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# Managerial Incentives in a Stock Market Economy

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ABSTRACT

This study presents an analysis of the managerial incentive problem in a stock market economy in which incentive contracts are structured in terms of security ownership. In our model, the manager's ownership share signals effort and is determined endogenously as the solution to a special portfolio decision problem. Managerial investment in the firm is evaluated under various security pricing arrangements. Our analysis indicates that, in general, stockholders should sell shares to a manager at a discount to ensure a Pareto efficient ownership (incentive) structure. However, efficient pricing (discount) schedules generally are nonlinear and, in many respects, isomorphic to discriminating price functions which have been considered in neoclassical models of monopoly.

THE THEORY OF THE firm has evolved rapidly over the past decade. With the emergence of research in property rights and the economics of uncertainty, the firm is no longer regarded as a black box. Jensen and Meckling [JM] [10] presented an elaborate theory of financial/ownership structure based primarily on agency costs and other studies by Campbell [2], Leland and Pyle [12], and Ross [19, 20] have developed theories of signaling to explain the financial structure of the firm.

While the issues and specific modeling approaches have varied, the problem of managerial incentives has been prominent in all these studies. In the JM [10] study, incentives were related directly to the manager's ownership proportion. Although the manager initially owned the entire firm, investment in new projects required outside financing and led to a managerial incentive problem.[1] Our model has implications directly related to the JM [10] study. The manager's ownership proportion also had an important role in the Leland and Pyle [12] model as a signal of the quality of new projects. In the Ross [20] study, the managerial incentive contract influenced the manager's activity choice and supported a signaling equilibrium.

Managerial incentive contracts have also been analyzed in the principal-agent literature (see Ross [17], Harris and Raviv [8], Holmstrom [9], Lewis [14], and

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[1] Such financing could involve the issuance of both debt and equity securities. The issuance of debt to outsiders creates incentives for the stockholders to accept risky projects which transfer wealth from bondholders to stockholders [JM 10]. Since the incentive effect associated with debt can be eliminated when bondholders simultaneously purchase an equal proportion of the firm's shares (see JM), we focus on risk aversion.

Shavell [23]). The approach in these studies has been to solve directly for the optimal share or bonus that the principal should assign the agent. In this paper, we instead focus on the optimal pricing of securities. Of course, shadow prices are implicit in the payoffs derived in principal-agent models (see Holmstrom [9]) and so the problem can be approached from either direction. We examine the principal-agent problem in a specific institutional setting, namely the corporation.

The objective of this paper is to investigate the managerial incentive problem in a simplified stock market economy in which incentives are provided by stock ownership. Two primary contributions of this study are: first, to explain how a manager's ownership share emerges as the solution to a special portfolio choice problem; and second, to determine how securities should be priced to a manager to achieve a Pareto efficient incentive contract.[2]

The security pricing decision in our model is determined by an owner/investor, while the purchase decision is made by the manager. Thus, the manager's ownership share emerges as the endogenous solution to a portfolio choice problem. By explicitly considering the manager's portfolio decision, limited managerial ownership is explained without relying upon budget constraints. Risk aversion deters the manager from becoming or remaining a 100% owner. In equilibrium, the manager's ownership share depends jointly on anticipated productivity gains and risk aversion.

Our study indicates that, for optimality, securities must generally be priced differently to the manager than to other stockholders. JM [10] have argued that when the manager sells shares, the market takes into account the diminution in managerial incentives by lowering the firm's share price. When the manager initially is not a 100% owner, however, the reduction in value is borne partially by the other initial owners. Thus, it is optimal for the manager to reduce his ownership share below the Pareto efficient level. Alternatively, when the manager initially has less than a Pareto efficient ownership share, our analysis indicates that managerial ownership must be subsidized by discounting share prices. However, a simple (single price) discounting scheme will not achieve Pareto efficiency. The Pareto optimal pricing function resembles a perfectly discriminating price schedule, but actually is consistent with a competitive environment in which the manager and stockholders jointly maximize their own expected utility.

The remainder of this paper is organized in five sections. Section I provides an introduction to the model. In Section II, we derive a constrained Pareto efficiency condition as a benchmark for evaluating various security pricing arrangements. Rational expectations security pricing is considered in Section III and two incentive pricing plans are investigated in Section IV. The first is a simple (discount) pricing scheme, while the second is a cumulative price function. Concluding remarks are presented in Section V.

[2] Grossman and Hart [7] have observed that businesses experience a life cycle. New fledgling firms often are closely held and operated by an owner-entrepreneur. In mature firms, a new executive may initially own relatively few, if any, shares. An important problem, therefore, is to induce the manager to make an efficient investment in the firm. This problem is the focus of Section IV.

## I. The Model

The model in this section is patterned after Diamond's [4] model of a stock market economy as extended by Zorn [24]. Investors trade securities in a timeless world in which firms face technological uncertainty, the investment opportunity set is represented within a state-preference framework, and investors' payoffs are a linear combination of firm profits in all states of nature. Unlike the original Diamond model, the firm's payoff in our model is affected by an owner/manager (OM) whose actions are unobservable and not necessarily consistent with the interests of other stockholders. To simplify the analysis, we let one owner/investor (OI) represent the market or a syndicate of investors whose common objective is to structure an efficient managerial incentive contract.[3]

The following notation is adopted:

- $U(\cdot)$: the OM's strictly concave utility function
- $V(\cdot)$: the OI's weakly concave utility function
- $Z$: the total number of units of the riskless security
- $z$: the number of units of the riskless security held by the OI after trading
- $N$: the total number of units of the risky security
- $n$: the total number of units of the risky security held by the OM after trading
- $X_s(n)$: the payoff accruing to each share of the risky firm in state of nature $s$ ($s = 1, \cdots, S$), which is a function of the OM's ownership share, $(n/N)$[4]
- $\{1, 1, \cdots, 1\}$: the state contingent payoff vector of the riskless security which is numeraire
- $\{\pi_s^m\}$, $\{\pi_s^o\}$: the $S$-dimensional probability vectors representing the beliefs of the OM and OI, respectively[5]

### A. Assumptions

The general assumptions of the model are:

(A.1) There are no transactions costs or taxes.

