogeneous. In the two-period case, there will be no social value if, in addition, signal beliefs are homogeneous and utilities are time-additive. That is, (8) is impossible if (12) and (1a) hold in the single-period case, and (12), (1a), (14), and

$$p_i(y) = p_1(y) \qquad \text{all } i \geq 2, \text{ all } y \tag{13a}$$

hold in the two-period case.

To prove the above in the two-period case, define the "average" allocations $(\hat{c}_{is}, \hat{w}_{is})$ as follows:

$$(\hat{c}_{is}, \hat{w}_{is}) \equiv \sum_y (c_i^y, w_{is}^y) p_1(y \mid s), \qquad \text{all } i, s$$

These allocations are clearly feasible since

$$\sum_i (\hat{c}_{is}, \hat{w}_{is}) = \sum_i \sum_y (c_i^y, w_{is}^y) p_1(y \mid s)$$

$$= \sum_y \sum_i (c_i^y, w_{is}^y) p_1(y \mid s) = \sum_i (c_i^y, w_{is}^y), \qquad \text{all } s$$

Denoting the expected utility of these "average" allocations by $\hat{V}_i$, Jensen's inequality gives

$$\hat{V}_i \equiv \sum_s \pi_{is}^o U_{is}(\hat{c}_{is}, \hat{w}_{is}) \geq \sum_s \pi_{is}^o \sum_y p_1(y \mid s) U_{is}(c_i^y, w_{is}^y) = V_i(Y) \tag{20}$$

[11] This corrects a previous statement of Fama and Laffer [1, p. 292], who appear to have assumed homogeneous information structures but did not assume a complete market (or full allocational efficiency).

Define the additional "average" first period allocations, using (13a), as follows:

$$\hat{\hat{c}}_i \equiv \sum_s \hat{c}_{is}\pi^o_{is} = \sum_y c^y_i p_1(y) \qquad \text{all } i$$

These allocations are also feasible since

$$\sum_i \hat{\hat{c}}_i = \sum_i \sum_y c^y_i p_1(y) = \sum_y \sum_i c^y_i p_1(y) = \sum_i c_i$$

Applying (14) and Jensen's inequality one more time gives

$$\hat{\hat{V}}_i \equiv f_i(\hat{\hat{c}}_i) + \sum_s \pi^o_{is} g_{is}(\hat{w}_{is}) \geq \sum_s \pi^o_{is}(f_i(\hat{c}_{is}) + g_{is}(\hat{w}_{is})) = \hat{V}_i \tag{21}$$

Since the no-information allocations (c^o_i, w^o_i) are fully allocationally efficient with respect to the prior beliefs π^o_i when (12) holds, we must have either $\hat{\hat{V}}_i = V^o_i$ for all i or $\hat{\hat{V}}_i < V^o_i$ for some i. Thus, by (20) and (21),

$$V_i(Y) = V^o_i, \qquad \text{all } i \tag{22}$$

or

$$V_i(Y) < V^o_i, \qquad \text{some } i$$

which concludes the proof for the two-period case.

The proof for the single-period case is now immediate: set $f_i \equiv 0$ and since $c^o_i = c^y_i = \hat{c}_{is} = \hat{\hat{c}}_i \equiv 0$, it is readily seen that f_i is irrelevant and that $p_i(y)$ is unconstrained.

Comparing the sufficiency part of Lemma 2, which yields social indifference, with the conditions of Lemma 3, note that the former presumes that the endowments are equilibrium allocations under no information and that the similarity restrictions on beliefs are somewhat weaker. Thus, disregarding the endowment condition, the conditions of Lemma 3 cover a strict subset of those of Lemma 2.

Note also that previous results concerning the lack of social value of information under pure exchange in previous papers (e.g. Hirshleifer [3], Marshall [5], Ng [6], Wilson [10], and Green [2, p. 340]) are in fact special cases covered by Lemma 3.[12]

When is *everyone* worse off with information? A partial answer is given in the following

Corollary: A sufficient condition for

$$V_i(Y) < V^o_i \qquad \text{all } i. \tag{23}$$

to hold is that (for all s)

$$(\hat{c}_{is}, \hat{w}_i) = (c^o_i, w^o_i) \neq (c^y_i, w^y_i) \qquad \text{all } i, \text{some } y \tag{24}$$

The first part of (24) implies that $\hat{\hat{V}}_i = \hat{V}_i = V^o_i$ for all i and the second part insures that (20) holds with strong inequality, which in turn yields (23).

[12] The Lemma can be viewed as an extension of Marshall's results. Basically, there are three generalizations in this paper: (1) the distinction between one and two periods is covered; (2) (completely) homogeneous beliefs are not necessary; and (3) only full allocational efficiency, as opposed to complete markets, is necessary.

Theorem 1 and Lemma 3 now provide our second main result.

Theorem 2: For non-null public information to have social value under pure exchange, it is necessary but not sufficient that either

the market is not fully allocationally efficient (iia)

or that

information structures are nonhomogeneous (iibb)

or that

signal beliefs are nonhomogeneous (iiiaa)

or that

the (two-period) utility function is nonadditive (iiib)

where (iiiaa) and (iiib) do not apply in the signal-period case. That is, for (8) to be valid it is necessary that *one* of conditions (12), (1a), (13a), or (14) is violated.

