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BY JOHN NAGORNIAK*

INTRODUCTION

Several recent writings have raised questions concerning the effectiveness of commonly used analytical measures and techniques which have their basis in Modern Portfolio Theory. Questions have been raised about the use of beta in the interpretation of risk adjusted equity performance and about the usefulness of the capital asset pricing model (CAPM) as a tool for measuring and interpreting investment performance. This paper restricts itself to questions concerning beta and CAPM as they relate to performance measurement. In the process of this examination it discusses some basic principles of performance measurement as they relate to judgments of investment management skill and ability.

It has been shown that simple applications of beta used in adjusting equity performance for risk can possibly produce *misleading or incorrect* interpretations of risk adjusted performance. Errors have been shown to be related to the market index used as a basis for the determination of beta. These findings are independent of the validity of CAPM, but the line of reasoning which leads to these conclusions is not totally antithetical to the derivation of CAPM. This paper has three major sections:

1. The capital asset pricing model is assumed to be valid, and a broader index of capital assets is constructed. This index is "better" in the sense that it includes many more assets than the S&P 500 and hence is more complete.
2. The better index is then used to "game" performance measurement using current investment technology and the Standard & Poor's 500.
3. Some conclusions are drawn as to the effectiveness of gaming and the ways in which performance measurement numbers should be interpreted in general.

General issues are discussed in the text. A parallel, more detailed and more technical argument is presented in the appendix.

Choosing a Market Index

While the capital asset pricing model is developed with respect to all capital assets, applications have traditionally been restricted to equity securities, and usually to a subset of equity securities in the United States. One such subset of equities, the Standard & Poor's 500 Index, represents about half of the total equity value outstanding in the United States and a much smaller proportion of all capital assets. Arguments advanced in support of this procedure have usually been as follows: (1) the present application is "good enough" for the practical

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purposes of performance measurement and portfolio management, (2) a more complete and correct model would be too costly, and (3) since the true index is impossible to measure, complete theoretical correctness cannot be attained. This paper attempts to examine the performance measurement questions by choosing a broader substitute for the Standard & Poor's 500 Index.

The original development of CAPM by Sharpe [1964] and Lintner [1965] showed the pricing of assets given the appropriate assumptions with respect to a market index of all capital assets. The problems associated with validating the theory have been apparent since the beginning. A market of all capital assets does not include only equities, let alone only a subset of equities outstanding. It includes holdings of all assets which represent investment for future consumption. Housing, automobiles, and other difficult to value assets which represent investment are included in the market of all capital assets. Because of the measurability problem alone, it seems likely that the capital asset pricing model cannot be definitely validated or invalidated. Our alternative index to the Standard & Poor's 500 Index is a much broader index of assets. It includes all outstanding equities, government and corporate bonds, treasury bills, and a proxy for investment real estate. This index is called the Complete Index throughout the rest of this paper and in the appendix. Since the Complete Index is not in fact complete, the findings of this paper must be accepted with a due amount of skepticism concerning their definitiveness.

Beating the Performance Measurement Services

After the Complete Index was constructed, we determined a "gaming" strategy for producing favorable results based on traditional performance measurement techniques. The following procedures were used and the following assumptions were made:

1. The capital asset pricing model was assumed to be applicable to the Complete Index.
2. The theoretical "alpha" which would be calculated using the present technology was determined from our knowledge of the Complete Index. (The mathematics of this process are shown in the appendix.)
3. A high and a low projected alpha portfolio were selected. An assumption was made in the construction of these portfolios that the Complete Index would have a positive risk premium. It was hypothesized that performance measurement during a period following our alpha projections using traditional methods would indicate that we were able to make superior investment judgments and select positive and negative alpha securities. In fact we had no knowledge, but merely projected the bias which would be reflected in traditional performance measurement methods.

Results of Our Analysis

After our efforts at constructing portfolios with a positive and negative bias, we applied methods of calculating risk adjusted equity performance which utilized

Standard & Poor's 500 average. Performance was tested for a two year period *after* the bias calculations. We used a commercially available performance analysis service which represents currently available investment technology. The service chosen utilizes beta as a measure of risk adjusting performance, but it also recognizes the presence of other factors in the determination of equity performance. Using such methodology, superior and inferior investment performance can be decomposed into sources. The sources are specific security selection, selection of the types of securities held, and selection of superior performing industries. Selection capabilities are measured relative to the positions held in the market portfolio which, in this case, is the Standard & Poor's 500 Index.

