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RETURN EXPECTATIONS IN ACTIVE INVESTMENT MANAGEMENT

Presiding: THOMAS FARQUHAR*

Factor-Related and Specific Returns of Common Stocks: Serial Correlation and Market Inefficiency

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INTRODUCTION

One of the most basic tests of market efficiency is the test for serial correlation of returns. If the return on a typical stock in period t is correlated with its return in period $t-1$, then the best (unbiased minimum mean square error) prediction for the return in period t equals the prior return multiplied by the correlation coefficient. An investor concerned only about the mean and variance of portfolio return would use the prediction to improve the mean/variance trade-off of portfolio return by shading the portfolio toward stocks predicted to have high returns and away from stocks predicted to have low returns. Hence, using knowledge about the previous month's returns, the investor produces a portfolio with superior (mean/variance) performance. This violates the weak form of market-efficiency (Fama [1]), in the sense that the expected return for each stock, based upon knowledge of the previous month's return, is different from the expected return based only upon the current stock price.

The same argument may be applied to any component of returns, as distinct from total return. For example, in a linear multiple-factor-plus-specific-return model of security returns, total excess return is decomposed into factor-related return and specific return. The empirical results in this paper find positive serial dependence in the factor-related component and negative serial dependence in the specific component which nearly offset one another, resulting in zero correlation in total excess returns. Formally,

$$\rho_r = \frac{V_c\rho_c + V_u\rho_u}{V_c + V_u}, \tag{1}$$

where ρ_r, ρ_c, and ρ_u are, respectively, the first-order serial correlations in excess returns, factor-related returns, and specific returns, and V_c and V_u are the variances of common-factor and specific returns. Thus, the serial correlation of total return is a weighted average of the correlations of the components.

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It follows that if the serial correlation of total returns is zero, then the market is not necessarily efficient with regard to lagged return. If either of the two components has a significant correlation, then, since the components are independent of one another, the investor can exploit the serial correlation of that component for abnormal gains. However, the investor who is unaware of the distinction between components of return, and who studies total return only, will miss this opportunity. Therefore, it is interesting to test whether there is serial correlation in either or both of the factor-related and specific components of return.

In the next section we specify a multiple factor model for security returns. We then discuss possible explanations for observing serial dependency in the components of return. Section IV describes our empirical results while the final section provides a discussion of the implications of the results.

I. Factor-Related and Specific Returns

The logarithmic or continuously compounded excess rate of return for stock n in month t is defined as:

$$r^*_{nt} = \ln(p^*_{n,t} + DIV_{n,t}) - \ln(p^*_{n,t-1}) - \ln(1 + i_{Ft}), \tag{2}$$

where $p^*_{n,t}$ is the "efficient market price" (i.e., the price that would clear the entire market in the absence of transaction and information costs, and other impediments to trading) for stock n at month-end t, $DIV_{n,t}$ is the value of dividends earned during month t as of month-end t, i_{Ft} is the riskless rate of interest of month t, and r^*_{nt} is the "efficient excess rate of return." A set of "descriptors" for stock n, which are predetermined in the sense of being available to market participants at the beginning of period t, can be used as factor coefficients or factor loadings to define a set of common factors, f^*_t, in month t. The factor-related return for each stock is then:

$$c^*_{nt} = x_{1nt}f^*_{1t} + \cdots x_{Jnt}f^*_{Jt}, \tag{3}$$

where the x_{jnt}, $j = 1, \cdots, J$ are the descriptor values for firm n at the beginning of month t for the J descriptors, and the factors, f^*_{jt}, $j = 1, \cdots, J$, are the common factors of return that influence all assets. Hence c^*_{nt} is the factor-related return arising from the sum of the J factors' contributions. Again, a superscript * has been used to denote an "efficient market" value of each factor and of the total factor-related return. The excess return on the asset is then decomposed as:

$$r^*_{nt} = c^*_{nt} + u^*_{nt},$$

where u^*_{nt} is that part of the excess return that is not accounted for by the common factors and which may be viewed as "specific" to the stock. Therefore, u^*_{nt} is the "efficient-specific return."

