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The Relative Price Volatility of Taxable and Non-Taxable Bonds: A Note

FRED D. ARDITTI and MILES LIVINGSTON*

BECAUSE THE YIELDS TO maturity on municipal bonds are generally more volatile than the yields on taxable bonds, previous authors have concluded that the prices of municipal bonds are also more volatile.[1] This paper shows that this conclusion does not follow. The highly volatile nature of municipal bond prices is generally attributed first to the institutional phenomenon that, for banks, loans dominate municipal bonds as assets and secondly to the manner in which municipals are taxed.[2]

The institutional explanation for the greater price volatility of municipals is that banks prefer loans to municipals as investments on account of the special nature of the bank-customer relationships, namely that a customer may be both a depositor and a borrower. Therefore, in periods of expansionary monetary policy and slack loan demand, banks enter the market with funds and bid up municipal bond prices. During periods of restrictive monetary policy, deposits grow more slowly relative to loan demand and banks, having little or no excess funds to invest in municipals, withdraw from the municipal bond market. Thus, it is contended that commercial banks, the owners of approximately 45 percent of outstanding municipals [1], sell municipals when interest rates are rising and buy them when interest rates are falling, accentuating the cyclical swings in municipal bond prices.

The tax explanation of municipal bond price volatility rests upon the argument that the yield volatility of municipals is increased by their special tax treatment.[3]

* Chicago Mercantile Exchange and University of Wisconsin, Parkside respectively. We would like to thank George Kaufman and Michael Brennan for their thoughtful comments.

[1] See [2], pp. 103–104, [5] pp. 81–84, and [7] pp. 520–21.

[2] A third explanation is changing default risk on municipals. A fourth explanation of the greater price volatility of municipals would be their longer Macaulay's duration. For example, if we take 10-year par bonds, with a taxable yield of 10% and a municipal yield of 5%, the duration of the taxable is 6.76 years and the duration of the municipal is 8.11 years. But this paper shows that for price declines below par both bonds will have the same price change, while for price increases above par the taxable bond will have a greater price increase. Duration is a misleading measure of volatility because it is in before-tax terms, whereas price changes result from changes in after-tax yields.

[3] See Stigum [7] p. 521. As an illustration, consider the case of a flat term structure, a personal tax rate of 50%, a capital gains tax rate of 20%, a 10-year bond. Assume that a taxable par bond initially has a 12% yield (and coupon) and that a par municipal has a 6% yield (and coupon). Then interest rates rise so that a taxable par bond has a yield of 16%. The 12% coupon taxable bond will have a new yield to maturity of 14.9% and the 6% coupon municipal will have a new yield to maturity of 8.2%. At the new level of interest rates, each bond will have an after-tax rate of return of 8%. The change in yield on the municipal is proportionately bigger even though both bonds will sell at the same price. An intuitive explanation of this greater volatility of municipal yields is that changes in bond price have a bigger relative impact upon the municipal yield because the municipal has a lower coupon.

It is then concluded that, if the yield to maturity of a municipal bond changes by a greater proportionate amount than that of a taxable bond, then the municipal's price will be more volatile than the price of the taxable bond.

In this note, we demonstrate that the tax explanation of municipal bond price volatility cannot be true. By comparing municipal and taxable bonds with the same initial price, we prove that taxable and municipal bonds that sell at a discount from par must exhibit identical price variability, while the price of a premium taxable bond will change by more than that of a premium municipal bond in response to a shift in interest rates. Thus, the greater volatility of tax-exempt pretax interest rates is not necessarily associated with greater volatility in municipal bond prices.

To compare the price changes of taxable and nontaxable bonds in response to a change in interest rates, it is necessary to consider both discount and premium bonds and the taxation of capital gains and losses on bonds for both banks and individuals. Consequently, we look at the four cases obtained by combining the two treatments of capital gains and losses for discount and premium bonds.

All bonds are assumed to be noncallable, default free, originally issued at par. The municipal and taxable bonds compared are assumed to be selling initially at the same price and have the same maturity. All investors are assumed to be in the same tax bracket.

Case 1. If (i) both bonds sell at a discount and (ii) capital gains taxes are paid at maturity, then for any change in interest rates, the changes in the prices of the two bonds will be equal.

Proof:

Define

A = the current price of a one-dollar n-year, tax-free annuity;
D = the current price of a tax-free, zero coupon bond that matures in N years and pays one dollar;
t_p = the tax rate on interest income;
t_g = the tax rate on capital gains income;
C_t = the periodic dollar coupon payment on a taxable bond;
F = the principal payment on both taxable and municipal bonds;
P_t = the price of a taxable bond; and
P_m = the price of a municipal bond.

The price of the two bonds may be written:

$$P_t = \frac{C_t(1 - t_p)A}{(1 - t_g D)} + \frac{(1 - t_g)FD}{(1 - t_g D)} \tag{1}$$

$$P_m = \frac{C_m A}{(1 - t_g D)} + \frac{(1 - t_g)FD}{(1 - t_g D)} \tag{2}$$

If the prices are equal, then $C_m = C_t(1 - t_p)$ and it follows that $P_t = P_m$ for all values of A and D.