Portfolios which we expected to have positive alpha had positive alpha over the test period, and portfolios which we had expected to have negative alphas had negative alphas. When the results were reviewed on a disaggregated basis, the differences in performance manifested themselves mostly on an industry level. The positive alpha portfolio had an overweighting (relative to the S&P) in industries which performed well over the test period, and the negative alpha portfolio was overweighted in industries which performed poorly. This was the most significant pattern to emerge from the data.

The conclusions emerging from this paper are numerous. It seems clear that simple-minded performance measurement approaches could be beaten by an appropriate "gaming" strategy. The results of the decomposition of investment performance are less clear. A simplistic analysis of the data would suggest that our positively biased portfolio performed well through superior portfolio management skills, evidenced by the portfolio's overweight in high performing industries. A more subtle interpretation is also possible. The measurement of sector selection skills (security type and industry) should be calibrated to positions other than those of the S&P 500. A calibration could exist which would be appropriate in terms of our broad market index and which would show our gaming strategy to have no skill. We have not explored this issue, but the conclusion that the S&P is not an appropriate benchmark for gauging equity performance is certainly consistent with the results of our analysis.

APPENDIX

Historical Data Utilized

The "Complete Index" was calculated from data as follows: Treasury bill returns were obtained from the study by Ibbotsen and Sinquefield [1979]. Returns on bonds were obtained from Lehman Brothers Kuhn Loeb, Inc. and included both government and corporate issues. Returns on equities were obtained from a broad index of securities maintained by Barr Rosenberg Associates. This index covers approximately 5,000 stocks.

Real estate returns were obtained from a study by James Hoag [1980]. The returns data had a substantial amount of serial correlation which we assumed to be measurement error. This was corrected before combining the data into the complete index. The correction for serial correlation involved a simple autocorrelation adjustment.

The amount outstanding of each of these asset types was obtained from calculations as of 12/31/79. Amounts outstanding for Treasury bills and bonds were obtained from Lehman Brothers Kuhn Loeb, Inc. Equities outstanding were obtained from a valuation calculated by Wilshire Associates. Real estate outstanding was estimated by Charles Kelso of Buck Consultants, Inc.

Five years of historical data for all these series were available prior to June of 1978. An index of bonds, bills, stocks and real estate was calculated using these returns and weights.

Projecting Performance Analysis Bias[1]

Prospective betas were assumed to be equal to those obtained from regressions using five years of historical data. The following notation is used:

$$\begin{aligned} M &= \text{S\&P 500 Return} \\ C &= \text{“Complete” Index Return} \\ F &= \text{Risk free return} \\ i &= \text{Return for asset (or portfolio) } i \\ E(j) &= \textit{Excepted return for } j \end{aligned}$$

Let

$$\begin{aligned} B(i) &= \text{Beta of } i \text{ on Complete Index} \\ B(M) &= \text{Beta of S\&P 500 with respect to the Complete Index} \\ b(i) &= \text{Beta of } i \text{ on S\&P 500} \end{aligned}$$

Assuming the Capital Asset Pricing Model, we see that:

$$E(i) - E(F) = B(i) \cdot [E(C) - E(F)] \tag{1}$$

Define the variable DIF to be the bias present in CAPM analysis using the S&P 500 instead of the "Complete Index":

$$E(i) - E(F) = b(i) \cdot [E(M) - E(F)] + \text{DIF} \tag{2}$$

Note that in equation (2), DIF can have any value. Since we have assumed CAPM, equation (4) holds for S&P 500 returns:

$$E(M) - E(F) = B(M) \cdot [E(C) - E(F)] \tag{3}$$

Equations (1)–(3) imply:

$$\text{DIF} = [B(i) - B(M) \cdot b(i)] \cdot [E(C) - E(F)] \tag{4}$$

The interpretation of this equation is that the expected bias present in any CAPM application solving for the alpha of i based on the S&P 500 can be related to four quantities:

—The Beta of i on the Complete Index (i.e., its "true" beta)
—The Beta of the S&P 500 on the Complete Index (i.e., the S&P's "true" beta)
—The Beta of i on the S&P (its "false" beta)
—The expected risk premium of the true market index

[1] This derivation is developed in a different manner by Roll [1980].

Given a positive risk premium for the market of capital assets, we can project that alphas calculated using the S&P for purposes of performance measurement will be proportional to:

$$B(i) - B(M) \cdot b(i) \tag{5}$$

Testing for Performance Analysis Bias

For a diversified list of 400 securities, expression (5) was calculated using historical regressions. These values were then sorted from high to low. The top quintile and bottom quintile were selected as portfolios likely to exhibit favorable and unfavorable risk adjusted performance using the S&P 500 as the index. On an *ex-ante* basis, these top and bottom quintile portfolios could be expected on average to exhibit a large difference in measured risk adjusted performance in periods of a normal risk premium for the market. Expression (5), when calculated, produced a value of .6 (equal weighted) for the top quintile of securities and −.72 for the bottom quintile. A performance analysis of these two portfolios was performed encompassing experience from July 1978 through June 1980, which was after the calculation of parameters.