This framework is a linear multiple-factor-plus-specific-return model of logarithmic returns. The model is operationalized by providing values for the predetermined descriptors. For example, when indicator variables for industry classifications are included in the list of descriptors, the model accommodates industry factors (e.g., King [3]). When the model draws upon predetermined data to incorporate numerical descriptors of size and yield, factors for the "size effect" and "yield effect" result (see, e.g., Rosenberg and Marathe [5]).

The net effect of all common factors on any particular stock, c^*, captures those influences on the stock that are common across many stocks. Specific return is, by construction, that part of the return for the company that is unrelated to other companies. It is obtained as a residual, subtracting the estimated factor-related return from the total return on the company.

II. Possible Explanations of Negative Serial Correlation in Specific Returns

One likely source of negative serial correlation in specific returns is any discrepancy between the reported price and efficient market price for a stock. This discrepancy is typically independent of events for other stocks, and hence specific to that stock. Such a discrepancy might arise, among several possible reasons, from (i) reporting error or data-entry error in the price or in the capital adjustment to the price due to a stock split, or (ii) a random deviation between the transaction price and the true efficient price. Very small deviations of this latter kind certainly do arise from indivisibilities in pricing (pricing in eighths, for example). More significant discrepancies can arise, as a practical matter, from transitory demand or supply pressures in the market, when information barriers or transaction costs bar most investors from promptly responding to force the transaction price toward equilibrium.

If we assume that discrepancies between transaction price and efficient price do arise, but then work themselves out within a month so that discrepancies at the ends of successive months are independent, then the consequence is negative serial correlation in monthly returns. Suppose that observed stock price is equal to an efficient price p_t^* which obeys a random walk (in logarithms), and a pricing discrepancy $p_t^* \epsilon_t$, where ϵ_t is the proportional error, and ϵ_t is independent of the true price. Suppose further for convenience that the price discrepancies are independent of one another from one month-end to the next. Thus reported price is $p_t = p_t^* \epsilon_t$. With suitable adjustments for the dividend and riskless rate processes, it can be shown that the correlation between successive observed specific returns, ρ_u, is given by:

$$\rho_u = \frac{-V_\epsilon}{V_{u^*} + 2V_\epsilon} = \frac{-V_\epsilon}{V_u}, \tag{4}$$

where V_ϵ is the variance of the logarithmic price discrepancy, and V_{u^*} is the variance of the efficient specific return. The negative correlation arises in a way that is intuitively obvious. When there is a pricing error at the end of one period—say, for example, a positive error—one result is to make the return in that period abnormally positive. There is an equal tendency for the return in the next period to be abnormally negative, since the next period's return starts off from too high a base. Thus, an abnormally positive return is followed by an abnormally negative one. Conversely, when the pricing discrepancy is negative, an abnormally negative return is followed by an abnormally positive one.

When the discrepancy between efficient and market price is serially independent from one month-end to the next, the resulting specific return is negatively correlated with the previous month's value and uncorrelated with earlier months. On the other hand, when the pricing discrepancies can themselves be serially correlated over several months due to a longer-lived phenomenon, then the serial correlation of specific returns can also extend over a number of lagged months. Hence, study of the time pattern of historical serial correlation is one good clue as to the source of the effect.

Another source of possible negative serial correlation in specific returns occurs if there is a nonsynchronous response of securities to common factors. Nonsynchronous response gives rise both to positive serial dependence in observed factor-related returns and to negative serial dependence in specific returns. The explanation is as follows. When a subgroup of the securities responds to a factor in period 1 and the balance responds in period 2, the true factor return is spread over the estimated factors of both periods. Hence successive factor returns tend to be alike and exhibit positive serial correlation. Roll [4] has recently discussed such effects in indices of rarely traded securities, and has stressed the importance of the implied short-term variance reduction. Moreover, those securities that fully respond in period 1 to the true factor will seem to overrespond to the estimated factor in period 1, because the estimated factor is less than the true factor. The overresponse shows up as a specific return in that month. These same securities will seem to underrespond in period 2. This underresponse shows up as a specific return of reversed sign. The other group of securities, which fully responds in period 2, exhibits specific returns of opposite sign, but these returns again reverse sign from month 1 to month 2. The result is negative serial dependence for the specific return of both groups.