(A.2) All individuals have a single period horizon and maximize their expected utility of wealth.[6]

[3] A representative investor obviously exists when all stockholders have identical beliefs, tastes, and endowments. These conditions can be weakened by making assumptions about the distribution of returns, firms' production technology, and perfect competition (see De Angelo [3], Ekern and Wilson [5], Leland [11], Ross [18], and Rubinstein [21]).

[4] The incentive effect also depends on the prominence of the investment within the manager's portfolio. Managerial risk aversion also creates an incentive for portfolio diversification (see Mayers [15], JM [10], and Fama [6]).

[5] Although the beliefs of the OI and OM are allowed to differ, it is assumed that the differences are not due to "insider information." One problem that could arise in that case is adverse selection, in which it could be Pareto efficient to offer a menu of contracts to the OM as a means of acquiring additional information (see Baron [1]).

[6] The model can be extended to encompass two periods, so that the beginning of the period

(A.3) A riskless security is available to investors.
(A.4) No borrowing or short selling is permitted.[7]
(A.5) There is one risky firm whose payoffs are distributed in proportion to share ownership and whose scale of operations cannot be altered.
(A.6) $X_s(n)$ is a nondecreasing concave function of $n$ with continuous first and second derivatives.

Assumptions (A.1)-(A.3) are standard assumptions in many stock market models and Assumption (A.4) is a simplification that facilitates our analysis of the managerial incentive problem. The linearity restriction on the sharing rule (A.5) is more restrictive than in most agency models, but is consistent with the constraint generally imposed on the feasible set of payoffs in many stockmarket models (see Diamond [4], Leland [11], and Ekern and Wilson [5]). Letting $X_s(n)$ be a function of the OM's ownership share is a simplification that does not affect the results, provided that the OM's labor/leisure decision problem has a unique solution for each $n$ and the contract is incentive compatible.[8] The assumption of a discrete state space is more restrictive than in many agency models (see Harris and Raviv [8], Holmstrom [9], and Shavell [23]), but has no qualitative effect on the results and the model can be modified readily.

The assumptions describing the information available to agents in the economy are:

(A.7) Neither the OI nor OM knows the state of nature when making the portfolio decision and structuring the incentive contract.
(A.8) The state of nature is neither observed nor inferred by the OI.[9]
(A.9) The OI and OM's prior probability beliefs are given and exogenous to the model.
(A.10) The OI cannot directly observe the OM's actual effort level (production

consumption decisions are modeled explicitly and the riskless security earns a positive rate of return. However, another decision variable complicates the model without providing any additional insight. A more important limitation is that multiperiod aspects of the managerial incentive problem such as managerial reputation are thus ignored (see Fama [6]). But reputation provides a solution to the managerial incentive problem only when the underlying distribution is sufficiently stable, allowing investors to separate statistically the manager's input from the stochastic environment. Our model is most applicable in those circumstances in which the underlying distribution is not stable over time. This is a strong assumption, but allows us to come to grips with the problem of managerial incentives when it cannot be completely resolved by managerial reputation.

[7] Alternatively, one could assume there are no incentive effects due to debt which play a prominent role in the JM [10] and Ross [19, 20] studies. In contrast with JM, we do not rely on wealth constraints to explain limited managerial ownership.

[8] The OM's production decision is suppressed to simplify the model. Implicitly, the OM (and OI) must compute the OM's optimal production decision (effort level) and $X_s(n)$ for each possible $n$. A rational expectations contract between the OI and OM must be made relative $X_s(n)$, since the OI knows that the OM's decision is based strictly on his private return. While both the OI and OM are able to make these ex ante calculations, the OI is unable to observe the state or OM's action ex post (see assumptions A.10 and A.12). Thus, the incentive contract is not based upon the state or the OM's claimed action.

[9] When the OM's action or the state is observable, the incentive contract can incorporate this additional information. Mirrlees [16] has shown that a first-best solution is obtainable by imposing a large penalty whenever the OM's action deviates from the optimum.

decision) or otherwise infer it from other observations and the OM knows this.

(A.11) Both the OI and OM observe the true payoff (profit) *ex post*.

(A.12) A rational expectations assumption: The OI knows *ex ante* the firm's payoff $X_s(n)$, and therefore the OM's productive decision as a function of $n$ in each state.

Assumptions (A.7)-(A.12) are consistent in most respects with the information assumptions in previous agency models.[10] Given (A.10) and (A.12), the familiar moral hazard problem arises because the OI is unable to determine whether a high (low) payoff resulted from a "good" ("bad") decision or a favorable (unfavorable) state realization. Since neither the OM's action nor the state is observed by the OI, only second-best sharing rules (which depend on the output alone) are feasible. The OI's prediction of $X_s(n)$ is based on his correct assessment of the OM's behavior given the latter's information advantage (A.12).

Although our model is timeless, the OI and OM's actions can be viewed from a sequential perspective. Initially, the OI and OM exchange shares of the risky and riskless securities. Once trading is completed and the OM's investment is determined, the OM makes a production decision. Then a state $s \in S$ is realized and the resulting payoff is allocated based on the OI's and OM's ownership proportions.

## II. Pareto Allocation

In this section, we derive the Pareto efficient allocation of shares subject to the constraints imposed by technology, incentive compatibility requirements implicit in the $X_s(n)$ functions, and the linearity of sharing rules characterizing a Diamond [4] stock market economy. This benchmark, established in the usual manner, will allow the efficiency of alternative security pricing institutions to be evaluated directly.

