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Author(s): Sheridan Titman
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The Effects of Anticipated Inflation on Housing Market Equilibrium

SHERIDAN TITMAN*

ABSTRACT

An increase in the anticipated rate of inflation causes distortions in the housing market due to a nonindexed tax system. Since nominal rather than real interest payments are tax deductible, an increase in inflation decreases the aftertax cost of capital for homeowners, which in turn increases the demand for housing and increases its real price. This tax gain is shown to be larger for rental housing than for owner-occupied housing. In a competitive market, this implies that although the real price of housing increases with a rise in anticipated inflation, real rental rates fall.

IN THE LAST DECADE the real price of corporate capital fell while the price of housing increased dramatically. During this same period, the rate of inflation increased substantially. Empirical studies have found a negative relationship between stock returns and the inflation rate,[1] while housing has been found to have been a good hedge against inflation.[2] Tax distortions related to the change in the rate of inflation provide one popular explanation for the negative relationship between stock prices and inflation.[3] The effect of similar tax induced distortions on the equilibrium in the housing market will be studied in this paper.

It is shown that the real price of housing increases with the anticipated rate of inflation while the real rental rate declines. These results are consistent with published data for the period 1967–77, during which the anticipated rate of inflation increased substantially.[4] The general price level, as measured by the consumer price index, increased 81.5 percent between 1967 and 1977.[5] In this

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[1] See for example, Nelson [16], Bodie [3], Jaffe and Mandelker [9], and Fama and Schwert [5].

[2] See Fama and Schwert [5].

[3] See for example, Nichols [18], Motely [15], Lintner [10], Hong [8], and Feldstein and Summers [6]; a different viewpoint is presented by Modigliani and Cohn [14].

[4] One measure of the anticipated rate of inflation, yields on U.S. Government long-term bonds, increased in the 1967–77 period from 4.85% to 7.06%. U.S. Bureau of the Census. Statistical Abstract of the United States: 1978, (99th edition), Washington D.C., 1978, p. 549.

[5] U.S. Bureau of the Census. Statistical Abstract of the United States: 1978, (99th edition), Washington, D.C., 1978, p. 490.

same period, the price (controlling for quality changes) of new houses increased 124.1 percent,[6] while rental prices increased only 53.5 percent.[7]

The popular explanation for the increase in the real price of housing is the sharp increase in the number of new households during the past decade. However, if new household formation had been the dominant force it would have manifested itself in a corresponding increase in rental rates. The demographic explanation cannot account for the decline in real rental rates; furthermore, it is inconsistent with informational efficiency in housing markets since it attributes the abnormal returns earned by property owners to factors which could have been easily predicted. The model developed in this paper explains both the increase in the real price of housing and the decline in the real rental rates; it is also consistent with informational efficiency in the housing market.

In this paper, an equilibrium model is developed and used to analyze the effect of a change in the anticipated rate of inflation on the market for housing. The model determines the amount of housing services each individual consumes, which individuals own and which rent their homes, the amount of rental property each individual owns, and the rental rate and purchase price of housing as a function of individual characteristics, the anticipated rate of inflation, and the relevant tax rates. Clientele effects based on differences in the individual marginal tax rates are shown to arise in a manner similar to the tax clientele effects discussed in the capital markets literature.[8] In this model the decisions of individuals to rent or to buy and to invest in rental property depend on their taxable incomes.[9]

The analysis shows that the cost for an individual obtaining housing services from a house with a given purchase price declines with an increase in the rate of inflation because of an induced decrease in his aftertax cost of capital. This decrease in the cost of capital increases the demand for housing and therefore its price. Similarly, the cost of providing rental housing from a house with a given purchase price declines with an increase in the rate of inflation. The net result of the decrease in the aftertax cost of capital for landlords, and the corresponding increase in the purchase price of housing, is that rental rates decline with an increase in the anticipated rate of inflation.

These results are derived within a partial equilibrium framework in which the nominal rate of interest is determined exogenously in the manner specified by Fisher [7], so that r, the real rate of interest is unaffected by changes in I, the anticipated rate of inflation. This assumption might be considered suspect in a model in which inflation distortions, due to a nonindexed tax system, affect the demand for borrowing. However, it leads to tractable analysis and facilitates comparisons between results in this paper and results presented in the previously mentioned capital markets literature.

[6] U.S. Department of Commerce. "Pricing Index of New One Family Houses Sold." (July 1978).

[7] U.S. Bureau of the Census. Statistical Abstract of the United States: 1978, (99th edition), Washington, D.C., 1978, p. 491.

[8] See, for example, Miller [12] and Blume, Crockett, and Friend [2].

[9] These choices were previously examined by Shelton [19], Mills [13], and Litzenberger and Sosin [11].