III. Possible Explanations of Positive Serial Correlation in Observed Factor-Related Returns

The factor-related returns in each month t are computed as the fitted value in a cross-sectional regression involving the data of that month only. For each of the N firms, the observed logarithmic excess return, r_{nt}, is regressed on the J descriptors, x_{jnt}, for $j = 1, \cdots, J$, so as to estimate the observed factors for that month, $\hat{f}_{jt}, j = 1, \cdots, J$:

$$r_{nt} = \hat{f}_{1t}x_{1nt} + \hat{f}_{2t}x_{2nt} + \cdots + \hat{f}_{Jt}x_{Jnt} + \hat{u}_{nt}, n = 1, \cdots, N. \tag{5}$$

The fitting errors in the estimated regression, $\hat{u}_{nt}$, are the estimated specific returns. The computed factor-related return is:

$$\hat{c}_{nt} = \hat{f}_{1t}x_{1nt} + \hat{f}_{2t}x_{2nt} + \cdots + \hat{f}_{Jt}x_{Jnt}. \tag{6}$$

If equations (5) and (6) hold exactly, and if the factors, f^*, and specific returns u^*, are serially independent, for any predetermined descriptors, x, the estimated factors, $\hat{f}_t$, factor-related returns, $\hat{c}$, and estimated specific returns, $\hat{u}$, are all serially uncorrelated. This follows because the estimated factor values and the estimated specific risks in each month are linear functions of the returns in that month. Hence, if the returns in that month are independent of the returns in prior months, since the descriptors, x, are predetermined, the estimated factors and specific returns are uncorrelated with returns in prior months and hence uncorrelated with estimated factors and estimated specific returns in prior months. Therefore, when estimated factor-related returns are found to be serially correlated, this must result from some sort of serial dependence in the components of return.

We may conjecture that the serial dependence in factor-related returns results from an imperfect or partial adjustment of the market to events within the month. Thus, even though the "efficient factor returns," f^*, would not be serially correlated, the factor return seen in the market, f, may reflect a lagged response to f^*. For example, there might be a two-month lagged moving average:

$$f_t = k_0 f_t^* + k_1 f_{t-1}^* + k_2 f_{t-2}^*. \tag{7}$$

In this equation, the coefficients, k_0, k_1, and k_2 are, respectively, the responses of the factor return in any month, say t, to the true factors in that month and the two previous months. Presumably, $k_0 + k_1 + k_2 = 1$, so that the full response occurs over three months. In other words, of the true factor value, f_t^*, the portion k_0 is reflected in the same month's market movement, the portion k_1 in the next month's, and the balance, k_2 in the following month.

If we further suppose that the pattern of lagged response is similar for all factors of return, f_1 through f_J, and if the descriptors of firms change slowly over time (as they do in most multiple-factor models), then the same lagged pattern will show up in the total factor-related return. In other words, the observed return, c_{nt}, will be related to the efficient factor returns by the lagged relation:

$$c_{nt} = k_0 c_{nt}^* + k_1 c_{n,t-1}^* + k_2 c_{n,t-2}^*. \tag{8}$$

Such partial adjustment is thought to be unlikely in an active capital market: if market participants developed a behavior pattern that gave rise to such partial adjustment, anyone who noticed this pattern could profitably speculate by trading in anticipation of next month's adjustment. In so doing, the speculator would bring more and more of the response to f_t^* into the current period, t.

One rationalization suggests that common factors—especially those applying across a large number of firms—may reflect economywide events that are each one of a kind. In these conditions, as each new event occurs, market participants may not only be quite uncertain as to its magnitude, but may also treat it as a new puzzle to be solved, rather than one in a series of similar puzzles. The market may therefore underrespond in numerous separate instances, without participants' perceiving this as being a regular pattern. In the long run, of course, the series of underresponses would inevitably be recognized, but the long run might be surprisingly long.