Case 2. If (i) both bonds sell at a discount, (ii) capital gains due to the closing of the discount are linearly amortized, added to annual income, and taxed at the ordinary rate, t_p, then for any change in interest rates, the price changes of the two bonds will be identical.

Proof: Let N denote the number of years to maturity of each bond; then the annual amount that is added to income due to the amortization of the discount amounts to $(F - P_t)/N$ for the taxable bond. This amount is taxed at the personal rate, t_p, so that the pricing formula for the taxable bond is

$$P_t = \frac{C_t(1 - t_p)A}{1 - \frac{t_pA}{N}} + \frac{F\left[D - \frac{t_pA}{N}\right]}{1 - \frac{t_pA}{N}} \tag{3}$$

In an analogous manner, the municipal bond price is

$$P_m = \frac{C_mA}{1 - \frac{t_pA}{N}} + \frac{F\left[D - \frac{t_pA}{N}\right]}{1 - \frac{t_pA}{N}} \tag{4}$$

As in the previous case $P_t = P_m$ implies that $C_m = C_t(1 - t_p)$, which implies that the two bond prices are equal for all values of A and D.

Case 3. If (i) both bonds sell at a premium, (ii) the premium on the taxable bond is deductible as a capital loss at maturity while the premium on the municipal bond is not deductible against capital or ordinary income, then for any change in interest rates, the price volatility of the taxable bond will exceed that of the municipal bond.

Proof: Using the notation of the prior proofs, we have as the pricing formulas for the taxable and municipal bonds,

$$P_t = \frac{C_t(1 - t_p)A}{(1 - t_gD)} + \frac{(1 - t_g)FD}{(1 - t_gD)} \tag{5}$$

$$P_m = C_mA + FD \tag{6}$$

If $P_t = P_m$, then

$$C_m = \frac{C_t(1 - t_p)}{(1 - t_gD)} - \frac{t_gFD(1 - D)}{A(1 - t_gD)} \tag{7}$$

Define i as the interest rate indicator, where it is assumed that if i increases, then all interest rates increase, and if i decreases, they all fall. Taxable bonds will be more volatile in price if

$$\frac{d(P_t - P_m)}{di} < 0 \tag{8}$$

$$\frac{d(P_t - P_m)}{di} = \left[\frac{C_t(1 - t_p)}{1 - t_gD} - C_m\right]\frac{dA}{di} + \left[\frac{t_gP_t}{1 - t_gD} - \frac{t_gF(1 - D)}{1 - t_gD}\right]\frac{dD}{di} \tag{9}$$

If all interest rates move in the same direction, $dA/di < 0$ and $dD/di < 0$. It follows that (8) will hold if each term in the brackets in (9) is positive. Using (7), we see that $C_t(1 - t_p)/(1 - t_g D) > C_m$. The second bracketed term in (9) is positive if $P_t > F(1 - D)$, which always holds if P_t is a premium bond, since then $P_t > F$. The proof is complete.

Case 4. If (i) both bonds sell at a premium, (ii) the premium on the taxable bond is linearly amortized over the bond's remaining life, N, resulting in an annual deduction against ordinary income and the premium on the municipal bond is not tax deductible, then for any change in interest rates, the price volatility of the taxable bond will exceed that of the municipal bond.

Proof: The pricing formulas are given by

$$P_t = \frac{\left[C_t(1 - t_p)A + FD - \dfrac{t_p F A}{N}\right]}{\left[1 - \dfrac{t_p A}{N}\right]} \tag{10}$$

$$P_m = C_m A + FD \tag{11}$$

If $P_t = P_m$, then

$$C_m = \frac{\left[C_t(1 - t_p) - \dfrac{t_p F(1 - D)}{N}\right]}{\left[1 - \dfrac{t_p A}{N}\right]} \tag{12}$$

Taxable bonds will be more volatile in price if (8) holds.

$$\begin{aligned}\frac{d(P_t - P_m)}{di} = \frac{dA}{di}&\left[\frac{t_p P_t}{N\left(1 - \dfrac{t_p A}{N}\right)} + \frac{C_t(1 - t_p) - \dfrac{t_p F}{N}}{1 - \dfrac{t_p A}{N}} - C_m\right] \\ &+ \frac{dD}{di}\left[\frac{F}{1 - \dfrac{t_p A}{N}} - F\right]\end{aligned} \tag{13}$$

To see that the first term in brackets is positive, substitute for C_m from (12), simplify, and note that $P_t > FD$ for premium bonds. The second term in brackets is positive if $1 - t_p A/N < 1$, which always holds for $t_p > 0$ and $A > 0$. Thus (13) will be negative if $dA/di < 0$ and $dD/di < 0$. The proof is complete.

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