It is important to note that the "essentially similar" conditions on beliefs, (1b) and (13b), *cannot* replace the stronger "identical beliefs" conditions (1a) and (13a) in Lemma 3 and in Theorem 2. That is, if (1b) and (12) hold but (1a) does not (in the one-period case, say), it is possible to construct examples in which information has social value in the absence of (9).[13]

[13] Consider the following single-period example with two states and two Arrow-Debreu securities (which implies (12)) and two investors with the following information structures (which imply (1b)):

$$\begin{array}{ccc} .6 & .3 & .1 \\ .4 & .4 & .2 \end{array} \qquad \begin{array}{ccc} 12/22 & 9/22 & 1/22 \\ 8/22 & 12/22 & 2/22 \end{array}$$

If the prior beliefs are $\pi_1^\circ = \pi_2^\circ = (.5, .5)$, the implied signal beliefs are .5, .35, and .15 for Investor 1 and 20/44, 21/44, and 3/44 for Investor 2. The posterior beliefs are then (.6, .4), (3/7, 4/7), and (1/3, 2/3) for the three signals (for both investors). Suppose also that endowments are (200, 0) and (0, 400), respectively, and that the utility functions satisfy the following conditions:

For Investor 1:

Wealth	75	600/7	100	120	150	1200/7	200	230
Utility	0	43.7	101	180.5	274	317.7	375	434.5
Marginal Utility	5.04	4.02	4	3.642	2.1	2.01	2	1.9

For Investor 2:

Wealth	80	100	800/7	125	170	200	1600/7	250
Utility	0	81	138	180.5	344.5	405	462	504
Marginal Utility	4.552	4	3.98	3.96	2.375	2	1.99	1.65

The equilibrium allocations and prices are then as follows (note that (9) is violated):

	Investor 1	Investor 2	Prices
No information	(100, 200)	(100, 200)	(2, 1)
Signal 1	(120, 230)	(80, 170)	(2.4, .8)
Signal 2	(600/7, 1200/7)	(800/7, 1600/7)	(12/7, 8/7)
Signal 3	(75, 150)	(125, 250)	(1.5, 1.25)

The corresponding expected utilities are then $V_1^\circ = 238$, $V_1^1 = 282.1$, $V_1^2 = 200.27$, $V_1^3 = 182.67$, $V_2^\circ = 243$, $V_2^1 = 137.8$, $V_2^2 = 323.14$, and $V_2^3 = 396.17$. Consequently, $V_1(Y) = 238.55 > V_1^\circ$ and $V_2(Y) = 243.87 > V_2^\circ$.

IV. Discussion and Summary

We have demonstrated that the oft-made assertion that information does not have social value under pure exchange is incorrect unless

the financial market achieves full allocational efficiency, (iia′)

and

information *structures* are homogeneous (iibb′)

When *either* of these conditions is violated in the single-period case, public information *may* yield Pareto-superior allocations; Pareto-superiority is, except for singular preference structures, guaranteed (Theorem 1) whenever

endowments represent equilibrium allocations *without* information (i)

and either (iia′) or

information structures are essentially homogeneous (iib′)

is violated. It is noteworthy that none of the preceding depends directly on preferences (e.g. whether they are state-dependent or not) or on the homogeneity or nonhomogeneity of prior or posterior beliefs. Only the allocational efficiency of the financial market, the information itself, and initial allocations matter.

In the two-period case, the "no social value" conditions (iia′) and (iibb′) must be supplemented with the following relatively strong requirements:

prior beliefs are homogeneous (iiiaa′)

and

the (two-period) utility functions are time-additive (iiib′)

Since it is rather implausible that *all* of conditions (iia′) (iibb′), (iiiaa′), and (iiib′) will be satisfied at the same time, it does not appear to be unrealistic to view public information as potentially valuable in a pure exchange environment. In different language, it is clear that previous authors reaching the traditional negative conclusion (Hirshleifer [3], Marshall [5], Ng [6], Wilson [10]) have in fact employed a strong and closely related set of assumptions. In addition, the significant differences between the single-period and the two-period cases have not previously been brought to light.

When *all* of the four conditions (iia′), (iib′),

prior beliefs are essentially homogeneous (iiia′)

and (iiib′) hold, trading is never optimal whenever (i) is valid (Lemma 2). Thus, these five conditions represent something of a watershed, in that information becomes a matter of indifference when they are all true. When *one or more* of (iia′) through (iiib′) is (are) violated, everyone is better off *with* information than without as long as (i) holds. Clearly, we can expect the same to be true for *some* endowments which also violate (i) but involve only "limited" trading or strongly state-dependent preferences.

When (i) alone is violated but (iia′), (iibb′), (iiiaa′), and (iiib′) hold, trading will occur and at least *some* individuals will be worse off with information than without. Some may also be better off except when everyone's *average* allocations of *securities* with information is in some sense *close* to the allocation that is achieved without information, assuming that the first-period allocation is relatively unaffected; when this happens, the corollary suggests that *everyone* will be worse off in the information case.

When none of the five conditions (i)–(iiib′) hold, it follows from the preceding that the outcome is unconstrained. The availability of public information may result in Pareto-superior allocations or it may lead to Pareto-inferior allocations, as well as various intermediate cases. This is because preferences and beliefs now enter the picture even more explicitly than via (iiia′) or (iiiaa′) and (iiib′). General statements must rely on the results of this paper or how "close" conditions (i)–(iiib′) are to being satisfied.

For those who find heterogenous beliefs offensive, all of the results in the present paper can of course be specialized to preclude differences in beliefs by simply striking all references to prior beliefs and information structures, i.e. to (1a), (1b), (13a), (13b), (iib), (iibb), (iib′), (iibb′), (iiia), (iiiaa), (iiia′), and (iiiaa′).

Finally, a brief note concerning the production case may be in order. When production decisions are introduced, public information may yield allocations which range from Pareto-inferior to Pareto-superior. However, we would also expect the set of conditions which yield Pareto-superior allocations to "expand" and the conditions which imply Pareto-inferior allocations to "shrink".

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