For example, the time period covered by the empirical study reported below—from January 1, 1973, through December 31, 1980—might be viewed in retrospect as a period in which a few major economic tendencies went to previously unanticipated extremes, such that predictions of their impact were being continually adjusted upward. The shocks to energy prices, the increased rate of inflation, and the progressive rejection of aggregate demand management would be such tendencies.

Another quite different source of positive serial dependence in estimated factors was presented in the previous section, where it was explained that positive correlation results from nonsynchronous responses of individual asset prices to the factor.

There is, finally, one explanation for serial dependence in factor-related returns that does not violate market efficiency. Computation of the serial correlation of returns assumes—although this assumption is generally implicit—that the expected return is constant throughout the sample period. However, it is quite possible that, due to changes in the price of risk, the expected excess return itself may change from time to time. If the expected excess return changes, so that a few periods of higher expected return follow periods of lower expected return, the average factor-related return in the former interval would be less than in the latter. When differences from the global mean are taken, this shows up as positive serial correlation in the returns.

However, since the expected excess return seems likely to change slowly over time, the resultant serial correlation of returns will be almost the same for several months' lag as for one month's lag. Also, this phenomenon can be put in perspective by considering how large a variation in expected returns can plausibly occur. Reasoning this way, we can perhaps distinguish this source of serial correlation from the others.

IV. An Empirical Study

A sample of monthly data for common stocks is prepared for eight years, from January 1973 through December 1980. Total return data are taken from Compustat files and from the Interactive Data Corporation Analystics data base. To estimate a factor model, descriptors of two classes are used. First are characterizations of the company's fundamentals and of the past behavior of its stock price, based upon predetermined Compustat and Analystics data. These variables result in thirteen "risk indices". The data on which the risk indices are based would, in all cases, have been available prior to the quarter (three monthly returns) for which they are applied. The second class of data are allocations of the firm's business activity across fifty-five industry groups. These result in fifty-five descriptors which are industry proportions. These data are almost entirely predetermined, but for some firms in early years, there is a small amount of "backcasting" involved. The formulas for the descriptors were largely established in 1974 (Rosenberg and Marathe [5]), and the modifications in the present study were not importantly influenced by the security returns in the present sample.

The sample of firms for the factor-model estimation is comprised of the 1,000 largest companies, by capitalization at the beginning of each quarter, that are in the Compustat data base. By merging current Compustat data bases with the Compustat Research Files that contain all deletions since 1970, a complete population without significant retroactive exclusion is obtained. A company is deleted from the cross-section when a monthly return for the present or four prior months is unavailable.

In summary, the factor estimation study closely approximates a true experiment, in the sense that the definition of descriptors, the values of the descriptors, and the choice of sample size are essentially independent of the monthly returns in each cross-sectional sample.

In advance of inspecting the data, we designed an experiment in which lagged factor-related returns of up to four months lag would be used, so that the first four months of data were set aside to initialize the lagged returns. Hence, the study is concerned with the analysis of ninety-two monthly cross-sections of returns. There are just under 1,000 stocks each month, for a total of about 85,000 observations.

The tests are carried out under the following procedure. Ninety-two independent cross-sectional relations are estimated, one for each month of returns, in which that cross-section of returns is related to lagged values of returns. The cross-sectional regressions are generalized least squares, with the weights being appropriate for heteroscedastic data—that is, inverse to the specific variances. Specific variances are estimated as each stock's variance of specific return in the ninety-six month sample history. Extreme values of weights are truncated so that the smallest is ⅑ the largest: the largest weight is given to American Telephone and Telegraph (the lowest specific risk firm in the sample); the smallest weights, ⅑ as great, apply to extremely high specific variance companies, which are usually among the smallest in capitalization.