Since neither the OM's action nor the state is observable by the OI, the allocation rule in our model cannot depend explicitly upon either of these. The allocation is also subject to an additional restriction, because the assets are allocated via security shares in our stock market economy. The OM receives $n$ shares and the remaining $N - n$ shares are assigned to the OI. The riskless asset must be allocated simultaneously: $z$ shares to the OI, and $Z - z$ to the OM. The OM's *end-of-period* wealth in each state is

$$W_{ms} = Z - z + nX_s(n)$$

The conservation constraint on total social output implies that, in each state, the wealth of the OI is

$$W_{os} = z + (N - n)X_s(n)$$

Given $N$ and $Z$, the pair $(n, z)$ completely defines an allocation in this economy.

[10] One technical difference between our model and some agency models is that we define beliefs over states of nature. In other models (e.g., Holmstrom [9]), beliefs are defined directly over the payoffs and depend on the agent's action.

Therefore, a (constrained) Pareto allocation is obtained by finding a pair $(n, z)$ maximizing the expected utility of one individual subject to a minimum constraint on the other's expected utility. Formally

$$\mathrm{Max}_{n,z}\{\textstyle\sum_s \pi_s^0 V(W_{os})\}$$

subject to

$$\textstyle\sum_s \pi_s^m U(W_{ms}) \geq \bar{U} \tag{1}$$

where $\bar{U}$ is an exogenously determined level of utility. The Lagrangian is

$$L = \textstyle\sum_s \pi_s^0 V(W_{0s}) + \lambda(\bar{U} - \sum_s \pi_s^m U(W_{ms})) \tag{2}$$

where $\lambda$ is an undetermined constant.

Differentiating (2) with respect to $z$ and $n$, the first-order (necessary) conditions are:

$$\frac{\partial L}{\partial z} = \textstyle\sum_s \pi_s^0 V' + \lambda \sum_s \pi_s^m U' = 0 \tag{3}$$

$$\begin{aligned}\frac{\partial L}{\partial n} &= \textstyle\sum_s \pi_s^0 V'[-X_s(n) + (N-n)X_s'(n)] \\ &\quad - \lambda \textstyle\sum_s \pi_s^m U'[X_s(n) + nX_s'(n)] = 0\end{aligned} \tag{4}$$

Solving (3) for $\lambda$,

$$\lambda = -\frac{\sum_s \pi_s^0 V'}{\sum_s \pi_s^m U'} \tag{5}$$

Substituting (5) into (4), the following Pareto efficiency condition is obtained:

$$\frac{\sum_s \pi_s^0 V'[X_s(n) - (N-n)X_s'(n)]}{\sum_s \pi_s^0 V'} = \frac{\sum_s \pi_s^m U'[X_s(n) + nX_s'(n)]}{\sum_s \pi_s^m U'} \tag{6}$$

The two quotients in (6) can be interpreted as marginal rates of substitution between risky and riskless assets ($MRS_{RL}$). Thus, the Pareto efficiency condition in (6) implies that

$$MRS_{RL}^0 = MRS_{RL}^m \tag{7}$$

where the superscripts 0 and $m$ designate the OI and OM. Notice that the OI's and OM's marginal utilities are multiplied both by probability "weights" and a second group of terms inside the square brackets. The former reflect their beliefs about the states of nature, while the latter terms represent the output and incentive effects.

### *Risk-Neutral OI*

The incentive or productivity effect is readily dentifiable when the OI is risk-neutral. A risk-neutral OI is not only convenient to model, but also of theoretical interest. Fama [6] has argued that stockholders contribute an important factor of production in their role as residual risk-bearers because of their opportunities for diversification. In our model, a risk-neutral OI is a surrogate for a class of fully diversified stockholders. Alternatively, the OI could be viewed as a syndicate of owners or a representative of the board of directors.

Assuming risk neutrality, (6) can be written as

$$\sum \pi_s^0[X_s(n) - (N - n)X'(n)] = MRS_{RL}^m \tag{8}$$

Taking expectations term-by-term, (8) is equivalent to

$$\bar{X}(n) - (N - n)\bar{X}'(n) = MRS_{RL}^m \tag{9}$$

where the bars denote expected values.[11]

The first term in (9) is the expected payoff accruing to each risky share, while the second term represents the fraction of the expected marginal payoff received by the *OI*. Thus, (9) indicates that $MRS_{RL}^m$ is not equated with the full actuarial value of the risky shares, $\bar{X}(n)$. There is a "discount" based on the expected marginal productivity effect. The Pareto allocation recognizes that, in the neighborhood of an interior optimum, an increase in the OM's share also increases the expected payoff to the *OI*. This result has important implications for security pricing and provides a benchmark for the analysis in Sections III and IV.

## III. Rational Expectations Pricing

The first security pricing scheme investigated is rational expectations pricing (REP) which is of interest for the following reasons. First, the theory of rational expectations underlies many important models of competitive equilibria. Second, REP provides a foundation for the analysis of more complicated incentive pricing schemes which are considered in Section IV. Third, since the JM study relied upon risk-neutral REP, a more direct relationship is established between the studies.

Since $X_s(n)$ is a positive function of $n$, the value of the firm's stock will be a positive function of the OM's ownership share. As in the JM model, REP assumes that the OI (or more generally the market) can forecast the OM's post-trade ownership and the implied incentive effect. Moreover, REP assumes that the OM trades at the equilibrium price per share even though he is not at the equilibrium. Rational expectations pricing in this sense implies that at equilibrium no one has an incentive to change the price from its pre-equilibrium level.

