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Source: *The Journal of Finance*, Vol. 37, No. 2, Papers and Proceedings of the Fortieth Annual Meeting of the American Finance Association, Washington, D.C., December 28-30, 1981 (May, 1982), pp. 357-369
Published by: Blackwell Publishing for the American Finance Association
Stable URL: http://www.jstor.org/stable/2327337
Accessed: 27/11/2009 08:14

THE JOURNAL OF FINANCE • VOL. XXXVII, NO. 2 • May 1982

IMPACT OF REGULATION ON MONEY MARKET INSTITUTIONS

Presiding: JACK GUTTENTAG†

A General Equilibrium Money and Banking Paradigm

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IN THIS PAPER we propose and analyze a new paradigm for monetary analysis that is considerably richer than the standard money multiplier models found in the literature. A general equilibrium approach to financial assets is developed that leads directly to a multiplier system that transcends the traditional dependence on demand deposits and is easily amenable to exposition and manipulation. The model incorporates financial innovation and regulatory changes easily and allows for the general equilibrium effects of the variation in financial ratios that result from induced movements in income, interest rates, or prices.

The current analysis differs from traditional multiplier presentations in two distinct ways. First, the traditional approach to monetary expansion and multiplier analysis relies upon the ratios of various financial assets to demand deposits. Over the last decade decreased financial regulation and subsequent innovation have substantially changed the nature of depository institution liabilities and their substitutability with demand deposits. This has led to the reduction in emphasis on demand deposits *per se* and an increasing dependence upon transaction accounts other than demand deposits. Thrift institutions' NOW accounts, automatic transfer accounts and overdrafts, money market mutual funds, and overnight repurchase agreements have made the dependence on demand deposits as a basis for multiplier analysis somewhat obsolete and artificial. The model developed here can accommodate any number of transaction assets by specifying general asset demand ratios relative to total household wealth.

The second and more fundamental way in which our model differs from existing ones is the way in which it weds general equilibrium considerations with simple multiplier analysis. The traditional approach to multipliers is to assume that any ratio between two financial assets vying for a role in the aggregate wealth portfolio is exogenous and invariant to other variables. In today's economy this seems rather inappropriate. Any regulatory or asset preference change will result in changes in the endogenous variables in the economy which will further impact financial demands. The exogeneity of asset preferences and reserve ratios is no

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longer justified, and a general equilibrium setting is necessary to analyze the effect of disturbances. Further, as regulations as to the type of liabilities that may be issued by an institution change and regulatory ceilings on interest rates are phased out, a model which is capable of analyzing the changes in income, interest rates, and prices as well as their effect on asset demands is needed. We conclude the analysis by examining the likely bias that may be introduced by ignoring these induced effects on the ratios that appear in money multipliers to see the nature of the errors associated with a simpler form of analysis.

I. Current Techniques Available to Derive Financial Aggregates

Before turning to the new paradigm, a few words are in order to outline the presently available techniques of multiplier analysis. There appear to be two. The first views the monetary process as a simple, usually constant, money multiplier model, namely:

$$M^J = m^J H^s, \tag{1}$$

where M^J is the j^{th} monetary aggregate (the sum of the first j assets), m^J is the related money multiplier, and H^s is the supply of high-powered, or base money. The textbooks are replete with examples using this approach and some seminal and insightful papers have used this methodology, (e.g. Meade (1934), Burger (1971), Pierce-Thomson (1972), and Burger-Kalish-Babb (1971)). Although the money multiplier is defined under various asset scenarios, the usual solution for the traditional M1 multiplier, m, (currency plus demand deposits), is

$$m = \frac{1 + c}{k_D + k_T t + c}, \tag{2}$$

where k_D is the required reserve ratio on demand deposits, k_T is the required reserve ratio on a composite time deposit liability, and t and c are the ratios of time deposits and currency, respectively, to demand deposits. More sophisticated approaches allow m to vary with the market rate of interest, or with some time series process.