The first group of results concerns first order serial correlation in the components of return. This is estimated through the predictive relation:

$$y_{nt} - \bar{y} = \hat{b}_t(y_{n,t-1} - \bar{y}) + e_{nt} \qquad n = 1, \cdots, N. \tag{9}$$

where y_{nt} is the element of return being studied for stock n in month t, $y_{n,t-1}$ is the lag return for that stock, $\bar{y}$ is the grand mean of all returns in all periods, and $\hat{b}_t$ is the estimated regression coefficient in month t, where $t = 5, \cdots, 96$. In other words, a series of cross-sectional regressions is juxtaposed to analyze the time variation of the coefficient $\hat{b}_t$. The analysis is run three times, with y being set, respectively, to actual return, factor-related return, and specific return.

These results are given in Table 1. The mean coefficients for the ninety-two cross-sections are −.013 for actual return, +.134 for factor-related return, and −.099 for specific return. The time series standard deviation of coefficients for specific return is relatively small (.083), but for actual return it is .27 and for the factor-related return it is .73. As a consequence, the standard error of the mean for actual return and particularly for factor-related return becomes quite large. The mean autoregressive coefficient $\bar{b}$ is similar to, but not identical to the first-order autocorrelation for the entire sample; autocorrelations are −.016, .072, and −.104 for the total, factor-related and specific return respectively.

Studying these results we see that for actual return the serial dependence is indistinguishable from pure randomness. For factor-related return there is substantial positive serial dependence which is nevertheless not quite statistically significant (t-statistic = 1.77). Finally for specific return there is negative serial correlation which is stable over time and highly significant (t-statistic = −11.43).

The pattern of signs of the monthly coefficients bears out these results. For actual return there are forty-four positive and forty-eight negative; for factor-related return there are fifty-four positive and thirty-eight negative, a ratio which

Table 1

Time-Series Results: 92 Coefficients

	Actual Return	Factor-Related Return	Specific Return
Mean Coefficient	−.013	.134	−.099
Std. Err. of Mean	(.028)	(.076)	(.009)
t-statistic	−.47	1.77	−11.43
Std. Dev. of Coefficients	.27	.73	.083
Maximum Coefficient	.51	3.14	.06
Minimum Coefficient	−1.00	−2.30	−.38
Number of Positive Coefficients	44	54	9
Number of Negative Coefficients	48	38	83

Table 2

Time Series Results: 92 Cross-Sectional Regressions of Return on Lagged Factor and Specific Returns

	Factor Return		Specific Return	
	Lagged Once	Lagged Twice	Lagged Once	Lagged Twice
Mean Coefficient	.108	.004	−.112	−.029
Std. Err. of Mean	(.050)	(.065)	(.011)	(.010)
t-statistic	2.14	.06	−10.4	−2.85
Std. Dev. of Coefficients	.483	.645	.103	.098
Maximum Coefficient	1.30	1.07	.09	.17
Minimum Coefficient	−1.18	−1.97	−.51	−.46
Number of Positive Coefficients	58	50	9	34
Number of Negative Coefficients	34	42	83	58

is not quite significantly different from 50%; for specific return there are eighty-three negative coefficients and only nine positive, a highly significant deviation from 50%.

The next study concerns the predictability of actual returns using the factor-related and specific returns from previous months. The estimated equation is:

$$r_{nt} - \bar{r} = \hat{b}_{1t}(c_{n,t-1} - \bar{c}) + \hat{b}_{2t}(c_{n,t-2} - \bar{c}) + \hat{b}_{3t}(u_{n,t-1} - \bar{u}) + \hat{b}_{4t}(u_{n,t-2} - \bar{u}) + e_{nt} \quad (10)$$

so that current lagged return is regressed on fitted factor-related and specific returns from the two previous months. All variables are expressed as differences from their grand means.

Table 2 gives the time-varying results. The lagged factor-related returns have positive mean coefficients, and the lagged specific returns have negative mean coefficients. For factor-related returns the first-order lag is significant at the 95% level (t-statistic = 2.14, 58 out of 92 coefficients positive) and the second-order lag is insignificant. For specific returns, the first-order lag is highly significant (t-

statistic = −10.4, 83 out of 92 coefficients negative) and the second-order is significant at the 99% level (t-statistic = −2.85, 58 out of 92 coefficients negative). Identical conclusions are reached when monthly returns are defined as proportional (arithmetic) return relatives rather than logarithmic returns. (Results for proportional returns were reported in the version of this paper presented at the Conference.)

