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Variance and Lower Partial Moment Measures of Systematic Risk: Some Analytical and Empirical Results

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ABSTRACT

As a measure of systematic risk, the lower partial moment measure requires fewer restrictive assumptions than does the variance measure. However, the latter enjoys far wider usage than the former, perhaps because of its familiarity and the fact that two measures of systematic risk are equivalent when return distributions are normal. This paper shows analytically that there are systematic differences in the two risk measures when return distributions are lognormal. Results of empirical tests show that there are indeed systematic differences in measured values of the two risk measures for securities with above average and with below average systematic risk.

LOWER PARTIAL MOMENT IS a measure of portfolio risk that depends on only those portfolio returns that fall below some target level of returns. It is defined and measured as

$$LPM_h(R_p) = \int_{-\infty}^{h} (R - h)^2 f_p(R)\, dR \tag{1}$$

where h is the target level and $f_p(R)$ represents the probability density function of returns for portfolio p.[1]

For expected utility maximizing investors, Bawa [3] has provided a theoretical rationale for using lower partial moment as the measure of portfolio risk. Stochastic Dominance (SD) rules can be used to derive the admissible set of portfolios for expected utility maximizing investors under far less restrictive assumptions regarding utility functions and probability distributions of returns than those required to rationalize the use of the mean-variance portfolio selection rule.[2] Bawa related the mean-lower partial moment (EL) portfolio selection rule

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[1] Semivariance of returns is defined and measured in exactly this same way. In this sense, semivariance and lower partial moment are interchangeable terms. However, as we shall see, the literature does distinguish between the two in terms of their theoretical origins. (See Footnote 3)

[2] See Quirk and Saposnik [20] for an early development of a stochastic dominance selection rule as an optimal selection rule. Hadar and Russell [7, 8] and Hanoch and Levy [9] imposed an assumption of risk aversion and derived the Second Order Stochastic Dominance (SSD) rule. Whitmore [22] obtained the Third Order Stochastic Dominance (TSD) rule by restricting the risk averse investor to a utility function for which the third derivative is positive.

to the *SD* rules and concluded that, at least on theoretical grounds, the *EL* rule is preferable to the popular mean-variance (*EV*) selection rule. The basis for this preference is that while the *EL* rule can be derived from the *SD* selection rules under very general assumptions regarding distributions of returns and utility functions, the *EV* rule can be so derived only under very restrictive assumptions (i.e., normal distributions and/or quadratic utility functions).

Even before Bawa's contribution, lower partial moment had it supporters, although these early supporters referred to their risk measure as semivariance rather than as lower partial moment. Markowitz [16] showed that if the investor utility function is quadratic up to h and linear beyond h, then expected utility maximizers rank portfolios in terms of their means and semivariances, where semivariance is defined as in Equation (1).[3] In addition, Markowitz argued that, due to its exclusive concern with downside deviations in returns, semivariance, or lower partial moment, is intuitively more appealing than is variance as a risk measure.[4]

Despite the strong theoretical support and intuitive appeal of lower partial moment, interest in abandoning the more familiar variance as a portfolio risk measure depends on how significantly different the inferences are from applying the two alternative risk measures. Our interest in this paper centers on differences in the equilibrium pricing relationships implicit in the *EV* and *EL* models. After stating the capital asset pricing models as derived from the two alternative portfolio theories, Section I analytically relates the mean-lower partial moment measure of systematic risk to its counterpart from mean-variance theory. Having established analytically the conditions under which the two measures of systematic risk are not identical, Section II investigates empirically the relationship between the two measures of systematic risk, and it presents some possible interpretations of the empirical results. Section III provides a summary.

