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On Unit Roots and the Empirical Modeling of Exchange Rates

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ABSTRACT

Tests are conducted for the presence of unit roots in the autoregressive representations of the logarithms of spot and forward exchange rates. The results from these tests provide one explanation for some of the conflicting conclusions which emerge from recent empirical papers on the foreign exchange market.

ESTIMATION AND INFERENCE IN rational expectations models of exchange rate determination require assumptions about the stationarity of exchange rates and their determinants for at least two important reasons. First, most theoretical rational expectations models rely on a stationarity assumption to derive time invariant representations of exchange rates in terms of conditional expectations of the explanatory variables. Second, the validity of the large-sample inference procedures often requires that the variables be stationary and ergodic.[1] Yet in spite of the importance of stationarity assumptions, little effort has been directed towards formally investigating whether spot and forward exchange rates are covariance stationary processes.

The purpose of this paper is to consider one potentially important deviation from stationarity. Specifically, we formally test for the presence of unit roots in the autoregressive representations of spot and forward exchange rates. There is presently disagreement (at least implicitly) in the literature about whether exchange rate processes have unit roots. On the one hand, Frenkel [7, 8, 9, 10], Bilson [1], and Frankel [6] estimate models that assume exchange rates are stationary in the levels or logarithms of the levels. On the other hand, Bilson [2], Cornell [3], Cumby and Obstfeld [4], Geweke and Feige [12], Hakkio [16], Tryon [29], and Hansen and Hodrick [17, 18], among others, consider models of differences of spot and forward exchange rates.[2] Whether or not levels or differences of exchange rates are assumed to be stationary can lead to substantially different

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[1] A time series $\{x_t\}$ is said to be covariance stationary if its first and second moments are finite and $\text{COV}(x_t, x_{t+s})$ depends only on s. For a heuristic discussion of the concept of ergodicity, see Nerlove, Grether, and Carvalho [26], pp. 23-26.

[2] Both the time difference of a single exchange rate (e.g. $s_{t+n} - s_t$) and the difference between spot and forward rates at different points in time (e.g., $s_{t+n} - f_t$) have been considered.

conclusions (e.g., when testing whether forward rates are unbiased predictors of future spot rates) and, hence, a test for unit roots seems warranted.[3]

The remainder of this paper is organized as follows. In Section I we briefly discuss tests for the presence of unit roots, and then present results for the forward and spot dollar exchange rates with three other currencies. Some implications of our results are discussed in Section II.

I. Tests for Unit Roots

Testing for the presence of unit roots in the autoregressive representation of a time series amounts to testing whether certain coefficients are unity. The standard t- and F-statistics do not lead to valid large sample inferences under the null hypothesis, however.[4] The requisite large sample distribution theory for testing for unit roots has recently been developed by Fuller [11], Dickey and Fuller [5], and Hasza and Fuller [19]. We briefly describe their test procedures before presenting our results for exchange rates.

Suppose the time series $\{y_t\}$ has the representation

$$y_t = \beta_0 + \beta_1 t + \sum_{s=1}^{p} \alpha_s y_{t-s} + \epsilon_t \qquad t = 1, 2, \cdots, \tag{1}$$

where $\{\epsilon_t\}$ is a sequence of independent and identically distributed random variables with mean zero and variance σ^2. The initial conditions $(y_{-p+1}, y_{-+2}, \cdots, y_0)$ are assumed to be known constants.[5] Let

$$\lambda^p - \sum_{s=1}^{p} \alpha_s \lambda^{p-s} = 0 \tag{2}$$

be the characteristic equation of the y process. The roots of (2), denoted by λ_1,

[3] The nonstationarity of exchange rates and their underlying determinants do not necessarily result in inconsistent estimates of the parameters. In particular, let $y_t = z_t'\beta + \epsilon_t$, $t = 1, 2, \cdots$, where the ϵ_t's are independent, ϵ_t has zero mean and variance $\sigma^2 < +\infty$, and the $p \times 1$ vector z_t satisfies $\text{COV}(z_{t-s}, \epsilon_t) = 0$, $s \geq 0$. Also, let $A_T = \sum_{t=1}^{T} z_t z_t'$ and a_{ii}^T be ith diagonal element of A_T. If $a_{ii}^T \to \infty$ as $T \to \infty$, $i = 1, \cdots, p$, then the ordinary least squares estimator of β will be consistent. This condition will be satisfied, almost surely, when $\{z_t\}$ has a finite autoregressive representation with roots inside, on, or outside the unit circle.