The Fisher assumption of a constant real interest rate can be weakened considerably without altering the results. The main result, that an increase in anticipated inflation causes housing prices to rise, holds under the much weaker condition that real, aftertax interest rates decline with an increase in anticipated inflation. This condition is consistent with the theoretical and empirical conclusions in the Feldstein and Summers [6] study of the effect of anticipated inflation on the demand for corporate capital, and is also consistent with empirical results presented by Fama [5] and Nelson and Schwert [17]. These empirical studies indicate that nominal interest rates may actually adjust by slightly less than the change in the anticipated rate of inflation. For example, Nelson and Schwert found that on average, a one percent rise in anticipated inflation led to a .889 percent rise in interest rates; the standard error of this estimate was .065. This estimated change in the interest rate with a change in anticipated inflation is substantially below the level which would alter the results in this paper.

In the next section, we present a model of a market restricted to owner-occupied housing and analyze the effect of anticipated inflation on its equilibrium. The results provide intuition which is useful in understanding the complete model, presented in Section II, in which a rental market is introduced.

I. Owner-Occupied Housing

Housing is assumed to consist of infinitely divisible, homogeneous units which do not depreciate and may be purchased without transaction costs. Individuals have perfect foresight and all markets are in a stable long run equilibrium in which population, incomes, and tastes are fixed, and the inflation rate is the same in every period. The supply of housing is an increasing function of its real price and is not affected by the rate of inflation.

Each individual has an exogenously determined marginal tax rate, τ_i, which, for simplicity, is assumed to be invariant with respect to the amount of housing he owns. Nominal interest payments are allowed as a deduction in computing taxable income and the imputed returns and capital gains accruing to homeowners are not taxable.

The net cost of consuming housing services is the difference between the aftertax interest cost of ownership and the amount by which the price of the house appreciates. With a constant rate of fully anticipated inflation, nominal housing prices will appreciate at the same rate as the general price level, since all factors which influence the demand and supply of housing are assumed to be fixed. The net per unit cost of consuming housing services for each consumer i $(i = 1, \cdots n)$ is thus

$$C_i = P[(r + I)(1 - \tau_i) - I] \tag{1}$$

where P is the purchase price of housing and the bracketed term is the aftertax cost of capital of the homeowner.

The demand for housing of each individual is assumed to be a decreasing function of the net cost of obtaining the housing services.

$$H_i = H_i\{P[(r + I)(1 - \tau_i) - I]\} \tag{2}$$

The market demand is then

$$H_d = \sum_{i=1}^{n} H_i\{P[(r + I)(1 - \tau_i) - I]\} \tag{3}$$

The equilibrium price satisfies the market clearing condition:

$$\sum_{i=1}^{n} H_i\{P[(r + I)(1 - \tau_i) - I]\} = H_S(P) \tag{4}$$

where $H_S(P)$ is the aggregate supply of housing.

To determine how a change in the anticipated rate of inflation affects the equilibrium price of housing, Equation (4) is totally differentiated and rearranged to yield:

$$\frac{dP}{dI} = \frac{\sum_{i=1}^{n} \tau_i P H_i'}{\sum_{i=1}^{n} [(r + I)(1 - \tau_i) - I]H_i' - H_S'} \tag{5}$$

Assuming that the net cost of consuming housing services is positive, so that $(r + I)(1 - \tau_i) - I > 0$,[10] $\frac{dP}{dI} \geq 0$ since $H_i' < 0$ and $H_S' > 0$. This means that unless the supply function is perfectly elastic, i.e. $H_S' = \infty$, real housing prices will rise when the anticipated rate of inflation increases. The increase in housing prices occurs because an increase in the anticipated rate of inflation decreases the aftertax cost of capital of each homeowner, and this in turn increases their demand for housing at any given purchase price.

The cost of housing services for a given individual is the product of the purchase price of housing and his aftertax cost of capital. Since the former increases and the latter decreases with the rate of inflation, it is not possible to determine a priori the direction of change in the cost of housing services due to a change in the inflation rate. We can, however, demonstrate that the cost of housing services must decline with an increase in the anticipated rate of inflation for those individuals with the highest marginal tax rates and, if the supply of housing is fixed, the cost of housing services must increase for individuals with the lowest marginal tax rates.

Using the chain rule,

$$\frac{dH_i}{dI} = \frac{dH_i}{dC_i} \cdot \frac{dC_i}{dI} \tag{6}$$

where

$$\frac{dC_i}{dI} = \frac{dP}{dI}[(r + I)(1 - \tau_i) - I] - \tau_i P \tag{7}$$

If the supply of housing is fixed, the aggregate change in housing demanded must be zero.

$$\sum_{i=1}^{n} \frac{dH_i}{dI} = 0 \tag{8}$$

[10] The Fisher effect implies that real, aftertax interest rates become negative when the anticipated rate of inflation is sufficiently high. If this occurs, the cost of housing services declines as the price of housing increases, and a stable equilibrium in the housing market will not exist. For this reason, the model is valid only for inflation rates sufficiently low so that real, aftertax interest rates are positive.