One important attempt to endogenize asset and reserve ratios is found in the early work of Brunner and Meltzer, [Brunner and Meltzer (1964), and (1968)]. Using a rather small general equilibrium model, these authors specified the functional dependence of the multiplier and explained the use of equations (1) and (2) as a first approximation to the true solution. However, their model still depended upon the central role of demand deposits and was restricted to an analysis of a small subset of financial assets.

The second alternative to the multiplier analysis is a fully developed general equilibrium model. The work of Brainard and Tobin, [Brainard (1964), Tobin and Brainard (1963), and Tobin (1969)], is the central contribution in this area. The financial markets are defined as a set of equilibrium conditions whereby demands from various sectors are equated to the fixed stock of real and nominal monetary assets via a vector of interest rates and output or the price level. Budget constraints impose adding-up conditions on cross equations summations, and the system is usually specified as exhibiting gross substitutability and homogeneity in wealth. Multiplier analysis of the form given in Equation (1) above can only be viewed as a extreme reduced form of the equation system of excess demand equations. The problem with such a system for multiplier analysis is its inability to be easily solved and manipulated. Therefore, little has been done to use this analysis to track the monetary process or empirically investigate the relationship between the quantity of some assets and the monetary base.

II. The General Equilibrium Model

We propose to synthesize the two approaches above by starting with a general equilibrium model of the Tobin-Brainard form which can be used to shed some insight on the multiplier analysis. Section A develops the analytic framework. Although this development involves some complicated notation, the reader should forbear since the model is significantly simplified in Section B, and this simplified form is used for multiplier analysis in Section C.

A. The Basic Structure of the Model

Assume a financial environment in which there are $n + 2$ assets: high-powered, or base money; $n - 1$ types of intermediary deposits; bonds; and equity. There are, therefore, $n + 2$ markets, numbered as follows.

—High-powered money (H) market is market 0.

—Depository markets (D_i) are markets $1 \ldots n - 1$.[1]

—Bond (B) market is market n.

—Equity (E) market is market $n + 1$.

There are four types of economic agents in the economy: households, financial institutions, firms, and the government. The household is assumed to have demand functions for all assets subject to the aggregate budget constraint for the sector. The depository institutions or banks may hold high-powered money, other depository institutions' deposits, and bonds (loans). Firms issue equity to support capital ownership, and because of the Modigliani-Miller Theorem their bond issuances (if any) are suppressed. The sole net supplier of bonds and high-powered money is the government, and in the present analysis these bonds are assumed to be viewed as net wealth.[2]

This view of the financial sector can be characterized as a set of equilibrium conditions for each of the above assets. In all the equations which follow, all demands and supplies (except the supply of high-powered money), income, and wealth are expressed in real terms. For the high-powered money market, H, the market equilibrium condition is

$$H_h^d(r_{Di}, r_b, r_E, Y, W) + \sum_{i=1}^{n-1} H_{F_i}^d(r_{Di}, r_b) = H^s/P, \tag{3-1}$$

[1] Assume, for example, demand deposits are market 1, time deposits at commercial banks are market 2, etc.

[2] If government bonds are not considered net wealth, the analysis proceeds with open market operations equivalent to high-powered money issuance and government debt has no effect.

where subscript h denotes household demand functions, r_{Di} is the vector of i deposit rates, r_b is the bond rate, r_E is the equity rate, Y is income, W is wealth, $H^d_{F_i}$ is the demand of the i^{th} financial institution (assumed, for simplicity, to issue the i^{th} depository liability) for high-powered money (reserves), H^s is the nominal supply of base money, and P is the price level. The equilibrium condition for the market for each depository asset D_i, is,

$$D^d_{ih}(r_{Di}, r_b, r_E, Y, W)\sum_{\substack{j=1\\ j\neq i}}^{n-1} D^d_{iF_j}(r_{Di}, r_b) = D^s_i(r_{Di}, r_b); \quad i = 1 \ldots n-1. \tag{3-2}$$

where $D^d_{iF_j}$ is the demand of the j^{th} financial institution for the i^{th} deposit. The equilibrium condition for the bond market is

$$B^d_h(r_{Di}, r_b, r_E, Y, W) + \sum_{i=1}^{n-1} B^d_{F_i}(r_{Di}, r_b) = B^s_g, \tag{3-3}$$