I. Analytical Relationship

A. Specifying the Alternative Measures of Systematic Risk

Hogan and Warren [10] and Bawa and Lindenberg [4] independently developed a mean-lower partial moment capital asset pricing model (*EL-CAPM*).[5] The

[3] The literature refers to the measurement defined in Equation (1) as lower partial moment when its theoretical support is based on Bawa's argument, and refers to it as semivariance when the theoretical support is based on the Markowitz argument. We find the Markowitz argument less appealing than the Bawa argument due to the former's partial reliance on a quadratic utility function, and due to the well known problem with this utility function and its increasing absolute risk aversion. Therefore, throughout most of this paper we refer to the measure in Equation (1) as the lower partial moment.

[4] Mao [14] presents survey evidence showing that decision makers tend to think of risk in terms of the possibility of outcomes below some target.

[5] We have restricted attention throughout this paper to what Bawa and Lindenberg refer to as second order lower patial moments. Given this, their pricing model and Hogan and Warren's pricing model are equivalent, notwithstanding the differences in the theoretical origins of their models (as mentioned in footnote 3).

equilibrium pricing relationship of this model is formulated as

$$E(R_i) = R_f + [E(R_m) - R_f] \frac{CLPM_{R_f}(R_m, R_i)}{LPM_{R_f}(R_m)} \tag{2}$$

where

$E(R_i)$ = equilibrium expected rate of return on asset i;
$E(R_m)$ = equilibrium expected rate of return on the market portfolio;
$LPM_{R_f}(R_m)$ = lower partial moment of returns below R_f on the market portfolio; and
$CLPM_{R_f}(R_m, R_i)$ = co-lower partial moment below R_f of returns on the market portfolio with returns on security i

$$= \int_{-\infty}^{+\infty} \int_{-\infty}^{R_f} (R_m - R_f)(R_i - R_f) f(R_i, R_m)\, dR_m\, dR_i \tag{3}$$

where

$f(R_m, R_i)$ = joint probability density function of returns on asset i and returns on the market portfolio.

In deriving Equation (2), the target rate in all cases was set equal to the risk-free rate.

In the *EL-CAPM*, *CLPM/LPM* replaces *COV/V* from the mean-variance pricing model (*EV-CAPM*) as the measure of systematic risk.[6, 7] It follows from our earlier discussion that *CLPM/LPM* is a more general measure of systematic risk than is *COV/V*. However, since *COV/V* is a more familiar measure than is *CLPM/LPM*, we would recommend replacement of *COV/V* with *CLPM/LPM* only if there are distinct and significant differences between the two measures of systematic risk for existing financial securities.

B. Relating the Alternative Measures of Systematic Risk

Normal Distributions. The issue of how these two measures of systematic risk relate to each other was addressed analytically by Bawa and Lindenberg [4] and by Nantell and Price [18] under the assumption that the joint distribution of returns between the market and the security is bivariate normal. Although $CLPM \neq COV$ and $LPM \neq V$, both sets of authors conclude that, under the bivariate normal distributional assumption, $CLPM/LPM = COV/V$. Hence, if a normality assumption is the most reasonable one to make for financial securities, the theoretical preference for *CLPM/LPM* as a measure of systematic risk is an empty one. Since $CLPM/LPM = COV/V$, we may as well refer to our measure of systematic risk in its most familiar form, *COV/V*.

[6] Whenever it is not confusing, we shall use *CLPM/LPM* for $CLPM_{R_f}(R_m, R_i)/LPM_{R_f}(R_m)$ and *COV/V* for $COV(R_i, R_m)/V(R_m)$ where $COV(R_i, R_m)$ = covariance of returns on the market portfolio with return on security i, and $V(R_m)$ = variance of returns on the market portfolio.

[7] See Nantell and Price [18] for an interpretive discussion of the relationship between *CLPM/LPM* and *COV/V*.

Skewed Distributions. There is, however, some reason to believe that ex ante return distributions are not symmetrical. Because security and portfolio returns are bounded on the low side due to limited liability and unbounded on the high side, a reasonable hypothesis is that ex ante distributions are better represented by positively skewed distributions than by symmetrical ones. The reasonableness of this hypothesis in terms of before personal-tax returns is reinforced by the existence of progressive taxation. Although the empirical evidence does not unanimously support a specific magnitude of skewness, there does seem to be some agreement that skewness does exist in the ex post distributions of monthly security returns.[8] Even for portfolios of securities, Lintner [13] finds that the distributions tend to be more lognormal than normal. Finally, Kraus and Litzenberger [12], in testing a three parameter pricing model, find that the market index exhibited positive skewness and that the market price of skewness for securities was negative and significant.