Therefore, if the housing stock is fixed, some individuals must increase their housing consumption and others must decrease their housing consumption as the anticipated rate of inflation changes. A change in anticipated inflation must, therefore, result in an increase in the cost of housing services for some individuals and a decrease in the cost for others.

By differentiating Equation (7) with respect to τ, we can determine whose costs will increase (or decrease) the most.

$$\frac{d^2C}{dI\,d\tau} = -\frac{dP}{dI}(r + I) - P < 0 \tag{9}$$

This means that the cost of housing services for individuals with relatively low marginal tax rates increases more (or decreases less) with an increase in inflation than the cost for the individuals with relatively high marginal tax rates. Since $\frac{dC_i}{dI}$ cannot have the same sign for all individuals, it must be negative for the ones with higher marginal tax rates and positive for the ones with lower marginal tax rates. It follows from Equation (6) that an increase in the inflation rate causes individuals with relatively high marginal tax rates to increase their housing consumption while individuals with relatively low marginal tax rates decrease their housing consumption.

If the supply of housing is not fixed, an increase in anticipated inflation will increase the total stock of housing as suppliers increase their output in reaction to the price increases. In this case, the cost of housing services for each consumer will decrease more (or increase less) than it would if the supply of housing were fixed. The cost of housing services for individuals with low marginal tax rates may, in this case, increase or decrease with a rise in anticipated inflation depending on the relative elasticities of the supply and demand functions.

In this section, it has been demonstrated that an increase in anticipated inflation causes the aftertax cost of capital for homeowners to decline, and this decline in turn causes the price of housing to increase. The decline in the aftertax cost of capital more than offsets the increase in purchase price for individuals with high marginal tax rates, so their cost of consuming housing services declines with an increase in anticipated inflation. The decline in the aftertax cost of capital does not, however, compensate individuals with low marginal tax rates for the increase in purchase price, so their cost of consuming housing services increases with the inflation rate. These distributional effects may differ if low income individuals are primarily renters and the high income individuals are primarily owners. This issue is addressed in the next section in a model in which the decision to own or to rent a home, along with the decision to purchase rental property is endogenous.

II. The Complete Model

In the previous section, we examined the effect of a change in the anticipated rate of inflation on a market restricted to owner-occupied housing. The model in this section is expanded to include rental housing. The model determines the amount of rental property each individual owns along with each individual's decision to buy or rent housing for personal consumption. It is shown that only those individuals with the highest marginal tax rates choose to invest in rental property, while the individuals with relatively low marginal tax rates become renters. Having established an equilibrium in this economy, comparative static analysis is used to determine the effect of a change in the anticipated rate of inflation on housing prices and rental rates.

A. *The Rental Property Investment Decision*

Individuals are infinitely lived and receive a constant endowment of taxable income in each period. They are in a steady-state in which their real, aftertax income equals consumption in every period, so that net savings equals zero. Any individual can potentially invest in rental property, but as we show, only those individuals with a high amount of taxable income will do so. Investment in rental property is financed exclusively through borrowing.

Although housing does not depreciate, landlords are allowed to deduct a fraction ψ of the book value of their property from their taxable incomes every period. Since the book value is based on historical cost, it declines in real terms due both to inflation and to the deductions claimed. Upon selling the property, one-half of the nominal capital gain, the difference between its current market value and its depreciated book value, is added to the landlord's taxable income.[11] Since the real book value of a house declines over time, landlords have an incentive to buy and sell property continually to keep the book value close to market value and maximize the depreciation tax shield. On the other hand, the capital gains tax creates an incentive to delay sales indefinitely. The optimal holding period minimizes the present value of the net taxes. Although this holding period, in reality, is a function of the anticipated rate of inflation, in our model it is determined exogenously. This assumption simplifies our analysis and, as we show, does not significantly affect our results.

Let X_t and x_t be the representative landlord's rental housing purchases and sales in period t. Note that x_t equals X_{t-s}, where s is the exogenously specified holding period. The landlord's total stock of rental property in period t, denoted H_t, then equals $\sum_{k=t-s+1}^{t} X_k$. The real book value of this property is denoted by B_t, and equals $\sum_{k=t-s+1}^{t} PX_k\left(\frac{1-\psi}{1+I}\right)^{t-k}$.

