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Sufficient Conditions for Public Information to Have Social Value in a Production and Exchange Economy

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ABSTRACT

Conditions are derived under which all consumers in a production and exchange economy will prefer (at least weakly) disclosure of public information to no such disclosure. The conditions involve consumer endowments, the allocative efficiency of the financial market, and value maximizing behavior by firms. Cases exist where consumers will prefer disclosure of public information in a production and exchange economy, although they would be indifferent to such disclosure in an otherwise similar pure exchange economy. The difference in results is due purely to the fact that in production and exchange economies, information may be used to reallocate resources across time and firms, thus highlighting the fundamental difference between the role of information in pure exchange and in production and exchange economies.

A NUMBER OF WRITERS have investigated the conditions under which public information (information that is simultaneously made available to all consumers in the economy) will have social value in pure exchange economies; that is, when public information will make no consumer worse off, and at least one consumer better off. See for example, Hirshleifer [4], Fama and Laffer [1], Marshall [8], Ng [9, 10, 11], Jaffee [6], Hakansson, Kunkel, Ohlson [3], Ohlson [12], and Ohlson, Buckman [13]. Under some circumstances public information will have social value in pure exchange. Under other circumstances it will not.

In pure exchange economies, public information serves the function of altering the risks perceived by the consumers and of altering the final risk sharing arrangements. Public information may not, by assumption, be used to alter the production plans of the firms in a pure exchange economy. Hence, the pure exchange analysis cannot capture one of the most important functions of information in any economy.

There exist very few results with respect to the social value of public information in production and exchange economies. Hirshleifer [4] gave an example in which perfect, public information made every consumer better off in a production and exchange economy where all consumers were identical. Starr [15] gave an example in which public information has no value if the production plans without information are still efficient production plans with information.

In this paper, we allow for endogenous determination of production plans, and provide conditions sufficient for public information to make no consumer worse

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off. Moreover, information will have social value (in a strict sense) if for at least one message from the public information system, firms change their production plans from what they would be in the absence of information.

These sufficient conditions for public information to have social value under production and exchange are essentially independent of consumers' preferences and beliefs, but they do depend upon consumer's endowments, the allocative efficiency of the financial market, and upon value maximizing behavior by firms.

One of our main insights is that there exist conditions under which public information will have social value in a production and exchange economy, even though there would be no social value in an otherwise similar pure exchange economy. The difference in results, which is due simply to the fact that in a production and exchange economy information may be used to alter and improve firms' production plans, highlights the fundamental difference between the role of information in pure exchange and in production and exchange economies.

I. Preliminaries

We consider a perfect, competitive production and exchange economy. The economy extends for two periods, and at the end of period one the economy will be in some state $s \in \{1 \cdots S\}$. There is a single commodity which may be consumed in period one, or may be invested in order to obtain state contingent consumption in period two.

There are I consumers (indexed $i = 1 \cdots I$) whose prior probability beliefs over the states are given by $\Pi_i^0 = (\Pi_{i1}^0 \cdots \Pi_{iS}^0)$, where $\Pi_{is}^0 > 0$ for all i and s.

The consumers' preferences are represented by the functions $T_i = \Sigma_s \Pi_{is}^0 U_{is}(C_i, W_{is})$, where C_i is the consumption of consumer i in the first period, and W_{is} is the consumption of consumer i in the second period if the economy is in state s at the beginning of that period. U_{is} is assumed to be monotone increasing in each argument and strictly concave; that is, consumers are nonsatiable and risk averse.

There are J firms in the economy (indexed $j = 1 \cdots J$). Each firm has a production function that is subject to technological uncertainty; that is, each firm's future output depends not only upon present input, but also upon the occurrence of one of the s possible states of nature. We denote the jth firm's production plan by $(x_j, a_{j1} \cdots a_{jS})$, where x_j is the input to the jth firm and a_{js} is the output of the jth firm in state s given input x_j. The jth firm's production in state s, a_{js}, is given by $f_{js}(x_j)$, where f_{js} is an increasing strictly concave function for all j and s.

Let A be the $J \times S$ matrix of firms' outputs across states. We assume that the rank of A is equal to S; that is, the financial market is complete relative to consumers' prior beliefs.

Consumers are endowed with claims to current consumption $\bar{C}_i$, as well as ownership shares of firms, $\bar{Z}_{ij}$, where $\Sigma_i \bar{Z}_{ij} = 1$. At time 0, consumers may trade from their endowed positions to purchase consumption in period one, as well as future consumption in every state s by purchasing shares of the firms in the economy. The future consumption in state s of consumer i is given by $W_{is} = \Sigma_j Z_{ij} a_{js}$, where Z_{ij} is his final ownership share of firm j.