These theoretical and empirical arguments regarding the presence of skewness suggest that, in relating *CLPM/LPM* to *COV/V*, it may be useful to move beyond Bawa and Lindenberg's [4] and Nantell and Price's [18] assumption of normality. In doing so, the result is the following theorem:[9]

THEOREM. If the joint density function of R_i and R_m is bivariate lognormal, then

(1) the systematic risk of security i as measured by the *EV-CAPM* is equal to its systematic risk as measured by the *EL-CAPM* if and only if security i is of average systematic risk or is riskless;
(2) the systematic risk of security i as measured by the *EV-CAPM* is less than its systematic risk as measured by the *EL-CAPM* when security i is of less than average systematic risk; and
(3) the systematic risk of security i as measured by the *EV-CAPM* is greater than its systematic risk as measured by the *EL-CAPM* when security i is of greater than average risk.

In other words, for low systematic risk securities $COV/V < CLPM/LPM$; for average systematic risk securities $COV/V = CLPM/LPM$; and for high systematic risk securities $COV/V > CLPM/LPM$.[10]

Hence, when distributions are lognormal, *COV/V* is a biased substitute for the more general measure of systematic risk, *CLPM/LPM*. If a significant number of securities exhibit significant lognormality in their returns,[11] then we can no longer

[8] See Arditti [1, 2], Francis [6], and McEnally [15].

[9] In order to treat the question analytically, one specific form of skewness was assumed. We do not know what form the theorem would take for more general forms of skewness.

[10] The proof of the theorem, which is available upon request to the authors, begins by developing, for an arbitrary bivariate lognormal distribution, the mathematical relationship between *CLPM/LPM* and the parameters of the bivariate distribution. It turns out that *CLPM/LPM* depends on, among other parameters, the mean of the security return and the covariance of security returns with the market. By putting this mathematical relationship together with the theoretical relationship of the *EL-CAPM*, it is possible to develop the relationship between *COV/V* and *CLPM/LPM* that must exist if the equilibrium returns are set by the *EL-CAPM*.

[11] Since skewness was captured in the proof of the theorem through the use of a lognormal distribution, we are actually referring to price relative distributions rather than rate of return distributions.

afford to continue using *COV/V* in place of its more general counterpart (*CLPM/LPM*). In this case, our implicit assumption in using *COV/V*, that *COV/V* = *CLPM/LPM*, no longer holds.

In the next section we examine the issue empirically. Whether skewness in the distributions (in the lognormal form) is significant enough or not depends, for us, not on the amount of skewness, but rather on how that amount of skewness affects *CLPM/LPM* relative to *COV/V*. If *CLPM/LPM* is insignificantly different from *COV/V* for high and low risk securities, whatever skewness does exist, it is not enough to warrant concern over continued use of the *COV/V* measure of systematic risk.

II. Empirical Relationship

A. Selecting the Relevant Securities

The theorem states that if returns are lognormally distributed, there are systematic differences between *CLPM/LPM* and *COV/V* only for those securities of above or below average risk. The first step in our empirical analysis is to derive one sample of securities with above average (high) risk and a second sample with below average (low) risk.

Data. The risk mesures of portfolio and capital market theories should be computed from ex ante distributions. Since such distributions are not directly observable, we follow the usual procedure of computing proxies for the ex ante parameters from ex post distributions. We divided the data on the *CRSP* monthly tapes into seven nonoverlapping 7-year periods. The sampling periods are listed in Table I. Their length was set at seven years in an effort to balance our concern for a short enough period to maintain stability in the parameters of the distributions and a long enough period to obtain a reasonable proxy for the ex ante distribution. For each sampling period, we identified all the securities on the *CRSP* monthly tapes for which there are returns available continuously throughout that sampling period and the one following it.[12] The number of such available securities for each sampling period is given in Table I.