The average response coefficient to the previous month's factor-return is quite substantial, whereas the response to the two-month lagged return is near zero. Taking these results at face value, the pattern suggests market inefficiency (partial adjustment to efficient factor returns, or nonsynchronous asset pricing) rather than efficient response to changing excess expected return; changes of the latter class would presumably produce gradual and trending changes in expected excess return, which would show up as positive coefficients for both one-month and two-month lagged factor returns. However, the instability of the monthly coefficients results in a large standard error for the estimated mean, so that any interpretive argument must be viewed with caution.

The coefficients for lagged specific returns are quite stable over time, and their mean values therefore admit of more solid interpretation. The negative coefficient for one-month lag implies substantial reversal of the previous month's return. If the effect arose from pricing discrepancies which were serially independent from one month to the next, the two-month lag coefficient would be approximately the square of the one-month coefficient. (Under the independent-pricing-error assumption, the simple regression coefficient of this month's specific return on a two-month lagged return is zero. However, in a multiple regression on both one-month and two-month lags, the two-month lagged return serves to predict the part of last month's specific return which arose from the pricing error at the inception of last month: only the pricing error at the end of last month shows up in this month's specific return, so the optimal prediction rule subtracts off the best prediction for last month's specific return derived from the prior month.) Thus serially independent monthly pricing errors would imply a two-month lag coefficient of $(-.11)^2 = +.012$. The actual estimate, −.029, is significantly more negative (t-statistic of −4.0). Thus the results reject serially independent pricing errors in favor of the alternative that a portion of the pricing error survives more than one month and disappears at the end of the second month. Nevertheless, the majority of the reversal occurs in the first month.

V. Selected Implications

These results reject the weak form of the efficient market hypothesis. The previous month's specific returns predict the current month's specific returns with a t-statistic of −11, and predict the current month's actual returns with a t-statistic of −10. The predictive variables are essentially predetermined, and there is not significant retrospective bias in the sample. With respect to this data base of returns, market efficiency can be rejected.

The result could be due to recording error in the prices—causing the data base of returns to be defective—and there is always a possibility that such is the cause. However, the data base is widely used, and we have cross-checked it to some degree and validated the largest outliers. At this point, it seems appropriate to accept as a working hypothesis that the returns in the data base are indeed the returns implied by month-end closing prices perceived by market participants.

The magnitude of the predictive power of the relations is not large. If relation (10) had been known in advance, it would have explained just over 1.2% of the variance of monthly returns. The standard deviation of predicted percentage rates of return is just over 1% (the fact that both items are near 1% is a coincidence). Nevertheless, in the absence of transaction costs, a portfolio strategy based upon these predictions would have shown statistically significant abnormal returns: although an "information ratio" of 0.11 or coefficient of determination of 1.2% in predicting return is of little value when available on a single asset, it can contribute to an improved diversified portfolio when predictions are available across 1,000 assets. However, it is quite possible that transaction costs would have swamped the benefit from the trading strategy. The predicted returns change a great deal from one month to the next, implying large portfolio turnover.

Perhaps the most interesting aspect of the results relates to the concept of absolute market efficiency. From equation (4), using the coefficient value $\rho_u = -.099$, we find that the variance of proportional pricing error is $V_\epsilon = .123 V_{u^*}$, or about 1/8 of the variance of efficient monthly specific return. Solving further, $V_\epsilon = .099 V_u$, about ⅒ of the variance of observed specific return. In our sample, using the weighting scheme which is inverse to specific variance and therefore weights the low variance stocks most heavily, we found average observed specific return variance is $V_u = .0046$ implying $V_\epsilon = 0.00046$. The square root of this is the standard deviation of logarithmic pricing error or .021.

This implies that the underlying or absolute inefficiency of the market in pricing individual securities is 2.1% of efficient price. This is actually only that part of the error which is worked out within a month. An extension of the model to allow higher order serial dependence would give a higher value for total error.