If no public information is made available to the consumers in the economy, the consumers trade at time 0 from their endowed positions using their prior beliefs.

Assume for the moment that the production plans $(x_j, a_{j1} \cdots a_{jS})$ are fixed for every firm j in the economy. Let P_s denote the implicit price of wealth in state s. Let $v_j = \Sigma_s P_s a_{js}$ be the value of the jth firm's aggregate output across states. Let $V_j = \Sigma_s P_s a_{js} - x_j = v_j - x_j$ be the *net* market value of the jth firm.

We define an exchange equilibrium relative to these production plans and relative to consumers' prior beliefs Π_i^0, and endowments $\bar{C}_i$, $\bar{Z}_{ij}$, to be a set of portfolio holdings C_i^0, Z_{ij}^0, such that every consumer solves the following problem:

$$T_i = \max_{C_i \{Z_{ij}\}} \Sigma_s \Pi_{is}^0 U_{is}(C_i, W_{is})$$

$$\text{subject to:} \quad C_i + \Sigma_j Z_{ij} v_j \leqq \bar{C}_i + \Sigma_j \bar{Z}_{ij} V_j$$

$$\text{where} \qquad W_{is} = \Sigma_j Z_{ij} a_{js} \quad \text{for all} \quad s.$$

The first order conditions for an optimal solution are given by:

$$\Sigma_s \Pi_{is}^0 \frac{\partial U_{is}(C_i^0, W_{is}^0)}{\partial C_i^0} = \lambda_i^0 \quad \text{for all} \quad i \tag{1}$$

$$\Sigma_s \Pi_{is}^0 \frac{\partial U_{is}(C_i^0, W_{is}^0)}{\partial W_{is}^0} a_{js}^0 = \lambda_i^0 v_j^0 \quad \text{for all} \quad i, j \tag{2}$$

$$C_i^0 + \Sigma_j Z_{ij}^0 v_j^0 = \bar{C}_i + \Sigma_j \bar{Z}_{ij}(v_j^0 - x_j^0) \quad \text{for all} \quad i \tag{3}$$

where λ_i^0 is the Lagrange multiplier associated with the ith consumer's budget equation.

The market clearing conditions are given by:

$$\Sigma_i \bar{C}_i = \Sigma_i C_i^0 + \Sigma_j x_j^0 \tag{4}$$

$$\Sigma_i Z_{ij}^0 = 1 \quad \text{for all} \quad j \tag{5}$$

$$\text{where} \quad \Sigma_i W_{is}^0 = \Sigma_i \Sigma_j Z_{ij}^0 a_{js}^0 = a_s^0 \quad \text{for all} \quad s$$

The budget equations need some interpretation. They may be written in the following form:

$$C_i^0 + \Sigma_j Z_{ij}^0 (\Sigma_s P_s^0 a_{js}^0 - x_j^0) + \Sigma_j Z_{ij}^0 x_j^0 \leqq \bar{C}_i + \Sigma_j \bar{Z}_{ij} (\Sigma_s P_s^0 a_{js}^0 - x_j^0)$$

The final shareholders of firm j may be viewed as purchasing a "shell" from the initial shareholders for an aggregate amount $(\Sigma_s P_s^0 a_{js}^0 - x_j^0)$. The final shareholders provide input x_j^0 to each firm j, in exchange for future output across states $a_{j1}^0 \cdots a_{jS}^0)$.

The budget equation may be given a second interpretation if we write it in the following equivalent form:

$$C_i^0 + \Sigma_j Z_{ij}^0 (\Sigma_s P_s^0 a_{js}^0) \leqq \bar{C}_i + \Sigma_j \bar{Z}_{ij} (\Sigma_s P_s^0 a_{js}^0 - x_j^0)$$

For each firm j, initial shareholders contribute an aggregate input x_j^0 in exchange for claims on future output across states with an aggregate value $(\Sigma_s P_s^0 a_{js}^0)$. These claims on future output may be sold (along with remaining endowed current consumption) to purchase new claims on present and future consumption.

It is well known that under our assumptions, net value maximizing firms acting as price takers in a financial market that is complete with respect to prior beliefs, choose production plans that are efficient with respect to prior beliefs.