Methodology. For each sampling period, the first step in our empirical analysis requires an identification of those securities that have high systematic risk and those with low systematic risk. To accomplish this we could simply compute *COV/V* for each security for which market returns are available throughout that period, and use this statistic to segment the entire sample into above and below risk samples. However, because *COV/V* is measured with error, using such a procedure in the security selection phase of our analysis would cause a sample selection bias in the second phase of our analysis (discussed in the following section), that of comparing each sample's *COV/V* to its corresponding *CLPM/LPM*.[13] In order to overcome this bias, we have made three adjustments to the security selection phase of our analysis. First, for a security to be included in the high (low) risk sample it is necessary that its *COV/V and* its *CLPM/LPM* be above (below) 1.0. Second, rather than select securities on the basis of the raw

[12] The rationale for requiring continuous listing throughout two sampling periods will be presented below in the methodology section.

[13] We are in debt to Stephen L. Brown for pointing out this problem and for suggesting a remedy.

measured values of COV/V and $CLPM/LPM$ relative to 1.0, for each estimate of COV/V and $CLPM/LPM$ we test the null hypotheses that they equal 1.0 against the alternate hypothesis that they are greater than 1.0 or less than 1.0. Using a five percent significance level, the high risk sample is formed from securities for which we reject the null hypotheses (for both risk measures) in favor of the alternative that risk is greater than 1.0. The low risk sample is formed from securities for which the null hypothesis is rejected in favor of the alternate (for *both* measures) that risk is less than 1.0. In this way we have eliminated from our high and low risk samples all those securities for which risk is not *significantly* different from average risk. We do not want securities in our samples for which measured risk does not equal 1.0, but for which true risk could easily equal 1.0.

The third step in our attempt to minimize any bias in the comparison of each sample's COV/V to its $CLPM/LPM$ (caused by the way in which the securities for those samples are selected) is to select the securities for the high and low risk samples based on one period's measured values for the two risk measures and to test for differences in COV/V and $CLPM/LPM$ using measurements from a second time period.[14]

The methodology described in this section is performed once for each of the sampling periods. An example of the methodology is given in terms of the 7-year sampling period, 1926–33. Adjustments from sampling period to sampling period must be made, but only in terms of the number of securities.

Of the 300 securities available on the *CRSP* monthly tapes throughout the 1927–33 and 1934–40 periods, we need to identify those that have high systematic risk and those with low systematic risk. For each of the 300 securities, COV/V was estimated as.

$$\left(\frac{COV}{V}\right)_i = \frac{\sum_{t=1}^{84} (R_{i,t} - \bar{R}_i)(R_{m,t} - \bar{R}_m)}{\sum_{t=1}^{84} (R_{m,t} - \bar{R}_m)^2} \qquad i = 1, \cdots, 300 \tag{4}$$

where

$\bar{R}_i$ = mean return on security i for the 84 months, and
$\bar{R}_m$ = mean return of the market index from the *CRSP* data,

and $CLPM/LPM$ was estimated as

$$\left(\frac{CLPM}{LPM}\right)_i = \frac{\sum_{t=1}^{84} (R_{i,t} - R_{f,t})[\min(0, R_{m,t} - R_{f,t})]}{\sum_{t=1}^{84} [\min(0, R_{m,t} - R_{f,t})]^2} \tag{5}$$

where

$R_{f,t}$ = the risk-free rate for period t,[15]

[14] It is this step of the methodology that makes necessary the requirement that for a security to be a candidate for the high or low risk samples, there must be returns available on that security throughout two consecutive sampling periods.