Initially we assume homogeneous beliefs, $\pi_s^0 = \pi_s^m$ for all $s$, and risk neutral REP. We subsequently show that these assumptions do not affect the main result of this section; REP does not in general result in Pareto efficient managerial incentives. Assuming that the OI is a surrogate for the risk neutral market, the

[11] Although we emphasize the interior solution case, corner solutions are also possible. When the productivity effect of incentives is very small and/or the OM is very pessimistic, $MRS_{RL}^m$ will become small, so the first-order condition in (9) may not be satisfied. Observe that when

$$\bar{X}(n) - (N - n)\bar{X}'(n) > MRS_{RL}^m \quad \text{for } n > 0$$

the Pareto efficient solution is for the manager to own zero shares. Alternatively, when the OM is much more optimistic than the OI and/or the productivity effect of incentives is very large, it may be the case that the corner solution, $n = N$, is optimal.

rational expectations price is $\bar{X}(n) = \sum \pi_s^0 X_s(n)$. $\bar{X}(n)$ is a competitive price - even though there are only two traders in the market. For the present, we ignore the fact that the OI might benefit by pricing "monopolistically."

*Proposition 1.* REP will result in an *interior* Pareto efficient managerial ownership structure *if and only if* the OM initially owns the entire firm.

Proof of Proposition 1 requires a formal definition of the OM's objective function. Also, the following wealth constraints for the economy must be defined:

$$z_e^0 + z_e^m = Z \tag{10}$$

where $z_e^0$ and $z_e^m$ are the OI and OM's endowments of the riskless security and

$$n_e^0 + n_e^m = N \tag{11}$$

when $n_e^0$ and $n_e^m$ are the OI and OM's respective endowments in the risky security. Incorporating these wealth constraints in the OM's objective function, the optimal portfolio decision is determined by

$$\text{Max}_n\{\sum \pi_s^m U(nX_s(n) + (N - n_e^0 - n)\bar{X}(n) + Z - z_e^0)\} \tag{12}$$

The first term inside the utility function in (12) is the payoff from the OM's investment in the risky security. Because $N - n_e^0 = n_e^m$, the second term represents the proceeds from the sale (or payment for a purchase) of shares of the risky security. When combined algebraically with the initial endowment of the riskless security ($z_e^m = Z - z_e^0$), the latter terms comprise the OM's total net investment in the riskless security.

*Proof.* It is convenient to prove the necessity (*i*) of Proposition 1 before demonstrating sufficiency (*ii*).

*Part i.* For technical and illustrative purposes, we establish necessity by separately analyzing cases where (1) the manager initially is a nonowner; and (2) the manager has an existing ownership interest. The former is representative of a situation in which an established firm hires an outside manager, while the latter can occur when the manager is promoted internally or the firm is not yet mature (see [7]).

Assuming the manager is a nonowner, $n_e^m = 0$, $n_e^0 = N$. Further, without loss of generality, we assume $z_e^m = Z$, therefore $z_e^0 = 0$.[12] Since the OM's utility function in (12) is assumed to be concave, Jensen's inequality indicates that for $n > 0$,

$$\sum \pi_s^m U(Z + nX_s(n) - n\bar{X}(n)) < U(\sum \pi_s^m(Z + nX_s(n) - n\bar{X}(n))) \tag{13}$$

Rewriting the right-hand side of (13) and simplifying,

$$U(\sum \pi_s^m(Z + nX_s(n) - n\bar{X}(n)) = U(Z + n(\sum \pi_s^m X_s(n) - \bar{X}(n))) \tag{14}$$

Since $\sum \pi_s^m X_s(n) = \bar{X}(n)$,

$$U(\sum \pi_s^m(Z + nX_s(n) - n\bar{X}(n))) = U(Z) \tag{15}$$

[12] Notice that by endowing the OM with the riskless security, he is in the most favorable position to absorb additional portfolio risk.

Thus, applying (13) and (15),

$$\sum \pi_s^m U(Z + nX_s(n) - n\bar{X}(n)) < U(Z) \tag{16}$$

Since the OM's expected utility is always less when he buys some of the risky security, he will remain at his endowment point.

While the above analysis provides one example in which REP is not Pareto efficient, one may question whether the inefficiency also occurs when the manager initially owns shares of the firm. In this case, the value of the OM's endowment varies with his ownership, so the Jensen's inequality logic is no longer applicable.[13]

We now consider the case in which the OM initially owns shares of the firm. Differentiating the OM's objective function using the chain rule, the necessary condition for an (interior) optimum is

$$\sum \pi_s^m U'[X_s(n) + nX_s'(n) - \bar{X}(n) + (N - n_e^0 - n)\bar{X}'(n)] = 0 \tag{17}$$

where $U'$ denotes the derivative of the utility function with respect to its argument.

Rearranging terms and factoring out the $\bar{X}(n)$ and $\bar{X}'(n)$ terms,

$$\bar{X}(n) - (N - n_e^0 - n)\bar{X}'(n) = \frac{\sum \pi_s^m U'[X_s(n) + nX_s'(n)]}{\sum \pi_s^m U'} \tag{18}$$

$$= MRS_{RL}^m \tag{19}$$

Observe that since the OM is endowed with less than 100% of the risky shares, $n_e^m < N$. But given (11), $n_e^0 > 0$, so the left-hand side of (18) is strictly larger than the Pareto efficiency condition, (9). Therefore, when the OM initially owns less than 100% of the firm, rational expectations pricing does not result in a Pareto efficient post-trade incentive structure. *Part ii.* The sufficiency part of Proposition 1 is established readily. When the OM initially owns the entire firm

$$n_e^m = N, \text{ so } n_e^0 = 0$$

Thus, the first-order condition in (18) simplifies to

$$\bar{X}(n) - (N - n)\bar{X}'(n) = MRS_{RL}^m \tag{20}$$

which is formally identical to the Pareto efficiency condition, (9). Q.E.D.[14]

Although not essential to our argument, Proposition 1 is valid under more general conditions. Specifically, risk neutrality and homogeneous beliefs need not

[13] The authors are indebted to Sudipto Bhattacharya for this point.