Assume that the firms in the economy maximize their net market value so that each firm solves the following problem:

$$V_j = \max_{x_j} \{v_j - x_j\}$$

$$= \max_{x_j} \{\Sigma_s P_s f_{js}(x_j) - x_j\}$$

The first order conditions are given by:

$$\Sigma_s P_s^0 \frac{df_{js}(x_j^0)}{dx_j^0} = 1 \quad \text{for all} \quad j \tag{6}$$

We define a production and exchange equilibrium relative to consumers' prior beliefs Π_i^0, and endowments $\bar{C}_i$, $\bar{Z}_{ij}$, to be a set of production plans $(x_j^0, a_{j1}^0 \cdots a_{jS}^0)$ for all j and consumer portfolio holdings C_i^0, $W_{is}^0 = \Sigma_j Z_{ij}^0 a_{js}^0$ for all i and s such that conditions (1)–(6) are satisfied.

Let $T_i^0 = \Sigma_s \Pi_{is}^0 U_{is}(C_i^0, W_{is}^0)$ be the ith consumer's expected utility given the optimal prior belief allocations.

We have defined this production and exchange equilibrium only for consumers' prior beliefs. In a similar manner we could define a production and exchange equilibrium relative to any set of consumers' posterior beliefs for a given endowment.

Now consider an otherwise similar economy except that at time 0 all consumers have the opportunity to simultaneously observe a message y (indexed $y = 1 \cdots Y$) from a public information system *prior* to trading from their endowed positions.

Let Π_{is}^y be the ith consumer's posterior probability that state s will occur given message y. We require that either $\Pi_{is}^y > 0$ for all i and s, or that if $\Pi_{is}^y = 0$ for some i, then $\Pi_{is}^y = 0$ for all i.

In general, there will be a different allocation for each message y, assuming that consumers trade at time 0 from their endowed positions. For any message y, let S^y be a number of states with positive posterior probability for all consumers. Let A^y be the $J \times S^y$ matrix of firms' output across those states with positive posterior probability. Following Ohlson, Buckman [13], we say the financial market is *conditionally complete* if the rank of A^y is equal to S^y for all y. If some states have zero posterior probability, then fewer securities will be needed to complete the financial market once a message is released from the information system.

We assume that firms maximize net market value for each message y. Let $\{C_i^y, W_{is}^y\}$ be the ith consumer's allocation of present consumption and state contingent future consumption implicit in portfolio choices Z_{ij}^y and firm output vectors $(a_{j1}^y \cdots a_{jS}^y)$ assuming that consumers have traded from their endowed positions, where $W_{is}^y = \Sigma_j Z_{ij}^y a_{js}^y$.

Let $T_i^y = \Sigma_s \Pi_{is}^y \, U_{is}(C_i^y,\, W_{is}^y)$ be the ith consumer's expected utility given message y. Then, the ith consumer's ex ante expected utility given the public information system is given by:

$$
\begin{aligned}
T_i(\text{info}) &= \Sigma_y \Pi_i(y) T_i^y \\
&= \Sigma_y \Pi_i(y) \{\Sigma_s \Pi_{is}^y U_{is}(C_i^y,\, W_{is}^y)\} \\
&= \Sigma_y \Sigma_s \Pi_i(s,y) U_{is}(C_i^y, W_{is}^y)
\end{aligned}
$$

where $\Pi_i(y)$ is the ith consumer's probability that message y will be released from the information system, and $\Pi_i(s,\, y)$ is the ith consumer's joint probability that s and y will occur.

Information makes no consumer worse off if, and only if

$$T_i(\text{info}) \geqq T_i^0 \quad \text{for all} \quad i$$

and information has social value if, and only if, the above equation holds for all i, with strict inequality for at least one i.

With these remarks in mind, we are ready to state our main result.

II. Social Value in Production and Exchange

Theorem

For public information to make no consumer worse off under production and exchange, it is sufficient that:

(1) endowments of current consumption and firm shares are equilibrium positions with respect to consumers' prior beliefs, i.e.,

$$\bar{\bar{C}}_i = C_i^0 \quad \text{for all} \quad i$$

$$\bar{\bar{Z}}_{ij} = Z_{ij}^0 \quad \text{for all} \quad i,\, j$$

where the $\bar{\bar{C}}i$, $\bar{\bar{Z}}_{ij}$, notation is used since the above endowment assumption assumes the ith consumer has already contributed $\Sigma_j Z_{ij}^0 x_j^0$ to the firms in the economy.

(2) No consumer is endowed with a negative (short) position in any firm, i.e.,

$$\bar{\bar{Z}}_{ij} = Z_{ij}^0 \geqq 0 \quad \text{for all} \quad i,\, j$$

(3) The financial market is complete with respect to prior beliefs, and conditionally complete.[1]

(4) Firms maximize their net market value.

Proof

The proof will be by contradiction. We shall show that if one consumer is made worse off by information, then this will contradict value maximization behavior on the part of firms.