where $B^d_{F_i}$ is the demand of the i^{th} financial firm for bonds and B^s_g is the supply of government bonds.[3] Likewise, the equilibrium condition for the equity market is

$$E^d_h(r_{Di}, r_b, r_E, Y, W) = E^s. \tag{3-4}$$

where E^s is the fixed capital stock K. The budget constraints for the household, depository institution, and firm sectors are

$$H^d_h + \sum_{i=1}^{n-1} D^d_{ih} + B^d_h + E^d_h = W = B^s_g + H/P + K, \tag{4-1}$$

$$H^d_{F_i} + \sum_{j=1}^{n-1} D^d_{jF_i} + B^d_{F_i} = D^s_i; \qquad i = 1 \ldots n-1, \tag{4-2}$$

$$E^d = E^s = K. \tag{4-3}$$

B. Simplification of the System

The view of depository and non-depository institutions in the general equilibrium framework of equations (3) is most general and as such suffers from the same difficulty of manipulation possessed by the Brainard-Tobin approach. To lend substance and to simplify the subsequent analysis, more specific assumptions about the determination of deposit rates are in order. In considering the nature of financial institutions' deposit liabilities, two distinct types emerge. First, there are deposits which have their rates set primarily by regulation at some fixed level, e.g., demand deposits and passbook savings accounts. Second, there are deposits that have yields which move closely with open-market rates. This latter dependence may be due to regulatory constraints on such liabilities, e.g., the money market certificate, or it may be due to the production and pricing pattern of the industry. For example, the institution may pay the depositor the return on the portfolio less some fee or cost. Money market mutual funds fall into this category.

A result of these pricing schemes is to make the quantity supplied of these depository assets solely dependent upon the demand for these deposits at the

[3] If the household or public sector is a net borrower from the financial institution sector, household bond demand will be negative. B^s_g need not be positive for any of the analysis.

vector of rates determined at either the fixed rate of return dictated by regulation or the net yield obtained from the portfolio assets. The size of these depository markets, therefore, is demand side determined, so that $D_i^s \equiv D_h^d$. Hence, the budget constraint for each institution may be written as

$$H_{F_i}^d + \sum_{j=1}^{n-1} D_{jF_i}^d + B_{F_i}^d = D_{ih}^d, \quad i = 1, \dots n-1, \tag{5}$$

where D_{ih}^d is evaluated at the fixed r_{Di} or at the deterministic relationship between r_{Di} and r_b (denoted $r_{Di} = \psi_i(r_b)$). In either case the number of equations in the system is reduced from $n + 2$ to 3.

Define the ratio of high-powered money demanded to total deposits supplied for each institution as $\hat{k}_i$, and assume it is a function of the bond rate and deposit rates,

$$\hat{k}_i \equiv \frac{H_{F_i}^d}{D_i^s} = \hat{k}_i(r_b; r_{Di}). \tag{6}$$

The vector of deposit rates r_{Di} is parameterized on the degree of government regulation and the bond rate. Likewise, define the ratio of the j^{th} deposit demanded by the i^{th} institution $D_{jF_i}^d$ to the i^{th} supplied as d_{ji}, denoted

$$d_{ji} \equiv \frac{D_{jF_i}^d}{D_{F_i}^s} = d_{ji}(r_b; r_{Di}). \tag{7}$$

From the budget constraint in equation (5) we have

$$\hat{k}_i + \sum_{j=1}^{n-1} d_{ji} + b_i = 1 \quad \text{for} \quad i = 1, \dots n-1, \tag{8}$$

where $b_i \equiv D_{F_i}^d / D_{F_i}^s$ is the ratio of bonds demanded by the i^{th} institution to deposits supplied.