[15] In a separate run, $R_{f,t}$ in Equation (6) was replaced with $\bar{R}_f$, the average of the 84 $R_{f,t}$'s. The results were basically unaffected.

and

$$\begin{aligned}\min(O - R_{f,t}) &= 0 && \text{if} \quad R_{m,t} > R_{f,t} \\ &= R_{m,t} - R_{f,t} && \text{if} \quad R_{m,t} < R_{f,t}\end{aligned}$$

For each of the 300 *estimates* of COV/V risk, we need to determine if the corresponding security is of low, average, or high risk. This is accomplished by computing 300 t-statistics where

$$t_i = \frac{(COV/V)_i - 1.0}{s_i} \qquad i = 1, \cdots, 300 \tag{6}$$

where s_i is the estimated standard error of $(COV/V)_i$. Each security for which the t-statistic on COV/V is significant (at the 5 percent level) is tentatively placed into the high or low risk samples for the 1927–33 period, depending on its sign. For each of these securities a similar t-statistic is computed for its $CLPM/LPM$ measurement relative to 1.0.[16] If this t-statistic is insignificant or of the opposite sign of that for COV/V, that security is eliminated from the sample altogether.

Results. Table I summarizes the results of segmenting the available securities into high, low, and average risk samples. The results as summarized in Table I are from using a 5.0% significance level on the t-statistics. There are a number of observations to be made regarding these results.

First, the high and low risk sample sizes (it is these two samples on which the final tests will be run) fall well within the "large sample" range. Second, the percentage of total available securities whose systematic risk is not significantly different from the average risk reaches a high of 56% for the 1927–33 sampling period. Initially, the relative size of the average risk sample may seem unusually large. However, this result is due to the fact that we wanted small Type I error ($\alpha = 0.05$) in assigning securities to the high and low risk samples. When a security is assigned to a high or low risk sample, we are very sure that it is not an average risk security. We necessarily are accepting greater Type II error, and, thus, we are not as sure that all securities in our average risk sample are "true" average risk securities. In other words, our average risk sample includes securities whose "true" risk equals 1.0, and other securities whose "true" risk is so close to 1.0 that their measured risks are not significantly different from 1.0.

Finally, note that the high risk samples are uniformly smaller than the low risk samples. This is presumably because of our interest in only those securities listed continuously through each period and its corresponding subsequent period. Because of this data requirement, our total sample of available securities in every sampling period is biased towards the lower risk securities.

B. Testing for Differences in Systematic Risk Measures.

Having derived samples of high risk and samples of low risk securities, we are now in a position to test whether or not enough skewness exists in the market for

[16] The standard error for $CLPM/LPM$ was suggested by Jahankhani [11].

Table I

Sample Sizes

Sampling Period	Available Securities[a]	High Risk Sample[b]	Low Risk Sample[b]	Average Risk Sample[c]
1927–1933	300	52 (17%)	80 (27%)	168 (56%)
1934–1940	556	140 (25%)	212 (38%)	204 (37%)
1941–1947	699	221 (32%)	244 (35%)	234 (33%)
1948–1954	749	234 (31%)	267 (36%)	248 (33%)
1955–1961	649	124 (19%)	270 (42%)	255 (39%)
1962–1968	699	101 (14%)	333 (48%)	265 (38%)

[a] Securities on the CRSP monthly tapes for which there are continuous returns throughout the sampling period and the following seven years.

[b] For the high (low) risk sample, the raw numbers represent the number of securities for which the test statistics on (COV/V-1.0) and on (CLPM/LPM-1.0) are both significantly positive (negative). The numbers in parentheses represent these raw numbers as percentages of the available securities.

[c] The raw numbers represent the number of securities for which the test statistics are insignificant or of opposite signs. The numbers in parentheses represent these raw numbers as percentages of the available securities.

equity securities to cause significant differences between the *EL-CAPM* and *EV-CAPM* measures of systematic risk.

Data. The samples were derived as described above. There are 6 high risk samples and 6 low risk samples (for a total of 12 samples ranging in size from 52 securities to 270 securities) for which we can test for differences between *COV/V* and *CLPM/LPM*.[17].

Methodology. As above, for ease of exposition, the methodology will be described for only two of the 12 samples—the high and low risk samples for the 1927–33 sampling period.[18] For the other 10 samples, the methodology is exactly the same, except for the number of securities.