The performance system is based on a model of equity returns and risk which allows for common elements of return due to the market, security characteristics and industry membership (Rosenberg [1978]). Returns and risk specific to individual securities are also allowed for. Performance analysis based on this system is conditional on the risk model. While the risk model goes beyond the simple beta adjustment for risk as suggested in Roll's work, it contains the single beta element as a component. For elements of return and risk, experience is analyzed in a multiple factor framework. The following equations show how this occurs:

RF = risk free return
B = portfolio Beta
M = market risk premium
f = vector of factor returns
a = vector of factor loadings
e = specific return
$V(M)$ = variance of market return
F = variance matrix of factor returns
$V(e)$ = variance of specific return

$$\text{Portfolio return} = B\,M + a^t f + e + RF$$
$$\text{Portfolio variance} = B^2 V(M) + a^t F a + V(e)$$

In this formulation, an old-fashioned alpha $= a^t f + e$. It has a variance of $a^t F a + V(e)$

As can be seen, the alpha can be decomposed into various factors and aggregates of factors. The same decomposition can be performed for the risk associated with the experienced alpha. Factor loadings and factor returns can be manipulated in such a way that they are residual to the market index. For components of the experienced alpha, the ratio of returns to the risk of these returns can be

Table 1

Annualized Increment to Return

	Equal Weighted Analyis		Cap Weighted Analysis	
	Positive Bias	Negative Bias	Positive Bias	Negative Bias
Alpha	9.54	.43	11.14	−6.56
Decomposition of Alpha:				
Industry Selection	1.50	−6.59	11.54	−8.72
Types of Stocks Emphasized	6.01	6.07	1.45	1.80
Individual Security Selection	2.43	1.15	−1.48	.14

Table 2

Significance Statistics for Deviations of Portfolios from S&P Index[a]

	Equal Weighted Analysis		Cap Weighted Analysis	
	Positive Bias	Negative Bias	Positive Bias	Negative Bias
Model Variance				
Alpha	2.07	.02	2.26	−1.33
Decomposition of Alpha:				
Industry Selection	1.04	−2.58	3.30	−2.78
Type of Stocks Emphasized	1.39	1.32	.75	.77
Individual Security Selection	1.08	.40	−.50	0
Experienced Variance				
Alpha	2.16	.03	2.38	−1.60
Decomposition of Alpha:				
Industry Selection	.79	−1.95	2.62	−2.08
Type of Stock Emphasized	2.08	1.91	1.22	1.44
Individual Security Selection	1.02	.48	−.56	0

[a] Tested as standard normal variables.

developed. Periods of experience can be aggregated and statistical measures of significance can be calculated. Performance analysis can be carried out using both predicted model variation and actual experience. Actual experience is, of course, calculated conditional upon the constructs of the risk model.

The positively biased and negatively biased portfolios were analyzed on both an equal weighted and capitalization weighted basis. This differentiation did not produce dramatically different results. Tables 1 and 2 display the actual performance from July 1978 through June 1980.

As can be seen, when the alpha is decomposed into individual components, some meaningful and "correct" differences in performance are due to industry selection. The phenomenon hypothesized by Richard Roll is consistent with our findings. We have been able to simulate some statistically significant investment skills.

In summary, the expected biases in performance measurement occur. When the source of alpha is analyzed in detail, it can be seen that these biases are primarily due to industry allocations of the portfolios with respect to the Standard & Poor's 500. An economic interpretation of these results is that the correlations of these industries with the broader market index are meaningfully different from the correlations of these industries with the Standard & Poor's 500, and that this differentiation produces a bias when results are analyzed with respect to the Standard & Poor's 500.

In addition to this interpretation, the data suggests that a different normal equity portfolio than the Standard & Poor's 500 could be used with this performance service to measure performance in a manner consistent with the broad market index determining performance. If this normal portfolio were appropriately constructed, the bias present in our numbers would have been removed. Using this procedure, current investment technology could more appropriately analyze equity performance.

The above exercise suggests that performance measurement procedures are complex and depend on many assumptions. Our findings, while ambiguous, support some of the contentions of Richard Roll. They suggest it is possible to game performance measurement utilizing current techniques. They also illustrate the difficulties of the performance measurement process, and the fact that any performance analysis is of necessity a complex and imperfect procedure. It is hoped that this is the primary impression left by this paper.

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