More structure is required, however, to obtain an asymptotic distribution theory for inference. In the context of this linear regression model, some of the weakest conditions on the moment matrix of explanatory variables for which asymptotic normality of the coefficient vector obtains are due to Grenander [14]. These conditons preclude exponential growth of any variable, so unstable processes are ruled out. Borderline nonstationarity (i.e., a unit root) is allowed if regressors are fixed or strictly exogenous. In addition, if one assumes that the regression disturbances are independent normal random variables, then normality of the estimators may obtain in the explosive root case; see for example, Fuller, Hasza, and Goebel [12]. Note that an independent error ϵ_t with finite variance is not sufficient for asymptotic inference; conditions on the regressors are also required.

Much of the empirical work cited above includes current and lagged values of the forward rate or $s_{t+n} - f_t$ as regressors in tests of the proposition that forward rates are unbiased predictors of future spot rates. Thus a relevant question is whether these predetermined, but not exogenous, regressors are stationary.

[4] In the regression equation $y_t = \rho y_{t-1} + \epsilon_t$, with $|\rho| < 1$ and ϵ_t a classical disturbance term, the variance of the limiting distribution of $\hat{\rho}$ (the OLS estimator of ρ) is $(1 - \rho^2)$. Clearly, an asymptotic t-test of $\rho = 1$ is not well defined.

[5] Because these initial conditions are assumed to be known constants, the tests discussed below are not the most powerful tests against the alternative that $\{y_t\}$ is a stationary autoregressive process. See Hasza and Fuller [19] for a further discussion of this point.

$\cdots$, λ_p, are the characteristic roots of the process. Consider first the problem of testing the null hypothesis that $\lambda_1 = \lambda_2 = 1$ and $|\lambda_j| < 1$, for $j = 3, 4, \cdots, p$. Let Δ denote the different operator, $\Delta y_t \equiv y_t - y_{t-1}$, and rewrite (1) as

$$y_t = \beta_0 + \beta_1 t + \beta_2 y_{t-1} + \beta_3 \Delta y_{t-1} + \sum_{s=1}^{p-2} \delta_s \Delta^2 y_{t-s} + \epsilon_t \tag{3}$$

where the roots of

$$\lambda^{p-2} - \sum_{s=1}^{p-2} \delta_s \lambda^{p-s-2} = 0$$

are less than one in absolute value. Then the characteristic equation (2) of $\{y_t\}$ will have two unit roots if and only if $\beta_2 = \beta_3 = 1$. Hasza and Fuller [19] have derived the requisite statistic (their $\Phi_3(2)$) for testing the null hypothesis $H_0^{(1)}: \beta_2 = \beta_3 = 1$. Further, they report estimates of the percentiles of the test statistics that can be used in actual applications.

The test of the null hypothesis of a single unit root proceeds in an analogous fashion. Equation (1) is rewritten as

$$y_t = \beta_0 + \beta_1 t + \beta_2 y_{t-1} + \sum_{s=1}^{p-1} \delta_s \Delta y_{t-s} + \epsilon_t \tag{4}$$

where the roots of

$$\lambda^{p-1} - \sum_{s=1}^{p-1} \delta_s \lambda^{p-s-1} = 0$$

are less than one in absolute value. Under the null hypothesis of a unit root, $\beta_2 = 1$. Fuller [11] and Dickey and Fuller [5] derive a test statistic ($\hat{\tau}_\tau$) for testing $H_0^{(2)}: \beta_2 = 1$, and tabulate empirical percentiles for this statistic.

We conducted tests of $H_0^{(1)}$ and $H_0^{(2)}$ for the univariate autoregressive (AR) representations of the logarithms of the spot ($\ln s_t$) and forward ($\ln f_t$) exchange rates.[6] The weekly data are Wednesday twelve o'clock Swiss franc-U.S. dollar, German mark-U.S. dollar, and Canadian dollar-U.S. dollar spot bid rates, and the midpoint of the twelve o'clock bid-ask spread on three-month forward rates. These data, collected by the Federal Reserve System as part of their daily monitoring of the New York foreign exchange market, are for the period beginning 7 January 1976 and ending 8 July, 2 July, and 24 June 1981 for Switzerland, Canada, and Germany, respectively. For each exchange rate, autoregressions of orders one through eight were estimated. In the regressions with $p > 1$, the estimated coefficients ($\delta_1, \cdots, \delta_{p-2}$) in (3) and ($\delta_2, \cdots, \delta_{p-1}$) in (4) were always small and insignificantly different from zero at conventional significance levels. Accordingly, we report only those results for the following special case of (3) and (4):

$$y_t = \beta_0 + \beta_1 t + \beta_2 y_{t-1} + \beta_3 \Delta y_{t-1} + \epsilon_t \tag{5}$$