[14] Although we have suppressed any fixed form of compensation, a fixed wage can be incorporated readily. Let C denote the OM's fixed compensation and $c = C/N$, the per share cost of the OM's compensation. The maximand in (12) can be written as:

$$\sum \pi_s^m U(n(X_s(n) - c) + (N - n_e^o - n)(\bar{X}(n) - c) + Z - z_e^o + Nc)$$

since the OM receives $C = Nc$ as a fixed wage, but the payoff accruing to each share (including those owned by the OM) is reduced by $c$. If $C$ is assumed to be constant, all of the terms involving $c$ will disappear upon differentiation, so the above result in (20) is obtained. Alternatively, given the more realistic assumption that $c$ is a function of $n$ (since $n$ signals effort), then it can be verified that all terms depending upon $c$ cancel out except $n_e^o c'(n)$. Once again, Pareto efficiency is obtained iff $n_e^o =$ 0.

be assumed in the above proof. We sketch how each assumption may be weakened.

If investors are risk averse, the market value of a share will not be the expected value of $X_s(n)$. But under certain circumstances it is possible to reinterpret $\bar{X}(n)$ in the above proof as the market value of each share, even though investors are risk averse. The value of each share as a function of $n$ must be well defined. Value maximization, appropriately defined, must be in the interests of the stockholders. Given these conditions it is possible to show that (9) characterizes Pareto optimality and Proposition 1 is still valid despite risk averse investors.[15] Only the strong result that a nonowner manager won't purchase any shares, no longer holds. But, this does not affect the negative optimality result.

Allowing for heterogeneous beliefs is somewhat more complicated. Differences in beliefs due to insider information seem inconsistent with a rational expectations framework. But it seems reasonable to suppose that some belief differences could arise despite the fact that no agent has superior information. The homogeneous beliefs assumption was utilized only in proving the strong result that the (nonowner) manager would purchase zero shares given the (risk neutral) REP. Obviously, this result also holds with heterogeneous beliefs when the manager is less optimistic than the OI (i.e., $\sum \pi_s^m X_s(n) < \bar{X}(n)$). When the OM is more optimistic than the OI, managerial investment in the firm may occur, but again the Pareto efficiency condition (9) is not satisfied given the $n_e^0$ term in (17).[16]

Proposition 1 demonstrates that, in general, REP is Pareto efficient only when the manager initially is a 100% owner. Otherwise, the OM has an incentive to reduce his investment in the firm below the Pareto efficient level. The difficulty arises because the OM does not bear the entire reduction in the value of the firm's shares, since the value of the other initial owner's holdings is reduced by the amount, $n_e^0\bar{X}'(n)$. Therefore, the OM does not internalize the total effect associated with reduced incentives.

Proposition 1 also indicates that there is a similar problem when the manager initially is a nonowner. The potential improvement in productivity accruing to the OI (due to the OM's increased ownership) is not shared with the OM. Since the OM is not fully compensated for assuming greater portfolio riskiness, he has an inadequate incentive to overcome his risk aversion. Of course, one might argue that the OI should recognize that the OM creates a favorable "externality" and bargain to improve productive incentives. One means of accomplishing this is for the OI to reduce the price of the shares to the OM. But discounting the share price results in incentive pricing rather than rational expectations pricing. In the following section, incentive pricing is investigated.

[15] Let $\bar{X}(n)$ now represent the value per share in an economy where investors are risk averse. If it is in the stockholders interests to maximize their shareholdings, namely $(N - n)\bar{X}(n)$ then the first term on the right-hand side of (2) can be replaced by $(N - n)\bar{X}(n)$. Proceeding as before, we can show that condition (9) characterizes Pareto optimality when stockholders unanimously support value maximization. To establish Proposition 1 we again proceed simply by re-interpreting $\bar{X}(n)$ as the value of the firm. Only the Jensen inequality argument is no longer valid, i.e., the result that a manager who is initially a nonowner will not purchase any shares is no longer valid. For conditions under which stockholders will unanimously support value maximization, see Footnote 4.

[16] If the OM were very optimistic relative to the OI, it could be optimal for him to own the entire firm.

## IV. Incentive Pricing

Incentive pricing explicitly recognizes the manager's impact on profits and, therefore, prices shares specially to the manager. The first of the two incentive pricing schemes considered in this section relies on a single (discounted) price per share, while the second is based upon a price schedule (function). Although the single-price (discount) contract is inefficient, the analysis provides important insights which help to suggest an efficient pricing contract.

### *A. Single Price Contract*

We now assume that the OI chooses a share price to maximize his own expected utility while taking into account the OM's reaction function. The resulting equilibrium has the property that both the OI and OM are optimizing given that the OI is choosing the price per share. In some respects, the role of the OI is similar to that of a board of directors. Formally, the OI's problem is:

$$\mathrm{Max}_{n,p}\{\textstyle\sum_s \pi_s^0 V((N-n)X_s(n)+np))\}$$

subject to

$$\text{(a) } n \in \mathrm{Argmax}\{\textstyle\sum_s \pi_s^m U(nX_s(n)-np)\}$$

$$\text{(b) } \textstyle\sum_s \pi_s^m U(nX_s(n)-np) \geq \bar{U} \tag{21}$$

where $\bar{U}$ is a minimum level of expected utility and Argmax denotes the set of maximizers.[17]

The first constraint in (21) requires the OI to recognize the OM's investment response to the pricing decision and the concomitant effect on productivity. The second constraint in (21) ensures that the OM's expected utility after the purchase must be at least as great as before the purchase. Otherwise, the OM would be unwilling to purchase the shares. It is apparent, therefore, that the second constraint places an upper bound on the price that can be charged the OM. That is, there exists a price, $p^*$, such that $p \geq p^*$, implies $n = 0$.

An important issue is whether it is possible for the OI to choose a single price satisfying constraints (a) and (b) that is also consistent with Pareto efficiency.

*Proposition 2.* An incentive pricing plan restricted to a single price will not satisfy the Pareto efficiency condition, (9).