The landlord's consumption (ζ_t) in a given period consists of his endowment (Y), plus the income from his rental property (RH_t), minus his interest expenses and the taxes he must pay. Taxes (T) are a convex function of taxable income, which consists of his endowment, the rent he receives, minus his nominal interest expense ($(r + I)PH_t$) and the depreciation allowance (ψB_t) plus one-half of the capital gains income he receives $\left(\frac{1}{2}X_{t-s}P\left[\left(1-\left(\frac{1-\psi}{1+I}\right)^{s}\right]\right)\right)$. The marginal tax rate, T', is bounded above by one. Each landlord will want to maximize his

[11] In reality, the gain in nominal market value is taxed as a capital gain; however, part of the gain due to past depreciation may be taxed as ordinary income.

utility from consumption over his infinite life time. His total utility (V) is represented as the sum of his utility (U) from consumption in each future time period discounted by his rate of impatience (β). The maximization problem for each landlord is represented below:

$$V_k(X_{k-s}, X_{k-s+1}, \cdots X_{k-1}) = \max_{\{X_t\}_{t=k}^{\infty}} \sum_{t=k}^{\infty} \beta^{t-k} U(\zeta_t)$$

$$s.t. \quad X_t \geq 0 \tag{10}$$

where

$$\zeta_t = Y + RH_t - rPH_t - T\Bigg[Y + RH_t - (r + I)PH_t - \psi B_t + \frac{1}{2} X_{t-s}\left(1 - \left(\frac{1-\psi}{1+I}\right)^s\right)P\Bigg]$$

It can be shown that an equilibrium steady-state exists for this dynamic process, in which a solution to (10) consists of $X_k = X_{k+1} \cdots = X = Q$ when $X_{k-s} = X_{k-s+1} = \cdots = X_{k-1} = Q$. Assuming that the process converges to a steady-state prior to Period $-s$, the optimal period 0 purchase of rental property satisfies the following first order condition:

$$\frac{\partial V_0}{\partial X_0} = \sum_{t=0}^{s-1} \beta^t \left\{R - rP - T'\left[R - (r + I)P - \psi)\left(\frac{1-\psi}{1+I}\right)^t P\right]\right\} - \beta^s\, T' \frac{1}{2}\left[1 - \left(\frac{1-\psi}{1+I}\right)^s\right]P \leq 0 \tag{11}$$

Since we have assumed a steady-state, the marginal utility term is the same for each period and can be factored out of the expression.

Condition (11) will hold as an equality for all individuals who invest in rental property. If their impatience factors (β) are the same, it follows that their marginal tax rates T′ must also be equal. Let τ^* denote the rate for which (11) holds as an equality. That this rate is the highest marginal tax rate in the economy will be demonstrated by showing that rental property acts as a tax shelter which decreases the investor's taxable income. Competition for this tax shield causes equilibrium rental rates to fall below the before tax cost of supplying housing units, so that only those individuals who stand to gain the most from reducing their taxable incomes invest in rental property.

We first show that an individual's taxable income falls if his rental property holdings increase. This is easier to see if the capital gains tax paid in period s is transformed into an annuity. Each payment of this annuity represents the presents value of that part of the capital gains tax which accrued during the period. Making this transformation, Equation (11) becomes

$$\sum_{t=0}^{s-1} \beta^t \left\{R - rP - \tau^*\left[R - (r + I)P - \psi\left(\frac{1-\psi}{1+I}\right)^t P + \frac{1}{2}\beta^{s-t}\left(\frac{\psi + I}{1+I}\right)\left(\frac{1-\psi}{1+I}\right)^t P\right]\right\} = 0 \tag{12}$$

since

$$\sum_{t=0}^{s-1} \beta^t \beta^{s-t} \frac{1}{2}\left(\frac{\psi + I}{1 + I}\right)\left(\frac{1 - \psi}{1 + I}\right)^t = \frac{1}{2}\beta^s\left(\frac{\psi + I}{1 + I}\right)\sum_{t=0}^{s-1}\left(\frac{1 - \psi}{1 + I}\right)^t$$
$$= \frac{1}{2}\beta^s\left[1 - \left(\frac{1 - \psi}{1 + I}\right)^s\right].$$

The coefficient of τ^* in Equation (12) is the marginal effect of rental property purchases on taxable income. A negative coefficient implies that an individual who increases his investment in rental property lowers his taxable income. The coefficient must be negative because

$$I + \psi\left(\frac{1 - \psi}{1 + I}\right)^t > \frac{1}{2}\beta^{s-t}\left(\frac{\psi + I}{1 + I}\right)\left(\frac{1 - \psi}{1 + I}\right)^t$$

since

$$\frac{1}{2}\beta^{s-t} < 1 \quad \text{and} \quad \left(\frac{1 - \psi}{1 + I}\right)^t < 1$$

implying that the coefficient of τ^* could only be positive if $R - rP$ is positive. If this were the case, the *LHS* of (12) would be strictly positive since $0 < \tau^* < 1$, and thus would violate the first order condition.