For each of the 52 securities in the high risk sample and 80 securities in the low risk sample, new estimates of *COV/V* and *CLPM/LPM* are computed using the 84 months of returns from the testing period for each security. The estimates are computed as indicated in Equation (4) and Equation (5), respectively.

Given these estimates of *CLPM/LPM* and *COV/V* for each of our 52 plus 80

[17] Although the period 1927–75 was divided into seven 7-year periods, our methodology of selecting securities using data from one time period and comparing the two risk measures using data from a second time period makes it impossible to use all seven periods; i.e. one of the periods can serve only as a selection period, not also as a testing period. It is for this reason that the final period, 1969–75, does not appear in Table I, leaving only six periods presented.

[18] In choosing the selection periods and testing periods, we had the option of having the selection period precede the testing period, or vice versa. The assumption in either case is that the true value of the variable to be measured is constant over the entire period represented by the two contiguous sampling periods. It's a matter of indifference, then, as to which 7-year period is used to select the securities into high and low risk samples and which is used to compare the values of the two risk measures. We chose to make the second period (calendar wise) the selection period and the first period (calendar wise) the testing period. Thus, for the 1927–33 sampling period, its selection period is 1934–40 and its testing period is 1927–33. We chose this option to maximize the range of "bull and bear" markets represented in our final six testing periods.

securities, we form the differences

$$x_i = \left(\frac{COV/V}{V}\right)_i - \left(\frac{CLPM}{LPM}\right)_i \tag{7}$$

The mean ($\bar{x}$) and standard deviation (s_x) of the x_i's are computed *within* each sample. For both samples, the null hypothesis that $\bar{x} = 0$ is tested against its alternate that $\bar{x} \neq 0$ using a t-statistic computed as

$$t = \frac{\bar{x} - 0.0}{s_x/\sqrt{n}} \tag{8}$$

where n is the sample size.[19]

In order to provide a more general, but weaker, test of the null hypothesis that the median difference in the x_i's is zero, we also ran a sign test on the values of the x_i's for each sample. For the high (low) risk sample, the null hypothesis that a positive difference is as likely to occur as a negative difference is tested against its alternate that a positive (negative) difference is more likely using a z-statistic computed as [20]

$$z = \frac{(k \pm .5) - .5n}{.5\sqrt{n}} \tag{9}$$

where

k = number of positive (negative) differences, and
n = sample size.

For each sample, we are testing whether there is enough skewness in returns to cause COV/V, on average, to be a systematic misestimate of the more general risk measure, $CLPM/LPM$.

Results. The results are summarized in Table II. In the majority of the samples (ten of twelve for the t-test and ten of twelve for the sign test), we are able to reject, at a 5 percent significance level, the hypothesis that $CLPM/LPM = COV/V$. In other words, regardless of the specific statistical significance test utilized, it appears that the EV-$CAPM$ measure of systematic risk is not a useful proxy for the more general measure of the EL-$CAPM$. For the low risk samples this is the case in all six sampling periods. For the high risk samples, there appear to be two or three periods in which skewness, at least in the lognormal form, is insignificant.[21]

Of the ten significant t-statistics, six are significant and of the sign indicated by

[19] Due to our large sample sizes, we are able to rely on the central limit theorem. The test statistic, assuming the differences are independent and the variance of the difference is constant across securities, should follow a student t-distribution with $n - 1$ degrees of freedom.

[20] In Equation (9), use +.5 when $k > n/2$ and use −.5 when $k < n/2$. For sample sizes of 11 or larger we use the normal approximation to the binomial, comparing z with the standard normal distribution 5 percent significance.

[21] The more significant results for the low risk samples may be due to the larger sample sizes and to the wider range of values for the risk measures.