*Proof.* Two important assumptions are invoked in proving Proposition 2. First, it is assumed that $p < p^*$. Second, we assume that the OM's maximization problem has a unique solution for each $p$, so that (a) is replaced by a first-order condition. Solving the first-order condition for the OM's (ordinary) demand function, $n = n(p)$ and substituting into the objective function, the following first-order (necessary) condition is obtained:

$$\frac{dEV}{dp} = \sum \pi_s^0 V'\left[(N-n)\frac{\partial X_s}{\partial n}\frac{dn}{dp} - \frac{dn}{dp}X_s(n) + n + \frac{dn}{dp}p\right] = 0 \tag{22}$$

[17] It is assumed that salary compensation is implicit in the price shares. One can easily make the salary explicit (see Footnote 14).

Rearranging terms,

$$\sum \pi_s^0 V'\left[(N-n)\frac{\partial X_s}{\partial n}\frac{dn}{dp}-\frac{dn}{dp}X_s(n)\right]=-\sum \pi_s^0 V'\left[n+\frac{dn}{dp}p\right] \tag{23}$$

or

$$\frac{\sum \pi_s^0 V'\left[(N-n)\frac{\partial X_s}{\partial n}\frac{dn}{dp}-\frac{dn}{dp}X_s\right]}{\sum \pi_s^0 V'}=-\left(n+\frac{dn}{dp}p\right) \tag{24}$$

Dividing by $-\frac{dn}{dp}$,

$$\sum \pi_s^0 V'\left[X_s(n)-(N-n)\frac{\partial X_s}{\partial n}\right]=p\left(1+\frac{1}{\varepsilon_p}\right) \tag{25}$$

where $\varepsilon_p$ is the OM's (own) price elasticity of demand for the risky security. In the case of a risk neutral OI, (25) becomes

$$\bar{X}(n)-(N-n)\bar{X}'(n)=p\left(1+\frac{1}{\varepsilon_p}\right) \tag{26}$$

This condition is equivalent to the Pareto efficiency condition if and only if $\varepsilon_p=-\infty$. This is clearly impossible since it requires the OM to have an infinite price elasticity of demand. Q.E.D.

The difficulty with the single share price (simple discount) contract is that incentive or productivity effects are not fully internalized. As all shares are discounted by the same amount, the marginal share price cannot be reduced without lowering the price of all shares sold to the OM. The lost revenue from the inframarginal shares will offset productivity gains accruing to the OI before the Pareto efficient point is reached, i.e., the discount per share is not large enough at the margin to compensate the OM for bearing additional portfolio risk.

In some respects, the OI's position is analogous to a single price monopolist. The right-hand side of (26) will be recognized as marginal revenue (MR), and the OI sets the left-hand side equal to MR rather than $p$ as required for efficiency. This result should not be misconstrued as a consequence of monopoly power on the part of the OI, but a consequence of restricting the OI to a single price, $p$. This point will be discussed further in the next section.

### *B. Cumulative Pricing Function*

We now consider an alternative pricing scheme that overcomes the problems with a single price incentive contract. We allow the marginal price to vary continuously with $n$. In this section we no longer restrict the OI to being risk neutral. The next proposition demonstrates that there is a cumulative pricing function that, within the context of the model, dominates any other feasible pricing scheme. We first state the following definitions:

*Definition 1.* A pricing function $\phi(n)$, is acceptable if Max $EU(\phi(n)) \geq \bar{U}$ where $\bar{U}$ is the OM's endowed or reservation level of expected utility, and Max $EU(\phi(n))$

is the OM's expected utility given his best response to the pricing function $\phi(n)$. A pricing function $\phi(n)$ is minimally acceptable at $n'$ if $EU(\phi(n')) = \bar{U}$.

*Definition 2.* A pricing function $\phi(n)$, is the residual gain maximizing pricing function if there is no function $\psi(n)$, such that

$$\text{Max } EV(\psi(n)) > \text{Max } EV(\phi(n))$$

The above definitions make precise the notion that the OI will optimally choose the pricing function that makes him best off, but it must be acceptable to the OM or no contract is possible. Such a pricing function exists and moreover is Pareto efficient as stated in the following proposition:

*Proposition 3.* There exists a differentiable cumulative pricing function $\phi(n)$, which is (1) Pareto efficient; (2) minimally acceptable; and (3) residual gain maximizing among those price functions which are acceptable.

*Remark:* We first establish that for any differentiable price function it is possible to characterize explicitly the OM's response.

*Proof.* Given an arbitrary pricing function $\bar{\phi}(n)$, the OM's optimization problem is

$$\text{Max}_{n,z}\{\sum \pi_s^m U(Z - z + nX_s(n))\} \tag{27}$$

Subject to

$$\bar{\phi}(n) + (Z - z) \leq Z \tag{28}$$

Constraint (28) ensures that the OM's total investment in the firm does not exceed his endowment of the riskless security.[18] We simplify and assume that (28) is always binding and substitute directly into (27). The first-order (necessary) condition is

$$\sum \pi_s^m U'[X_s(n) + nX_s'(n) - \bar{\phi}'] = 0 \tag{29}$$

Rearranging terms:

$$\frac{\sum \pi_s^m U'[X_s(n) + nX_s'(n)]}{\sum \pi_s^m U'} = \bar{\phi}' \tag{30}$$

The left-hand side of (30) is the $MRS_{RL}^m$ as defined by (6), so (30) indicates that the OM invests up to the point where

$$MRS_{RL}^m = \bar{\phi}' \tag{31}$$

*Remark:* For any acceptable differentiable function, (31) is the OM's optimal reaction function. We now show that it is possible to find a Pareto efficient price function.