In short, we have shown that an individual's taxable income declines with each additional unit of rental housing he purchases. The resulting tax savings are offset by the fact that landlords subsidize their tenants by the amount $rP - R$. If an individual's marginal tax rate is below τ^*, his tax savings from owning a unit of rental housing is less than the subsidy. Individuals of this type do not invest in rental property. If an individual's marginal tax rate is above τ^*, the reduction in taxes from owning an additional unit of rental housing more than compensates him for the subsidy. He continues to buy rental property, decreasing his taxable income, until his marginal tax rate falls to τ^*. Indeed, $\frac{\partial V_0}{\partial X_0}$ is negative for $\tau_i < \tau^*$, and positive for $\tau_i > \tau^*$.

Arbitrage by corporations will keep this critical tax rate equal to the corporate rate. If the critical tax is below the corporate rate, taxpaying corporations find it profitable to invest in rental property. Since the corporate tax rate is constant, this forces the critical tax rate to rise to the corporate tax rate. If the critical rate is above the corporate rate, corporations find it cheaper to lease rather than to own their equipment and buildings. They again continue to do so until the critical tax rate equals the corporate tax rate. When the rates are equal, corporations are indifferent between owning or renting their property and equilibrium exists.

B. *The Consumer*

An individual's demand for housing is inversely related to the relevant cost of housing services. This cost is either the cost of homeownership calculated in the previous section or the rental rate. Since Condition (11) holds as an equality for all individuals who own a positive quantity of rental property, it can be solved for the equilibrium rental rate. This can be accomplished by separating the terms unaffected by the time superscript, dividing through by $\sum_{t=0}^{s-1} \beta^t$ and solving for rent:

$$R = rP - \frac{\tau^*}{1-\tau^*} P \left[I + \psi \frac{\sum \beta^t \left(\frac{1-\psi}{1+I}\right)^t}{\sum \beta^t} - \frac{\frac{1}{2}\beta^s \left[1 - \left(\frac{1-\psi}{1+I}\right)^s\right]}{\sum \beta^t} \right] \tag{13}$$

This rental rate depends only on the rate of interest, the purchase price of housing, and the effect of the federal income tax laws. The cost of owning a home, given by Equation (1), is also only a function of these factors. The difference between this derived rental rate and the calculated cost of owning can, therefore, be attributed to the relative differences in the effective federal tax treatment of these two methods of acquiring housing services. Aaron [1] has argued that the federal income tax discriminates against renters. He failed, however, to account for the depreciation deduction landlords receive. The following analysis indicates that if allowable depreciation is greater than the real rate of interest,[12] and if the marginal tax rate of landlords is above 50 percent, the value of the deductions which landlords pass on to renters actually exceed the tax advantages from homeownership.[13]

This is established by comparing the periodic cost of ownership C, given by Equation (1), with the rental rate $\hat{R}$, which would prevail if landlords were constrained to sell the property after one period. $\hat{R}$ is obtained by setting $s = 1$ in Equation (13):

$$\hat{R} = rP - \frac{\tau^*}{1-\tau^*} P \left[I + \psi - \frac{1}{2}\beta\left(1 - \frac{1-\psi}{1+I}\right)\right]$$

Note that the equilibrium rental rate R will not exceed $\hat{R}$ since landlords do not face the holding period constraint imposed in deriving $\hat{R}$. It suffices, therefore, to show that $C > \hat{R}$ to establish that the federal tax subsidy to renters exceeds that to homeowners. This is accomplished by expressing $\hat{R}$ and C as

$$\hat{R} = P\left\{r - \frac{\tau^*}{1-\tau^*}\left[1 - \frac{1}{2}\beta\left(\frac{1}{1+I}\right)\right]\psi - \frac{\tau^*}{1-\tau^*}\left[1 - \frac{1}{2}\beta\left(\frac{1}{1+I}\right)\right] I\right\}$$

$$C = P[r - r\tau - I\tau]$$

The result that $\hat{R} < C$ follows from our assumption that $\psi \geq r$, and $\tau^* \geq .5$, since

$$\left[1 - \frac{1}{2}\beta\left(\frac{1}{1+I}\right)\right] > \frac{1}{2} \quad \text{and} \quad \frac{\tau^*}{1-\tau^*} \geq 2\tau$$

The preceding analysis indicates that all individuals should rent rather than

[12] For most buildings $\psi = 4.5\%$. Land cannot be depreciated but generally is worth less than 20% of the total property value. The real rate of interest is generally believed to average less than 3%.

[13] Two important aspects of this problem which were ignored in Aaron's work are also ignored in this analysis. They are the property tax deduction homeowners receive and the standard deduction that can be taken by renters. When these factors are added to the model, it is not clear whether our tax system discriminates against renters or owners.

own their homes. The derived rental rate and cost of homeownership do not, however, include all the relevant costs incurred in providing housing services. When these additional costs are taken into account, homeownership may be less expensive than renting for some individuals.