Table II

Summary of the Results from Testing for Differences in EV-CAPM and EL-CAPM Measures of Systematic Risk

Sampling Period	High Risk Sample		Low Risk Sample	
	t	z	t	z
1927–1933	+1.9[a]	+2.3[a]	−8.2[a]	−5.7[a]
1934–1940	+1.4	−0.8	−6.1[a]	−7.3[a]
1941–1947	−0.5	−2.2[b]	−4.7[a]	−6.2[a]
1948–1954	−4.4[b]	+0.2	+2.3[b]	+3.1[b]
1955–1961	−4.1[b]	−4.4[b]	+8.3[b]	+6.4[b]
1962–1968	+2.6[a]	+1.9[a]	−7.4[a]	−7.0[a]

[a] Significant at the 5% level and of the "correct" sign; i.e. the sign corresponds to the theorem of Section I.

[b] Significant at the 5% level but of the "incorrect" sign; i.e. the sign does not correspond to the theorem of Section I.

our theorem in Section I. Results for these six samples are consistent with the existence of a bivariate lognormal distribution. In these cases, COV/V systematically *overestimates* $CLPM/LPM$ for the high risk samples, and COV/V systematically *underestimates* $CLPM/LPM$ for the low risk samples.

There are four samples, however, for which the significant t-statistics are of the "incorrect" sign. That is, for the high risk sample for 1948–54 and 1955–61, the EV model actually *underestimates* EL systematic risk. According to our theorem, if the bivariate distributions were skewed in the lognormal form, the EV model would *overestimate* EL systematic risk for high risk securities. There are corresponding "incorrect" signs for the low risk samples for the 1948–54 and 1955–61 sampling periods. Our determination as to what are "correct" and "incorrect" signs depends on the assumption of our theorem regarding the bivariate disribution of returns. The theorem assumes a *bivariate* lognormal distribution. So, as a first step in trying to explain these "anomalous" results, we examine the characteristics of the market distribution of returns for the two troublesome sampling periods, 1948–54 and 1955–61, to see if they fit our assumption.

For all six sampling periods, skewness coefficients were estimated for the market index.

As an estimate of skewness, we used g:

$$g = \frac{m_3}{(m_2)^{3/2}} \tag{10}$$

where

$$m_d = \sum_{t=1}^{84} (R_{m,t} - \bar{R}_{\mathrm{m}})^d \qquad d = 2, 3 \tag{11}$$

The results of this test for skewness are reported in Table III.[22] For the periods

[22] See Snedecor and Cochran [21], pp. 86–89 for a discussion of the use of this test statistic for skewness for $T < 150$, where T is the number of months.

Table III

Results of Skewness Test for Market Index

Sampling Period	Value of Test Statistic[a]
1927–1933	1.83
1934–1940	0.36
1941–1947	0.02
1948–1954	−0.26
1955–1960	−0.23
1961–1968	0.30

[a] As defined in Equations (10) and (11).

in which we obtained "incorrect" signs in Table II, our estimate is that the market return distribution was not positively skewed, and, in fact, it may have been moderately negatively skewed.[23] For the other three sampling periods in which the market skewness is positive, the significant results from Table II are consistent with the analytical results of Section I.[24] These results invite the conjecture that negative market skewness causes the "incorrect" signs for 1948–54 and 1955–61 periods. Even with negative market skewness for these periods, we can reject the null hypothesis (on the basis of the t-test) that $CLPM/LPM = COV/V$ for high risk and for low risk samples of securities.

How one interprets the results for the 1948–54 and 1955–61 periods depends on their feelings about negative market skewness. We are using ex post distributions as proxies for ex ante distributions. To the extent a negatively skewed market represents a legitimate proxy, the results for those time periods should be taken at face value. On the other hand, in the absence of some theoretical rationale for negative market skewness, we might reject those results, arguing that for those time periods we did not have legitimate proxies for ex ante return distributions.

At any rate, our results indicate that, if our preference is based on insignificant differences in the risk measures, we are in a rather weak position when we use the more familiar COV/V rather than $CLPM/LPM$ as a measure of systematic risk.[25]

[23] Although the negative skewness coefficients are not significant at the 5% level, they are significant at the 15% level.