*Proof of (1):* Define a new pricing function $\phi(n)$ whose derivative (marginal price)

[18] For simplicity, we again assume that the OM's endowment is riskless and the OI's endowment is the risky security.

is pointwise equal to $MRS^m_{RL}$ thereby satisfying condition (31) for all feasible $n > 0$. Assuming that $\phi(n)$ is acceptable, we show that it is optimal for the OI to sell shares to the OM up to the point where $MRS^0_{RL} = \phi'$. Accordingly, $\phi$ now represents the amount received by the OI. The OI chooses the optimal number of shares to sell the OM by solving the following program:

$$\text{Max}_{n,z}\{\sum \pi^0_s V((N-n)X_s(n) + z)\} \tag{32}$$

subject to

$$z \le \phi(n) \tag{33}$$

and

$$\sum \pi^m_s U(Z - z + nX_s(n)) \ge \bar{U} \tag{34}$$

The first constraint (33) ensures that the OI receives no more than the OM pays. The second constraint (34) ensures that the OM obtains at least his endowed level of expected utility, $\bar{U}$. We simplify by assuming that constraint (33) is satisfied as an equality.

Substituting $\phi(n)$ for $z$ into (32) and differentiating with respect to $n$, the first-order condition is

$$\sum \pi^0_s V'[-X_s(n) + (N-n)X'_s(n) + \phi'] = 0 \tag{35}$$

Rearranging (34), one obtains

$$\frac{\sum \pi^0_s V'[X_s(n) - (N-n)X'_s(n)]}{\sum \pi^0_s V'} = \phi' \tag{36}$$

Thus,

$$MRS^0_{RL} = \phi' \tag{37}$$

Since $\phi' = MRS^m_{RL}$ for all $n$, it follows that there is some $n^*$ such that $MRS^m_{RL} = MRS^0_{RL}$. We have shown that $\phi(n)$ results in an equilibrium satisfying the Pareto efficiency condition (7). Q.E.D.

*Remark:* In the proof of Part (1) we assumed that $\phi(n)$ was acceptable. We now show that $\phi(n)$ is minimally acceptable.

*Proof of (2):* The price function defined implicitly by the following relation:

$$\sum \pi^m_s U\left(Z + nX_s(n) - \int_0^n P(t)\, dt\right) = \bar{U} \tag{38}$$

for all feasible $n$, is clearly minimally acceptable for all $n$, because the OM receives his endowment level of expected utility (see Definition 1). It can be shown that $\phi(n)$ is equivalent to $\int_0^n P(t)\,dt$ and $\phi' = P(n)$. This is done by differentiating (38) implicitly with respect to $n$ and verifying that, for all $n$, $P(n) = MRS^m_{RL}$. But that is identical with the definition of $\phi(n)$ in part (1). Q.E.D.

*Proof of (3).* To establish that of all acceptable pricing functions, $\phi(n)$ is the residual gain maximizing pricing function, we assume the contrary and derive a contradiction. Suppose that there is a price function $\psi(n)$ such that for some $n'$, $EV(\psi(n')) > EV(\phi(n^*))$, where $EV(\phi(n^*)) = \text{Max}\, EV(\phi(n))$. Suppose $n' = n^*$, then $\psi(n^*) > \phi(n^*)$. From (2), $\phi(n^*)$ is minimally acceptable, $\psi(n^*)$ therefore cannot be acceptable because the amount that the OI receives is the amount the OM pays. Alternatively, suppose that $n' \neq n^*$. If $EV(\psi(n')) > EV(\phi(n^*))$, it follows that $EV(\psi(n')) > EV(\phi(n'))$. But $\phi(n)$ is minimally acceptable for all $n$ including $n'$, and hence $\psi(n')$ cannot be greater than $\phi(n')$ and be acceptable. Q.E.D.

The cumulative pricing scheme leads to the Pareto efficient incentive structure because the OI need not discount the price of all shares in order to induce the OM to purchase an additional share. The discount is necessary in order to overcome the OM's risk aversion. The OI is willing to discount the price of the marginal share as long as the additional incentive effect accruing to the OI plus the revenue from the marginal share offsets the value to the OI of the marginal share. This can be seen more clearly when the OI is risk neutral, since (36) upon simplification and rearrangement becomes

$$\bar{X}(n) = \phi' + (N - n)\bar{X}'(n)$$

The term on the left is the value of the marginal share to the OI. The terms on the right-hand side are the price per marginal share paid by the OM and the marginal incentive effect accruing to the OI.

At the formal level, one can draw an analogy between the problem of pricing securities to a manager and the neoclassical monopolist's pricing problem. Previously, we considered the single price contract. The OI's problem in the case of a single price contract was to choose a price that maximized his expected utility given the OM's demand function. Similarly, a monopolist picks a price that maximizes profits subject to market demand. The comparison can be made even closer if we reformulate the monopolist's problem so that he faces an individual's demand function instead of a market demand curve. In both cases, a single share price is neither Pareto efficient, nor residual gain (value) maximizing. Of course, the pricing of shares to a manager is more complex than the monopolist's problem in that managerial ownership affects incentives.

One can also draw an analogy between the optimal cumulative pricing function $\phi(n)$ and perfect price discrimination of neoclassical monopoly theory (see Scherer [22]). Perfect or first degree price discrimination is not to be confused with interpersonal (second or third degree) price discrimination. Perfect price discrimination is intrapersonal price discrimination in which each individual consumer is faced with a price function that extracts all of his consumer surplus. The OI's problem is to choose a price function that maximizes his expected utility subject to the constraint that the OM's expected utility must be greater than or equal to some minimum level. Likewise, a perfectly discriminating monopolist must choose a price function that maximizes his wealth subject to the constraint that the consumer's utility must be greater than or equal to some minimum level. We showed that $\phi(n)$ is residual gain maximizing, Pareto efficient, and minimally acceptable. The perfectly discriminating price function is well known to result in an allocation that is (from the viewpoint of the monopolist) residual gain maximizing, Pareto efficient, and minimally acceptable to the consumer.[19] Also, by construction, $\phi'(n)$ equals $MRS^m_{RL}$ for all $n$. An analogous condition characterizes the perfectly discriminating pricing scheme. An important difference between the two pricing functions is that in the present model the amount of the good purchased, $n$, affects the output $X_s(n)$ in each state.