[1] In general, the financial market may be incomplete with respect to prior beliefs if negative positions in securities are not allowed, even though the market matrix A is of rank S. Hence, assumption (2) requires that consumer preferences and beliefs and firms' production technologies are such that no consumer desires to hold a negative position in any firm. An example of when no consumer desires to hold a negative position in any firm is when consumers have homogeneous beliefs and they have linear risk tolerance utility functions.

Suppose there exists a consumer i who is made worse off by the information so that

$$\Sigma_y \Pi_i(y)\{\Sigma_s \Pi_{is}^y U_{is}(C_i^y, W_{is}^y)\} < \Sigma_s \Pi_{is}^0 U_{is}(C_i^0, W_{is}^0)$$
$$= \Sigma_y \Pi_i(y)\{\Sigma_s \Pi_{is}^y U_{is}(C_i^0, W_{is}^0)\}$$

This implies that for at least one y

$$\Sigma_s \Pi_{is}^y U_{is}(C_i^y, W_{is}^y) < \Sigma_s \Pi_{is}^y U_{is}(C_i^0, W_{is}^0)$$

By assumption, the financial market A^y is complete for each y. Any vector of W_{is}^0 consumption claims with positive posterior state probability (i.e., $\Pi_{is}^y > 0$) must lie within the space generated by A^y. Since the consumer prefers (with respect to Π_{is}^y) his prior belief consumption plan across time and states, the reason he did not purchase his prior belief consumption plan must be that it was too expensive. Therefore, it must be true that

$$C_i^0 + \Sigma_s P_s^y W_{is}^0 > W_{0i}^y \quad \text{for at least one} \quad y \tag{7}$$

where P_s^y is the implicit price of wealth in state s given message y, and W_{0i}^y is the ith consumer's initial wealth given message y.

The ith consumer's initial wealth given message y may be written

$$W_{0i}^y = [C_i^0 - \Sigma_j Z_{ij}^0(x_j^y - x_j^0)] + \Sigma_j Z_{ij}^0(\Sigma_s P_s^y a_{js}^y)$$
$$= C_i^0 + \Sigma_j Z_{ij}^0 x_j^0 + \Sigma_j Z_{ij}^0(\Sigma_s P_s^y a_{js}^y - x_j^y)$$

Condition (7) implies:

$$C_i^0 + \Sigma_s P_s^y W_{is}^0 > C_i^0 + \Sigma_j Z_{ij}^0 x_j^0 + \Sigma_j Z_{ij}^0(\Sigma_s P_s^y a_{js}^y - x_j^y)$$

But $W_{is}^0 = \Sigma_j Z_{ij}^0 a_{js}^0$ which implies:

$$C_i^0 + \Sigma_j Z_{ij}^0(\Sigma_s P_s^y a_{js}^0 - x_j^0) > C_i^0 + \Sigma_j Z_{ij}^0(\Sigma_s P_s^y a_{js}^y - x_j^y) \tag{8}$$

But Condition (8) cannot be satisfied since by definition of value maximization $(\Sigma_s P_s^y a_{js}^y - x_j^y) \geqq (\Sigma_s P_s^y a_{js}^0 - x_j^0)$ for all firms j, and by assumption, $Z_{ij}^0 \geqq 0$ for all i, j.

This completes the proof.

We note that this theorem is the production and exchange analogue to Theorem 1 of Hakansson, Kunkel, Ohlson [3]. The fact that information can make no consumer worse off if consumers' endowments are equilibrium positions with respect to their prior beliefs is obviously true in the pure exchange economy analyzed in Theorem 1 of Hakansson, Kunkel, Ohlson [3], since for each message y, every consumer can always refuse to trade and hold his endowed position.

The proof is more subtle in a production and exchange economy since consumers are endowed with shares of firms rather than direct claims on future consumption. Owning a share of a firm does not guarantee one a certain pattern of consumption across states if information causes implicit prices to change, and hence causes value maximizing firms to change their production plans. However, if the financial market is conditionally complete, there will be equality across consumers of implicit prices of wealth in each state. Consumers will assign the same value to changes in firms' production plans, and no consumer can be made worse off by information.