In terms of the parameters of (5), the null hypotheses are $H_0^{(1)}: \beta_2 = \beta_3 = 1$ and $H_0^{(2)}: \beta_2 = 1, |\beta_3| < 1$.[7]

[6] Depending on the model of interest, one may wish to test for unit roots in the multivariate representations of $\ln s_t$, $\ln f_t$, and other variables. Unfortunately, this is a substantially more difficult problem than the one considered here, and to date the requisite distribution theory has not been developed.

[7] The results from the estimation of equations including additional lags were qualitatively the same as those reported below.

Table I

Tests for Unit Roots in the Autoregressive Representations of the Logarithms of Spot and Forward Rates

Country	y_t	T^a	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	R^{2b}	$\Phi_3(2)$	$\hat{\tau}_\tau$
Switzerland	$\ln s_t$	286	.2333	.0113	.9992	.0761	.9897	123.28	−.0861
			(.3839)	(.0010)	(.0093)	(.0661)			
	$\ln f_t$	286	.2160	−.0012	.9999	.0065	.9903	125.39	−.0111
			(.3784)	(.0010)	(.0090)	(.0602)			
Canada	$\ln s_t$	285	1.6774	−.0011	.9826	.1283	.9937	1357.5	−.9080
			(1.0969)	(.0008)	(.0109)	(.0591)			
	$\ln f_t$	285	2.0859	−.0013	.9784	.1353	.9926	1996.5	−1.826
			(1.1783)	(.0009)	(.0118)	(.0591)			
Germany	$\ln s_t$	284	−.1346	−.0015	1.0075	.0977	.9910	169.37	.8467
			(.3611)	(.0007)	(.0088)	(.0609)			
	$\ln f_t$	284	−.1606	−.0016	1.0082	.0883	.9916	184.53	.9554
			(.3514)	(.0007)	(.0086)	(.0609)			

[a] T = Sample size.
[b] R^2 = Coefficient of determination. Standard errors of the coefficients are reported in parentheses.

The results are presented in Table I. The critical value for a 1%-level test of $H_0^{(1)}$ with sample size $T = 250$ is 11.70 (Hasza and Fuller [19] p. 1116). Thus, the large values of $\Phi_3(2)$ in Table I indicate rejection of the null hypothesis of two unit roots at well below the one percent significance level. In contrast, the values of the test statistics $\hat{\tau}_\tau$ provide little evidence against the null hypotheses that these autoregressions have one unit root. The critical value for a 10% (5%)-level test of $H_0^{(2)}$ with $t = 250$ is $\hat{\tau}_\tau = -3.13$ (-3.43) (Fuller [11] p. 373), with the rejection region given by small values of $\hat{\tau}_\tau$. These results suggest that $\ln s_t$ and $\ln f_t$ do not have stable univariate AR representations, even after removing a linear trend.

For the purpose of comparison, we also conducted tests for unit roots in the series $\{\ln s_{t+n} - \ln f_t\}$, where n is the number of weeks to maturity of the forward contract.[8] This variable has been the regressand in some of the empirical studies of the proposition that forward rates are unbiased predictors of future spot rates cited in the introduction. Before conducting these tests, we first attempted to account for the institutional features of foreign exchange markets which induce irregularities in the alignment of the data. This alignment problem has been largely ignored in previous empirical studies of exchange markets. Misalignment derives from the fact that when two parties agree to a spot transaction of Swiss francs for dollars, for example, the transaction is for a value date two business

[8] Suppose the null hypothesis that the forward rate is an unbiased predictor of the future spot rate is true. Then, if the sampling interval is finer than the maturity of the forward contract, the series $\{\ln s_{t+n} - \ln f_t\}$ will follow a moving average process; see, for example, Hansen and Hodrick [17]. In this case, the univariate autoregressive representation of this series will be of infinite order. Since recent empirical work has cast doubt on the validity of the null hypothesis, we have no priors on the order of the univariate AR representation of $\{\ln s_{t+n} - \ln f_t\}$. For the three countries we consider, the estimated coefficients on lags of 4 or more in the AR representations are small and statistically insignificant from zero. For this reason, Table II displays only the results for 3 lags of the dependent variable.