The absolute inefficiency is certainly large enough to profit from by portfolio rebalancing. How is it that its variance is so much larger than the variance of predicted specific return? The explanation is that the predictive model cannot distinguish between that part of last month's specific return which was the efficient return, and that part which led up to the pricing error at month-end. Hence the prediction rule implies that ⅒ (.099) of total specific return will reverse itself, whereas an omniscient investor, knowing the exact value of the pricing error, would be able to predict that the entire error would be reversed. With perfect knowledge, the best Bayesian prediction is of course the known value of the error. With imperfect knowledge, using the past specific return to obtain an approximation to the pricing error, the Bayesian prediction has less explanatory power.

It is useful to extend this line of argument further. First, consider the investor who does not have available the distinction between specific and factor-related return, and hence must base his estimate of pricing error upon last month's total return. For this mode of prediction, the coefficient is .01 and the R-squared of the predictive relationship is 1/100 of one percent. Starting from this basis, the separation of specific from factor-related returns has improved predictive capacity one hundred fold. Following along this path from relative ignorance towards improved prediction, one can imagine the possibility of other analytical methods, yet to be determined, which isolate a still smaller component of total return containing the valuation error.

From this perspective, the factor-related return has served to distinguish the specific return, which contains an element of reversal arising from market inefficiency, from the component, related to other securities, which does not contain any reversal. More powerful valuation methods, by supplementing the factor model, may add further discriminatory power.

Another implication of these results has an entirely different thrust. This concerns the bias in estimated returns from strategies with constant investment weights, or any "exogenously determined weight strategy" in which the weights do not respond to price change. As Fisher [2] originally pointed out, when there is pricing error any exogenously weighted index will exhibit spuriously high returns. The explanation is as follows. When the investment proportions in the securities are exogenously determined, for example as equal weights, the portfolio is rebalanced each month based upon the prices at month-end. To attain the appropriate weights, the portfolio buys more shares of those securities with negative pricing errors, and fewer shares of those with positive pricing errors. Thus the portfolio is rebalanced each month to take advantage of the opportunity for abnormal returns promised by the pricing errors. Holdings are increased in those securities with negative errors, expected to perform abnormally well next month from this depressed base price, and holdings are reduced in securities with positive pricing errors, which are expected to perform abnormally poorly next month, from an inflated base.

If these are truly erroneous prices, then the rebalancing opportunity would not be available to a market participant, and the upwards increment to return is entirely spurious. If, on the other hand, these are actual market prices, then an effective trader might indeed have successfully rebalanced, and the increment to return is not spurious. Even in this latter case however, the increment is not reflective of a buy and hold strategy, but instead results from rebalancing with its implication for high turnover and perhaps high transaction costs.

By contrast, a capitalization weighted index shows no performance increment because it is essentially a portfolio of a fixed number of shares, and hence a buy and hold strategy. For any buy and hold strategy the compound return is the ratio of the values of those shares at the end of the history to the value at the beginning. Pricing errors, at intermediate month-end valuation points, have no effect upon cumulative return (excepting reinvestment of dividends).

In our simple model, the increment in performance over each holding period (month) may be shown to approximately equal the variance of the proportional pricing error. For the results of our simple model of market inefficiency, this increment is on the order of .00046 per month, or twelve times this amount annually. Thus the incremented return amounts to .55% per annum. Moreover, the increment is larger for those securities where pricing error is a larger fraction of value, i.e., those securities where the market is absolutely more inefficient. We have implicitly assumed that pricing error variance is a constant fraction of specific return, and therefore that there is greater pricing error among companies with higher specific variance. According to this assumption, the increment would be four or more times as great, or as much as 2% per annum, for the large number of high specific variance companies in the sample.

It is important to note that estimates of factors in almost all factor models are exogenously weighted average returns. Therefore these estimates may be subject to this rebalancing increment, in addition to the short-term variance reduction from positive serial dependence in factors which Roll has proposed. The increment is an upwards performance bias in the sense that it reflects a component of return which cannot be achieved by buy and hold, but only by frequent rebalancing. Further study of the extent of this effect is certainly warranted.

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