The system of equations (3) can now be written as

$$H_h^d(r_b, r_E, Y, W; r_{Di}) + \sum_{i=1}^{n-1} \hat{k}_i(r_b; r_{Di}) D_{ih}^d(r_b, r_E, Y, W; r_{Di})$$
$$+ \sum_{i=1}^{n-1} \sum_{\substack{j=1 \\ j \neq i}}^{n-1} \hat{k}_j(r_b; r_{Di}) d_{ji}(r_b; r_{Di}) D_{ih}^d(r_b, r_E, Y, W; r_{Di}) = H^s/P, \tag{9-1}$$

$$B_h^d(r_b, r_E, Y, W; r_{Di}) + \sum_{\substack{i=1 \\ j \neq i}}^{n-1} \sum_{\substack{j=1 \\ j \neq 1}}^{n-1} (1 - \hat{k}_i(r_b; r_{Di})$$
$$- \hat{k}_j(r_b; r_{Di}) d_{ji}(r_b; r_{Di})) D_{ih}^d(r_b, r_E, Y, W; r_{Di}) = B_g^s, \tag{9-2}$$

$$E_h^d(r_b, r_E, Y, W; r_{Di}) = K. \tag{9-3}$$

The third term in equation (9-1) includes the derived demand for high-powered money due to the intradepositing within the financial institution sector.[4, 5]

[4] This formulation captures the entire uniqueness of commercial banks' debate espoused by Guttentag and Linsay (1968) and Wood (1970). Given the 1980 Monetary Control Act which introduces uniform high-powered money reserve ratios, the importance of the uniqueness discussion has been significantly reduced. However, in as much as required reserves are held in pass-through balances at a correspondent bank and institutions continue to hold deposit liabilities of other institutions,

It is convenient to define a reduced form reserve ratio, which includes both the reserves supporting the household sector's deposits and the reserves supporting a financial institution's deposits at other institutions. Denote this general reserve ratio as k_i, where

$$k_i(r_b; r_{Di}) = \hat{k}_i + \sum_{\substack{j=1 \\ j \neq i}}^{n-1} \hat{k}_j d_{ji}. \tag{10}$$

Further, define D_0 as currency so that k_0 equals unity, and D_n as bonds with k_n equal to 0. Therefore equations (9) can be written compactly as

$$\sum_{i=0}^{n} k_i(r_b; r_{Di}) D_{ih}^d(r_b, r_E, Y, W; r_{Di}) = H^s/P \tag{11-1}$$

$$\sum_{i=0}^{n} (1 - k_i(r_b; r_{Di})) D_{ih}^d(r_b, r_E, Y, W; r_{Di}) = B_g^s \tag{11-2}$$

$$E_h^d(r_b, r_E, Y, W; r_{Di}) = K. \tag{11-3}$$

Equations (11) are the financial market equilibrium conditions in an overall macroeconomic model. The endogenous variables are Y, r_b, P, and r_e. We shall assume that the real sector determines P and r_e so that the model determines income, Y, and the rate of interest.[6] By Walras' Law only two of the three equations in system (11) are independent so we can write the system as equations (11-1) and (11-2), with (11-3) determined by the system's budget constraint. The two equilibrium conditions have more intuitive appeal if they are written as the excess demand for high-powered money, equation (11-1), and the excess demand for all liquid assets defined as all assets less equity, i.e., currency, plus all deposits and bonds, equations (11-1) plus (11-2). This is represented by

$$\sum_{i=0}^{n} k_i D_{ih}^d = H^s \tag{12-1}$$

$$\sum_{i=0}^{n} D_{ih}^d = H^s + B_g^s. \tag{12-2}$$

Equilibrium in the economy exists when the total demand for liquid assets (currency, all deposits, plus bonds) equals the supply (high-powered money plus government bonds) and the reserve weighted demand for these liquid assets equals the supply of high-powered money.

C. Money Multipliers From a General Equilibrium System

To use this system to derive multipliers some additional notation is required. Define a_i as the desired ratio of asset i demanded by the household to total wealth,

$$a_0 \equiv \frac{H_h^d}{W}, \quad a_i \equiv \frac{D_{ih}^d}{W}, \quad a_n \equiv \frac{B_h^d}{W}, \quad \text{and} \quad a_{n+1} \equiv \frac{E_h^d}{W}, \quad \text{where} \quad \sum_{i=0}^{n+1} a_i = 1.$$

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interinstitutional deposit ratios remain relevant in any realistic characterization of the financial markets.