Another very important difference between the pure exchange and the production and exchange analyses is as follows: in Lemma 2 of Hakansson, Kunkel, Ohlson [3] it is shown that if: (1) endowments are equilibrium positions with respect to prior beliefs; (2) consumers have essentially homogeneous conditional and unconditional message beliefs;[2] (3) the financial market generates Pareto efficient allocations with respect to prior beliefs; and (4) consumers have time additive utility functions, then information cannot have social value in a two period pure exchange economy, since these assumptions guarantee that consumers will not trade from their endowed positions. Indeed, if Conditions (1) and (3) hold, there will be equality across consumers of marginal rates of substitution between present and future consumption in any state, where marginal rates of substitution (implicit prices) are calculated using prior beliefs. If, in addition, Assumptions (2) and (4) hold, there will *still* be equality across consumers of marginal rates of substitution (using the posterior beliefs) after the release of a message but prior to trade. In Lemma 2 of Hakansson, Kunkel, Ohlson [3], a message from the information system causes marginal rates of substitution to change, but since these marginal rates of substitution remain equal across consumers, there is no room for trade.

In a production and exchange economy, output is not fixed. A message from the information system changes consumers' beliefs, and therefore the marginal rates of substitution. The change in consumers' marginal rates of substitution serves as a signal for value maximizing firms to change their production plans. Trade will take place and information will have social value even though the assumptions of Lemma 2 of Hakansson, Kunkel, Ohlson [3] hold. Thus we see a substantial difference in results due purely to the fact that in a production and exchange economy, information may be used to reallocate resources across time and firms.

Although we have only proven that no consumer can be made worse off by information in our production and exchange economy, in general, belief revision by consumers will change marginal rates of substitution and cause firms to change their production plans and information will have social value in the usual sense.

Perhaps the most "nonstandard" assumption is that endowments (which are equilibrium positions with respect to prior beliefs) are such that no consumer holds a negative position in any firm. The theorem may still hold if we only assume that consumers hold positive levels of consumption in every state so that short positions in some firms are balanced by long positions in other firms. However, even though under our assumptions value maximizing firms choose

[2] In Hakansson, Kunkel, Ohlson [3] unconditional message beliefs $\Pi_i(y)$, and conditional message beliefs $\Pi_i(y/s)$ are essentially homogeneous if

$$\Pi_i(y) = k_i(y)\Pi_1(y) \qquad i = 2 \cdots I$$

and if

$$\Pi_i(y/s) = k_i(y)\Pi_1(y/s) \qquad i = 2 \cdots I$$

where $k_i(y)$ is a constant.

production plans that are Pareto efficient with respect to each belief system, we do not expect value maximization behavior by a particular firm to benefit those consumers with negative holdings in that firm.

We have considered an economy in which there was no message-contingent trade *prior* to the release of a message. In such an economy, there is no opportunity for consumers to share risk across messages, nor is there an opportunity for firms to co-ordinate production plans across messages. The economy will have *local* efficiency properties in the sense that the allocations will be efficient (with respect to posterior beliefs) for each message, but there is no opportunity for consumers to equate their marginal rates of substitution across *both* the states $s = 1 \cdots S$, *and* the messages. Nor is there an opportunity for firms to co-ordinate their production plans across both the states $s = 1 \cdots S$ and the messages. This notion of efficiency, which is similar to what Grossman [2] has called Social Nash Optimality, was first applied in an information setting by Ohlson [12], and Ohlson, Buckman [13]. The above theorem would still hold with only minor modification if message-contingent trade of current consumption and firm shares were allowed *prior* to the release of a message. Such an economy would achieve a higher level of efficiency since it allows co-ordination of allocations across messages.

The theorem allows us to compare not only finer information to no (null) information, but also allows comparison of two non-null information systems, one of which is finer than another.[3] All consumers will prefer (at least weakly) more information (in a fineness sense) to less information, as long as endowments are equilibrium message-contingent positions with respect to the coarse information system; no consumer is endowed with a negative position in any firm for any message from the coarse information system; financial markets are conditionally complete for each information system; firms maximize value; and message-contingent trade takes place *prior* to the release of a message from the finer information system. Thus we see that the theorem is not dependent upon whether message contingent trade is allowed prior to or only after the release of a message. Nor is the theorem restricted to the comparison of non-null information to null information.

III. Summary

We have given conditions sufficient for public information to have social value in production and exchange. These conditions were essentially independent of consumers' beliefs and preferences. The conditions did involve consumer endowments, the allocational efficiency of the financial market, and value maximizing behavior by firms. One of our main insights was that there exist conditions under which public information will have social value in a production and exchange economy, even though there would be no social value in an otherwise similar pure exchange economy. The difference in results is due to the fact that in pure exchange economies, information may only be used to alter the risk sharing among consumers, while in a production and exchange economy it serves the

[3] See Marshak and Radner [7] for a discussion of the fineness concept.

additional function of allowing consumers to reallocate resources to those firms that are expected to be the "most productive."

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