The analogy between the managerial incentive pricing and neoclassical monopoly pricing models raises two important questions. First, since the OM is indifferent between all feasible $n$, will he in fact choose $n^*$, the Pareto efficient ownership level? Second, does this pricing scheme imply monopolistic exploitation of the OM? With regard to the first question, the OI can provide an inducement $\varepsilon$, for the OM to choose $n^*$. Since any $\varepsilon > 0$ is sufficient for the OI to choose $n^*$, $\varepsilon$ can be made arbitrarily close to zero. With respect to the second question, it may be assumed that $\bar{U}$ takes into account the market value of the OM's services. Although such considerations are outside the context of the present model, in a more general model the OI must attract the OM from alternative opportunities by ensuring that the OM receives at least the competitive wage for his services. In the present model, we simply assume that such considerations are implicit in $\bar{U}$.[20] An alternative interpretation of Part (2) of Proposition 3 is that, despite the existence of a managerial incentive problem, the OM need not receive more than the competitive level of compensation.

## V. Conclusions

This study presented a simple model of the managerial incentive problem in the context of a stock market economy. We established a correspondence between the pricing of securities to a manager and monopoly pricing. Our analysis indicated that shares should be priced differently to the manager than other investors. Generally incentive pricing is required to take into account the manager's effect on productivity and his risk aversion. But simple (single price) discounting was found to be inefficient. A cumulative share pricing function, however, provided an effective means of compensating the manager for increased portfolio risk exposure. By varying the marginal share price, the cumulative share price function overcame managerial risk aversion and simultaneously maximized the residual expected payoff to the OI. The cumulative pricing function resembles a perfectly discriminating price scheme.

Many theoretical issues and assumptions involving the design of efficient managerial incentive contracts deserve further analytical study. We believe, however, that empirical research predicated on the present model is of interest.

[19] Through a lump sum transfer, one can obtain Pareto efficiency without the residual gain maximizing or minimally acceptable properties in the case of both pricing schemes. In the theory of price discrimination, it is also well known that nonresidual maximizing pricing schemes (from the viewpoint of the monopolist) are able to achieve an efficient output (see Scherer [21]). These multipart pricing schemes are possible in the present context.

[20] Alternatively, one may assume that the OM's salary is implicit in $\int_0^{n^*} \phi(t)\, dt$.

## REFERENCES


1. D. Baron. "A Model of the Demand for Investment Banking Services for New Issues." Northwestern University Working Paper, October 1980.
2. T. Campbell. "Optimal Investment Financing Decisions and the Value of Confidentiality." *Journal of Financial and Quantitative Analysis* (December 1979), pp. 913–24.
3. H. De Angelo. "Competition and Unanimity." *American Economic Review* 71 (March 1981), 18–27.
4. P. Diamond. "The Role of a Stockmarket in a General Equilibrium Model with Technological Uncertainty." *American Economic Review* 57 (September 1967), 759–73.
5. S. Ekern and R. Wilson. "On the Theory of the Firm in an Economy with Incomplete Markets." *Bell Journal of Economics* 5 (Spring 1974), 171–80.
6. E. Fama. "Agency Problems and the Theory of the Firm." *Journal of Political Economy* 88 (April 1980), 288–307.
7. S. Grossman and O. Hart. "Takeover Bids, the Free Rider Problem, and the Theory of the Corporation." *Bell Journal of Economics* 11 (Spring 1980), 42–64.
8. M. Harris and A. Raviv. "Optimal Incentive Contracts with Imperfect Information." *Journal of Economic Theory* 20 (April 1979), 231–59.
9. B. Holmstrom. "Moral Hazard and Observability." *Bell Journal of Economics* 10 (Spring 1979), 74–91.
10. M. Jensen and W. Meckling. "Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure." *Journal of Financial Economics* 3 (October 1976), 305–60.
11. H. Leland. "Production Theory and the Stockmarket." *Bell Journal of Economics* 5 (Spring 1974), 125–44.
12. ——— and D. Pyle. "Informational Asymmetries, Financial Structure, and Financial Intermediation." *Journal of Finance* 32 (May 1977), 371–87.
13. W. Lewellen. *The Ownership Income of Management.* New York: NBER, 1971.
14. T. Lewis. "Bonuses and Penalties in Incentive Contracting." *Bell Journal of Economics* 11 (Spring 1980), 292–301.
15. D. Mayers. "Portfolio Theory, Job Choice and the Equilibrium Structure of Expected Wages." *Journal of Financial Economics* 2 (January 1974), 23–42.
16. J. Mirrlees. "Notes on Welfare Economics, Information, and Uncertainty," Balch, McFadden, and Wu. In *Essays on Economic Behavior Under Uncertainty.* Amsterdam: North Holland, 1974.
17. S. Ross. "The Economic Theory of Agency: The Principal's Problem." *American Economic Review* 63 (May 1973), 67–84.
18. ———. "Options and Efficiency." *Quarterly Journal of Economics* 90 (February 1976), 75–90.
19. ———. "The Determination of Financial Structure: The Incentive-Signalling Approach." *Bell Journal of Economics* 8 (Spring 1977), 23–40.
20. ———. "Some Notes on Financial Incentive Signalling Models, Activity Choice and Risk Preference." *Journal of Finance* 33 (June 1978), 777–92.
21. M. Rubinstein. "An Aggregation Theorem for Securities Markets." *Journal of Financial Economics* 1 (January 1974), 225–44.
22. F. M. Scherer. *Industrial Market Structure and Economic Performance.* Second ed. Chicago: Rand McNally College Publishing Company, 1980, p. 315.
23. S. Shavell. "Risk Sharing and Incentives in the Principal and Agent Relationships." *Bell Journal of Economics* 10 (Spring 1979), 55–73.
24. T. Zorn. "Information Differences, the Stockmarket and Managerial Incentives." Dissertation, University of California, Los Angeles, 1978.