[5] Required reserve calculations necessitate the netting of intra-industry demand deposit balances. For this type of liability $k_i = 0$.

[6] The neoclassical counterpart of this model appears in Santomero and Siegel (1981).

Note that these are simply scaled demand functions. Following Tobin (1969), we will assume the a_i is homogeneous of degree zero in aggregate wealth. Accordingly, the two equation system may be written as

$$\textstyle\sum_{i=0}^{n} k_i(r_b;\, r_{Di})a_i(r_b,\, r_E,\, Y;\, r_{Di})W = H^s \tag{13-1}$$

$$\textstyle\sum_{i=0}^{n} a_i(r_b,\, r_E,\, Y;\, r_{Di})W = H^s + B_g^s. \tag{13-2}$$

Assume that Equations (13) plus specifications of the real economy determine the equilibrium r_b, r_E, Y, and P. Then equation (13-1) is central to the multiplier methodology, and can be used to derive the money multipliers traditionally derived in monetary analysis.

From (13-1) one may define an aggregate wealth "multiplier" m_W as

$$W = m_W H^s, \quad \text{where} \quad m_W = \frac{1}{\sum_{i=0}^{n} k_i a_i}, \tag{14}$$

where a_i and k_i (and hence m_W) are evaluated at their equilibrium values determined by the solution of equations (13). Note that the wealth multiplier is the inverse of the portfolio-weighted reserve ratios of all assets in the economy. The real equilibrium quantity of each asset, D_i is then simply

$$D_i = a_i m_w H^s, \tag{15}$$

and the equilibrium quantity of any monetary aggregate M^J, which is the sum of the first j assets, can be written as

$$M^J = \textstyle\sum_{i=0}^{j} D_i = m_J H^s = \dfrac{a^J}{\sum_{i=0}^{n} k_i a_i} H^s, \tag{16}$$

where $a^J = \sum_{i=0}^{j} a_i$.

The value of the monetary aggregates M^J are homogeneous of degree zero in the first $n - 1$ asset ratios (a_i's) so that a proportional change in these asset demands has no effect on the nominal value of monetary aggregates. This property allows portfolio ratios to be expressed as ratios of any financial asset. As such it illustrates the equivalence of the solution from the present model *before consideration of general equilibrium effects* with the traditional multiplier approach.[7]

It is frequently convenient for empirical purposes to express portfolio demands as ratios to total liquid assets (all assets except equity) rather than ratios to total wealth, since data on the capital stock is not readily available. In this version of the model define

$$a_i' = a_i / \textstyle\sum_{i=0}^{n} a_i, \tag{17-1}$$

$$m_L = 1 / \textstyle\sum_{i=0}^{n} k_i a_i', \tag{17-2}$$

[7] The impact of general equilibrium considerations on the multiplier framework is considered in Section III below.

where m_L is the liquid asset multiplier, i.e., the ratio of all assets except equity to high-powered money. m_L, like m_W, is the inverse of the portfolio weighted average of reserve ratios, the weights being the asset's proportion to total liquid assets rather than wealth. Aggregate multipliers and asset ratios are easily obtainable in this form. Denoting the monetary assets as currency C, transactions deposits D, time deposits T, and large time deposits CD, the nominal values of M, $M2$, and $M3$ are[8]

$$M1 = m_1H^s = (a_C + a_D)m_WH^s = (a'_C + a'_D)m_LH^s, \tag{18-1}$$

$$M2 = m_2H^s = (a'_C + a'_D + a'_T)m_LH^s, \tag{18-2}$$

$$M3 = m_3H^s = (a'_C + a'_D + a'_T + a'_{CD})m_LH^s, \tag{18-3}$$

where

$$m_L = \frac{1}{a'_C + a'_Dk_D + a'_Tk_T + a'_{CD}k_{CD}}. \tag{18-4}$$

Equation (18-1) can be transformed into Equation (2) by aggregating time and CD's. However, note that the multipliers for *every* monetary aggregate are immediately calculated in this framework, since the denominator for every aggregate (Eq. 18-4) is identical and the numerator is just the desired portfolio fraction. A quick reference to published data suggest that these equations can obtain rather exact estimates of existing multipliers and asset totals.[9]