[24] Except for the positive skewness coefficient for 1927–33, they are not significant at the 5% level, but they are significant at the 15% level. Even more importantly, Price and Nantell [19] have derived results for the case where the security is skewed, but the market distribution is normal. The result is similar to that given in the theorem for the bivariate lognormal case, except that the break point is less than 1.0. In other words $COV/V < CLPM/LPM$ for all $CLPM/LPM < z$, $COV/V = CLPM/LPM$ for $CLPM/LPM = z$, and $COV/V > CLPM/LPM$ for $CLPM/LPM > z$, where $z < 1.0$ and depends on the parameters of the bivariate distribution. For the bivariate lognormal case, $z = 1.0$ and is independent of the parameters of the bivariate distribution. So, even without significant positive market skewness the results are as expected for the low risk and high risk samples.

[25] Jahankhani [11] presented some evidence on the similarity of the EL-$CAPM$ to the EV-$CAPM$ with respect to four hypotheses for the period 1951 to 1969. His hypotheses do not relate directly to the relationship between COV/V and $CLPM/LPM$ as our hypothesis does. Rather, his hypotheses relate to how well the two models perform as equilibrium pricing models with riskless borrowing. At the time of his tests there were no analytical results relating the two risk measures, and, as such, no particular attention was given to the importance of skewness to his results. Our results, which

III. Summary

Bawa and Lindenberg [4] developed a measure of security systematic risk that depends on deviations in security returns only *below the risk-free rate.* The theoretical support for this risk measure is based on the relationship between stochastic dominance portfolio selection rules and mean-lower partial moment portfolio selection rules. The relationship between these selection rules relies on fewer restrictive assumptions than does the relationship between mean variance rules and stochastic dominance rules. In this sense the lower partial moment measure of systematic risk is more general than is COV/V.

The position of the literature seems to be that because the two measures of systematic risk are basically equivalent, we should use the more familiar COV/V. In fact, Bawa and Lindenberg [4] and Nantell and Price [18] prove analytically that $COV/V = CLPM/LPM$ as long as the bivariate return distribution is normal. ($CLPM/LPM$ is the measure of systematic risk for Bawa and Lindenberg's model.)

After reference to some theoretical and empirical literature arguing that return distributions are more likely to be skewed than normal, we presented a theorem that concludes that for bivariate lognormal distributions, $COV/V = CLPM/LPM$ only for average risk securities. For above average risk securities $COV/V > CLPM/LPM$, and for below average risk securities $COV/V > CLPM/LPM$. We then tested some ex post distributions to see if there was enough skewness of the lognormal form in the market to cause the risk measures to diverge. For six nonoverlapping seven-year selection periods, all continuously available securities were divided into above average risk, average risk, and below average risk samples on the basis of both risk measures being significantly above or below 1.0. The values of the two risk measures for the high risk and low risk samples were then compared for each of six adjacent testing periods. For ten of the twelve samples we rejected the null hypothesis that $COV/V = CLPM/LPM$. For six of the ten significant samples, the relationship between the two risk measures was as hypothesized by our theorem for bivariate lognormal distributions. For the other four, the relationship was the opposite of that hypothesized. It was conjectured that these "anomalous" results were caused by the moderate *negative* skewness that existed in the ex post distributions of returns for the market index. The significance of these four samples depends on how good a proxy their ex post distributions are for the true ex ante distributions.

At any rate, the results do not allow us to rest easy with the assumption that $CLPM/LPM = COV/V$, and hence that the latter, more familiar, measure can be used as our measure of systematic risk. Rather, our results indicate that Bawa and Lindenberg's model is a meaningful alternative model. We hope that this result will rekindle an interest in and study of the mean-lower partial moment model.

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analytically (for one form of skewness) and empirically identify the importance of skewness to our hypothesis, suggest that a reworking of Jahankhani's tests over periods of varying market skewness and over periods of longer than he used (two years) might yield a different set of results than Jahankhani presents.

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