Shelton [18] listed the following costs which landlords incur: maintenance (M), insurance (Ins), management costs (MF), property tax (PT), and an allowance for vacant apartments (VA). Since these costs are proportional to the number of housing units rented, they can be directly added to the rent given in Equation (13) to determine the total per unit rental rate.

$$R^* = rP - \frac{\tau^*}{1 - t^*} P \left[I + \psi \frac{\sum \beta^t \left(\frac{1 - \psi}{1 + I} \right)^t}{\sum \beta_t} - \frac{\frac{1}{2} \beta^s \left[1 - \left(\frac{1 - \psi}{1 + I} \right)^s \right]}{\sum \beta^t} \right]$$
$$+ (M + Ins + PT + VA + MF) \tag{14}$$

Homeowners must also pay for maintenance, insurance, and property tax. Since homeowners can deduct property tax payments from their taxable incomes, they bear only the proportion $(1 - \tau_i)$ of the tax.[14] Another cost which has been largely ignored is due to the standard deduction (STD) which can be claimed by most renters but not by homeowners who claim interest and property tax deductions. This tax deduction is generally higher than the nonhousing related deductions (NHD) claimed by homeowners. The tax loss due to the difference between the standard deduction and the nonhousing deductions must, therefore, be considered a cost of homeownership. Given these additional costs, the total per unit cost of homeownership can be expressed as the following:

$$C_i^* = P[(r + I)(1 - \tau_i) - I] + (1 - \tau_i)PT$$
$$+ M + Ins + \tau_i[STD - NHD]/H \tag{15}$$

The term $[STD - NHD]/H$ must be less than $[(r + I) + PT]$ if homeowners choose to itemize their deductions. An individual's cost of owning a home is thus a decreasing function of his marginal tax rate. The cost of renting a home, however, is the same for all individuals. This implies that if individuals choose the lowest cost method of obtaining housing services, those individuals with relatively high marginal tax rates will own their homes while those with relatively low marginal tax rates will rent. Also notice that an increase in property tax tends to increase the rental rate relative to the cost of homeownership. This indicates that there should be relatively more homeowners in areas with high property tax than in areas with low property tax.

C. Equilibrium

The additional costs of acquiring housing services which were discussed previously are considered exogenous to the equilibrium analysis that follows. These variables are assumed to be invariant with respect to changes in the anticipated

[14] See Mills [13] for a more thorough discussion of this.

rate of inflation, the price of housing, and the rental rate. The demand for housing services, by a homeowner or renter, can be expressed as a function of the cost of acquiring housing services given by Equations (1) and (13). The quantity of housing demanded by a renter is assumed to be a decreasing function of the rental rate, R.

$$H_j = H_j(R) \tag{17}$$

The aggregate demand for rental housing is obtained by adding the demands of the m renters.

$$H_{D,R} = \sum_{j=n+1}^{m+n} H_j(R) \tag{18}$$

The supply of rental housing can be expressed as a function of the rental rate and the purchase price of housing.

$$H_{S,R} = H_{S,R}(R, P) \tag{19}$$

By rearranging Equation (12) in the following manner, we can establish that

$$\frac{\partial H_{S,R}}{\partial R} > 0 \quad \text{and} \quad \frac{\partial H_{S,R}}{\partial P} < 0$$

$$\sum_{t=0}^{s-1} \beta^t \left\{ \frac{R}{P} - r - \frac{\tau^*}{1-\tau^*} \left[-I - \psi\left(\frac{1-\psi}{1+I}\right)^t + \frac{1}{2}\beta^{s-t} \left(\frac{\psi+I}{1+I}\right)\left(\frac{1-\psi}{1+I}\right) \right]^t \right\} = 0 \tag{12'}$$

An increase in R, holding P constant, causes the critical equilibrium marginal tax rate to decrease if the *LHS* of (12′) is to remain equal to zero. Landlords must, therefore, hold more rental property; otherwise, their taxable incomes and marginal tax rates rise. For the same reason, if the rental rate is positive, an increase in P, holding R constant, leads to a decrease in the supply of rental housing.

The equilibrium rental rate along with the equilibrium quantity of rental housing must satisfy the following market clearing condition:

$$H_{D,R}(R) = H_{S,R}(R, P) \tag{20}$$

The derived properties of the supply function $\left(\text{e.g.}\ \frac{\partial H_{S,R}}{\partial R} > 0\right)$ along with the assumed properties of the demand function $\left(\text{e.g.}\ \frac{\partial H_{D,R}}{\partial R} < 0\right)$ indicate that for a given purchase price of housing, a rental rate exists which satisfies the above equilibrium condition. Which individuals rent, and which own; the quantity of housing services consumed by each renter; the amount of rental property owned by each landlord; and the equilibrium rental rate is determined in this equilibrium as a function of the purchase price of housing. In order to complete this analysis total housing demand and its purchase price must be determined endogenously.