The model is also capable of analyzing any number of asset innovations such as money market mutual funds. Let a_M be the fraction of total wealth the public wished to hold as money funds. Since such funds hold a certain fraction of their portfolio $d_{M,CD}$ in bank certificates of deposit, then according to Equation (10), the effective reserve ratio on money funds is simply $d_{M,CD}k_{CD}$, so that the wealth multiplier can be expressed as[10]

$$m_W = \frac{1}{a_C + a_Dk_D + a_{CD}k_{CD} + a_Tk_T + a_Md_{M,CD}k_{CD}}. \tag{19}$$

The effect on the multiplier of shifts from any asset to money funds is readily ascertained. For example, shifts from time deposits to money funds are expansionary on the multiplier if and only if $d_{M,CD}k_{CD} < k_T$, so that the weighted average reserve ratio falls.[11]

[8] As of January, 1982, *M1A*, which included only currency and commercial bank demand deposits was discontinued. *M2* and *M3* also contain other assets. In particular *M2* contains money market mutual funds which are analyzed below.

[9] Estimating $k_D = .10$, $k_T = k_{CD} = .01$, we have (July, 1981) the ratios $a'_C = .0479$, $a'_D = .1233$, $a'_T = .4601$ and $a'_{CD} = .1451$, so m_L from (18-4) is 15.16. Since high-powered money averaged \$165 billion in July, 1981, our formula yields total liquid assets at \$2501 billion, very close to the \$2521 billion reported by the Federal Reserve for its broadest aggregate L. It is immediate from (18-1) through (18-3) that the multipliers for *M1* is 2.59, for *M2* is 9.57, and for *M3* is 11.31.

[10] It is imperative to recall that the a'_is are the *households'* desired portfolio weights. They may differ from the total outstanding issues if, as in the case of money funds, some assets are composites of others.

[11] Before Nov. 1980, 6 month *CD*'s, the largest competitor of money funds, had a 2½% reserve requirement while short-dated large *CD*'s had a 6% reserve. In that case expansion will result only if

III. General Equilibrium Considerations

A. Illustration of the Problem

Equation (16) is the general multiplier expression for calculating the monetary aggregates from the quantity of high-powered money and reserve and portfolio ratios embedded in m_W and a^J. Much multiplier analysis treats the variables a_i and k_i as exogenous, as we have illustrated above, so that a change in the value of a monetary aggregate resulting from a shift in a preference function, reserve ratio, or high-powered money, can be calculated directly by taking the partial derivative of (16) with respect to these parameters. As was indicated this approach ignores the fact that the preference parameters and possibly the reserve ratios frequently change in response to a shift in an exogenous variable. The relevancy of these general equilibrium effects can be easily seen by examining the case of an increase in the reserve ratio on a monetary asset, such as demand deposits. The effect of the increase in required reserves is an increase in the demand for high-powered money by banks and an equal decrease in the banks' demand for bonds. However, since the supply of bonds and high-powered money is unchanged, the general equilibrium or "induced" effect must be to reduce the public's demand for high-powered money and to increase its demand for bonds. This induced change is effected through the change in the endogenous variables, interest rates and income. Since the public must be induced to hold more bonds it must, through its budget constraint and fixed wealth, reduce its demand for currency and deposits taken as a whole. The reduced demand for currency is matched, in the new equilibrium, by the increase in the demand for reserves by banks to satisfy the reserve requirements.[12]

Therefore, in the new equilibrium, the portfolio fractions *cannot* remain constant, but must decrease, as a whole, for assets 0 through $n - 1$, i.e., currency and all deposits. It is for this reason that the narrowly defined monetary aggregates will decline in response to a reserve increase. In this model, where all deposits are demand determined, the contraction in the money supply is not because banks have less "loanable funds" to expand deposits, but because induced changes in income and interest rates will reduce the demand for these deposits.