Table II[a]

Tests for Unit Roots in the Autoregressive Representations of $\{\ln s_{t+n} - \ln f_t\}$

Country	T	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	R^2	$\Phi_3(2)$	$\hat{\tau}_\tau$
Switzerland	286	.9618 (.1578)	−.866−3 (.0010)	.9358 (.0212)	.1585 (.0597)	.9021	117.08	−3.028
Canada	285	−.1091 (.0910)	.256−3 (.545−3)	.8989 (.0249)	.0832 (.0587)	.8298	148.96	−4.057
Germany	284	.1614 (.1109)	−.1408−2 (.7374−3)	.9197 (.0224)	.1661 (.0594)	.9097	123.32	−3.590

[a] See explanatory notes for Table I.

days following the day the transaction was closed (Riehl and Rodriquez [27]). To be an eligible spot value date, it must be a business day in both the United States and Switzerland as both currencies are involved in the transaction. Similarly, today's U.S. dollar-Swiss franc three-month transaction involves settlement of a contract three months from the corresponding spot value date, provided it is an eligible day. The future spot rate that corresponds to this forward value date was contracted two business days prior to the forward value date. The spot and forward rates used in this study were aligned using back issues of Morgan Guaranty Trust Company's *World Calendar of Holidays.*

The results of the tests, which are displayed in Table II, suggest that the series $\{\ln s_{t+n} - \ln f_t\}$ for Canada and Germany have stable univariate AR representations. On the other hand, the results are consistent with the presence of a single unit root in the series for Switzerland.

II. Discussion

The values of the tests statistics for $H_0^{(2)}$, together with the fact that the coefficients in the autoregressive representations are small beyond the first lag, support Mussa's [24] conjecture that the processes generating the natural logarithms of exchange rates are well approximated by random walks.[9] Cornell [3] arrived at a similar conjecture for the levels of exchange rates.

Furthermore, and perhaps most importantly, these findings may have implications for a number of recent empirical studies of exchange rate behavior. Specifically, empirical tests of the null hypothesis that forward rates are unbiased predictors of future spot rates should be conducted using transformations of s_t and f_t which render these series stationary. If the results of Table I are taken to imply that $\ln f_t$ is a nonstationary process, then conventional tests of this null hypothesis based on a regression of $\ln s_{t+n}$ on the logarithm of the n-period forward rate, $\ln f_t$, may be inappropriate, since the asymptotic distribution theory employed may not be valid. Frenkel [7, 8, 9, 10] has frequently employed this

[9] Formal procedures for estimating lag length (Geweke and Meese [14]) also suggest that exchange rates follow first-order AR processes. These procedures assume the AR processes are stable and of finite but unknown order.

methodology and his test results are generally supportive of the null hypothesis. In contrast, those authors who have recently found evidence of systematic differences between forward rates and expected future spot rates (Bilson [2], Cumby and Obstfeld [4], Geweke and Feige [13], Hakkio [16], Hansen and Hodrick [17, 18], and Tryon [29]), use regressors of the form ($\ln s_{t+n} - \ln f_t$). The evidence from the unit root tests for the series $\{\ln s_{t+n} - \ln f_t\}$ suggest that the stationarity assumptions underlying the latter studies may be more appropriate.[10]

Similar issues may arise when conducting inference in "structural" models of exchange rates. Bilson [1] and Frankel [6] studied models of the exchange rate which assumed that logarithms of exchange rates were stationary. In the context of their models, the stationarity assumption amounts to assuming a linear combination of money, income, and interest rate variables, say x_t, is stationary. More recently, Huang [20] tested and rejected a version of Bilson's [1] monetary model in which $\{\Delta x_t\}$ and $\{\ln s_t - x_t\}$, but not $\{\ln s_t\}$, were assumed to be stationary processes.

[10] Another credible explanation of the recent findings is that many of the authors in the second group consider larger information sets than Frenkel considered. For example, both Geweke and Feige [13] and Hansen and Hodrick [17] employ series for several bilateral exchange rates in their empirical analyses. In addition, Cumby and Obstfeld [4], Hakkio [16], and Hansen and Hodrick [17] sampled exchange rates on a weekly or biweeky basis, while Frenkel used monthly data.

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