The equilibrium price of housing equates the aggregate demand for housing services with its aggregate supply. The aggregate demand function is calculated by deriving the demand for rental housing and adding it to the demand for owner-occupied housing determined in the previous section. If the calculated cost of homeownership is positive, then the demand for owner-occupied housing is a decreasing function of its purchase price. The demand for rental housing, which is an implicit function of its purchase price, is also a decreasing function of this variable. This is shown by defining a function $F(R, P)$, which equals the aggregate demand for rental housing minus its aggregate supply. The market clearing condition implies that $F(R, P)$ always equals zero.

$$F(R, P) = H_{D,R}(R) - H_{S,R}(R, P) = 0 \tag{21}$$

The implicit function rule states that

$$\frac{dR}{dP} = -\frac{F_P}{F_R} = \frac{\dfrac{\partial H_{S,R}}{\partial P}}{\dfrac{\partial H_{D,R}}{\partial R} - \dfrac{\partial H_{S,R}}{\partial R}} \tag{22}$$

The reader can verify that $\frac{dR}{dP}$ is positive if R is greater than zero. A direct application of the chain rule establishes that $\frac{dH_j}{dP}$ is negative if $\frac{dH_j}{dR}$ is less than zero, which establishes the result that renters consume less housing services when the purchase price of housing increases.

The preceding discussion indicates that the aggregate demand function, defined in the following manner, is downward sloping:

$$H_D(P) = \textstyle\sum_{i=1}^{n} H_i(P) + \sum_{j=n+1}^{m+n} J_j(P) \tag{23}$$

The function is continuous, but is not continuously differentiable. Kinks in the function appear because individuals change their buy/rent decision when housing prices change. However, both right- and left-side derivatives can still be easily obtained.

The housing market equilibrium can now be fully defined by imposing the following market clearing condition:

$$H_D(P) = H_S(P) \tag{24}$$

where $H_S(P)$ is again defined as the aggregate supply of housing.

The derived properties of the demand function $\left(\text{e.g. } \frac{dH_D}{dP} < 0\right)$ along with the assumed properties of the supply function $\left(\text{e.g. } \frac{dH_S}{dP} > 0\right)$ establish the existence of an equilibrium price. Having fully specified this equilibrium, comparative static analysis can now be used to determine how a change in the anticipated rate of inflation affects this market.

In a market limited to owner-occupied housing, housing becomes more valuable with increased inflation because homeowners deduct nominal interest payments from their taxable income and are not taxed on the subsequent capital gain. Landlords also receive the nominal interest deduction and are only taxed on part of the resulting capital gain. Inflation, however, lowers the real value of their depreciation tax shield by lowering the real book value of their property, which tends to increase their tax payments. The following analysis demonstrates that these two inflation-induced tax effects net an increase in the value of rental housing. Since both homeowners and landlords consider housing more valuable when the anticipated rate of inflation increases, its price must increase if its supply is not perfectly elastic. This can be established by taking the total derivative of the market clearing condition presented in the following form:

$$\sum_{i=1}^{n} H_i(C) + \sum_{j=n+1}^{m+n} H_j(R) = H_S(P) \tag{24'}$$

The analysis assumes that corporate arbitrage holds τ^* equal to the corporate tax rate so that $\frac{d\tau^*}{dI}$ equals zero. Taking this into account, the total derivative of the above equation is

$$\sum_{i=1}^{n} H_i'\,dC + \sum_{j=n+1}^{m+n} H_j'\,dR = H_S'\,dP \tag{25}$$

where

$$dC = [(r + I)(1 - \tau_i) - I]\,dP - \tau_i P dI \tag{25i}$$

$$dR = -\left(\frac{\tau^*}{1 - \tau^*}\right) P(1 - \theta)\,dI + \lambda\,dP \tag{25ii}$$

where

$$\theta = \frac{\psi \sum \beta^t \cdot t\left(\frac{1-\psi}{1+I}\right)^{t-1} \cdot \frac{1-\psi}{(1+I)^2}}{\sum \beta^t} + \frac{\beta^s \cdot s\left(\frac{1-\psi}{1+I}\right)^{s-1} \cdot \frac{1-\psi}{(1+I)^2}}{2\sum \beta^t}$$

and

$$\lambda = r - \frac{\tau^*}{1-\tau^*}\left[I + \psi\frac{\sum \beta^t\left(\frac{1-\psi}{1+I}\right)^t}{\sum \beta^t} - \frac{\beta^s\left[1 - \left(\frac{1-\psi}{1+I}\right)^S\right]}{2\sum \beta^t}\right]$$

The reader can verify that θ is positive but less than ½ for reasonable values of β, ψ, I, and s. Also note that λ must be positive if the rental rate is positive. Solving Equation (25) for $\frac{dP}{dI}$ yields:

$$\frac{dP}{dI} = \frac{\sum_{i=1}^{n} \tau_i P H_i' + \sum_{j=n+1}^{m+n} \frac{\tau^*}{1-\tau^*} P(1-\theta) H_j'}{\sum_{i=1}^{n} [(r + I)(1 - \tau_i) - I] H_i' + \sum_{j=n+1}^{m+n} \lambda H_j' - H_S'} \tag{26}$$

If our analysis is confined to inflation rates where real, aftertax interest rates and rental rates are positive, $\frac{dP}{dI}$ must be positive. It should be noted that $\frac{dP}{dI}$ is not defined at all points. Changes in the inflation rate may cause consumers to change their buy/rent decision, causing a kink in the price function. However, both right- and left-side derivatives are positive at all kinks since the preceding derivative was shown to be positive for a completely general n and m.

The effect of anticipated inflation on rental rates can also be determined within this framework. The approach used in the previous section to show how a change in the anticipated rate of inflation affects the relative cost for homeowners with different marginal tax rates can be employed to show that rental rates must decrease in relation to the cost of owning for all individuals if the anticipated rate of inflation increases. If the price of housing, and consequently its supply, increases with the anticipated rate of inflation, rental rates and the cost of homeownership cannot both increase. Thus, if $\frac{dR}{dI}$ is less than $\frac{dC}{dI}$ for all homeowners, rent must decrease if the anticipated rate of inflation increases.

Solving (25ii) for $\frac{dR}{dI}$ yields

$$\frac{dR}{dI} = -\frac{\tau^*}{1-\tau^*} P(1-\theta) + \lambda \frac{dP}{dI} \tag{27}$$

The expression $\frac{dC}{dI}$ was represented in Equation (7) as

$$\frac{dC}{dI} = \frac{dP}{dI}[(r+I)(1-\tau) - I] - \tau P \tag{7}$$

The coefficient of P in Equation (27) is less than its counterpart in Equation (7) since $\frac{\tau^*}{1-\tau^*} > 2\tau$ and the term in brackets is greater than ½ (recall $\theta < ½$). Since the derived rental rate is always below our calculated cost of owning, the coefficient of $\frac{dP}{dI}$ in Equation (27) is lower than the corresponding coefficient in Equation (7). This demonstrates that $\frac{dR}{dI}$ is less than $\frac{dC}{dI}$ for all homeowners which in turn implies that an increase in the anticipated rate of inflation causes the real rental rate to decline. Furthermore, since the real rental rate falls in relation to the cost of homeownership, the relative number of renters should increase when the anticipated rate of inflation increases.

In deriving these results, we implicitly assumed that the holding interval s remains constant when the anticipated rate of inflation changes. In reality this may not be the case, but the implications of this model are not substantially different when the holding interval is allowed to change. Allowing landlords to change the length of the holding interval when the inflation rate changes must lower their costs, which in turn reduces rental rates in a competitive market. This implies that the actual decline in rental rates associated with an increase in inflation is probably steeper than the preceding analysis indicates.

It should also be noted that the derived relationship between rental rates and the anticipated rate of inflation holds under quite strong conditions. The analysis implies that for a fixed supply of housing, an increase in inflation causes rental rates to decline more than the corresponding decline in cost for each homeowner. If the supply of housing is not fixed, then the rental rate declines even more with an increase in the anticipated rate of inflation.

III. Conclusion

In this study a housing market equilibrium was defined. It was demonstrated that because the U.S. tax system is not indexed for changes in the inflation rate, an increase in the anticipated rate of inflation causes real housing prices to rise, and real rental rates to fall. If the supply of housing is not perfectly inelastic, consumption of housing services increases.

The results in this paper complement the results pertaining to corporate capital obtained by Feldstein and Summers [6], Hong [8], Linter [10], Motely [15], and Nichols [18]. The analysis in these papers imply that if the Fisher effect holds, an increase in the anticipated rate of inflation results in a net outflow of capital from the corporate sector. The Fisher interest assumption in this paper implies that an increase in the anticipated rate of inflation results in a net inflow of capital into the economy's housing sector. These results, if considered separately, lead one to suspect the validity of the Fisher interest hypothesis in an economy with taxes. However, when the results are considered together, Fisher's theory of interest rates and inflation may be considered a reasonable approximation. This will be the case if the relative elasticities of demand and supply for housing and corporate capital are such that the increase in the demand for housing capital (given Fisherian interest rates), caused by an increase in anticipated inflation, approximately equals the corresponding decrease in the demand for corporate capital.

If the Fisher effect does not hold, the price of housing should still increase relative to stock prices with an increase in the anticipated rate of inflation. Real estate, in this paper, was shown to be more valuable with an increase in inflation since the owner's nominal interest tax deductions increase with inflation. Corporations, however, do not benefit by as much from an increase in nominal interest rates since only the payments to debt holders are tax deductible. The analysis in this paper seems to indicate that the effect of anticipated inflation on the value of a capital asset depends in part on its debt capacity. Additional research, however, is needed to further clarify these issues.

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