In order to explain how this occurs analytically, linearize the equilibrium conditions (13-1) and (13-2) to yield

$$\sum k_i \frac{\partial a_i}{\partial Y} dY + \sum \left(a_i \frac{\partial k_i}{\partial r_b} + k_i \frac{\partial a_i}{\partial r_b} \right) dr_b = - a_i \delta k_j \tag{20-1}$$

$$\sum \frac{\partial a_i}{\partial Y} dY + \sum \frac{\partial a_i}{\partial r_b} dr_b = 0. \tag{20-2}$$

$d_{M,CD}$ the ratio of *CD*'s to total assets held by money funds, was less than 5/12 or 42%. Under the Monetary Control Act the reserve ratio on both *CD*'s and time deposits will be phased over a 4 to 8 year period to be 3%. Then, shifts to money funds are unambiguously expansionary as long as such funds hold some nonreservable assets, such as commercial paper or government securities.

[12] In general, the drop in demand for monetary assets will reduce, but not eliminate, the bank's increased demand for high-powered money. If the demand for currency is totally insensitive to the endogenous variables, then the public will drop its demand for deposits by the same percentage as the rise in the reserve ratio, so as to keep total reserve demanded constant.

Solving for the endogenous effects of an exogenous change in the j^{th} reserve ratio yields

$$\frac{dr_b}{dk_j} = \frac{-a_j \sum \partial a_i/\partial r_b}{D} \tag{21-1}$$

$$\frac{dY}{dk_j} = \frac{-a_j \sum \partial a_i/\partial Y}{D} \quad \text{where} \tag{21-2}$$

$$D = (\sum k_i \partial a_i/\partial r_b + \sum a_i \partial k_i/\partial r_b)(\sum \partial a_i/\partial Y) - \sum \partial a_i/\partial r_b \sum k_i \partial a_i/\partial Y. \tag{21-3}$$

Assuming gross substitutability between assets ($\partial a_i/\partial r_j \leqslant 0$ for $i \neq j$, otherwise $\partial a_i/\partial r_i > 0$), and assuming that the monetary assets' demands, on the whole, respond positively to income ($\sum \partial a_i/\partial Y > 0$), D will be negative and $dr_b/dk_j > 0$ and $dY/dk_j < 0$. A rise in the required reserve ratio increases the bond rate and reduces income. Now it is clear how the general equilibrium solution is obtained. The rise in the bond rate induces the increase in the households' demand for bonds necessitated by the drop in the bank's demand caused by the increased reserve requirements. Furthermore, the rise in the bond rate, as well as the drop in income, reduces the households' demand for currency and deposits, therefore lowering the demand for narrow monetary aggregates. The households' demand for equity remains unchanged, as it must in the new equilibrium since the capital stock is unchanged. The lower equity demand resulting from the rise in the bond rate is exactly offset by the increased demand resulting from the reduced level of income. All aspects of the general equilibrium solution are satisfied by the movements of the endogenous variables.

B. Bias in the Exogenous Multiplier Model

The fact that preference ratios must change in response to an exogenous shift does not necessarily invalidate the standard multiplier analysis employing exogenous parameters. From Equation (16) the response of the J^{th} monetary aggregate to a shift in an arbitrary exogenous variable x_0 is:

$$\frac{dM^J}{dx_0} = \frac{a^J}{\sum a_i k_i}\frac{dH}{dx_0} + H\left[\frac{\partial(a^J/\sum a_i k_i)}{\partial x_0} + \frac{d(a^J/\sum a_i k_i)}{dx_0}\right] \tag{22}$$

where the first term in the brackets indicates the impact, or exogenous effect on the multiplier, and the second term is the induced effect caused by changes in interest rates and income. It is immediate that if induced changes in each asset demand (except bonds and equity) are proportional and reserve ratios are fixed, that is:

$$\begin{matrix} da_i = \lambda a_i \\ dk_i = 0 \end{matrix} \quad i = 0, \ldots n-1 \tag{23}$$

then the second bracketed terms of (22) is zero. Condition (23) means that if, as interest rates and income change, the *relative* demands for currency and all deposits remain unchanged, and reserve ratios are fixed, then the general equilibrium induced effects are zero and the multiplier may be manipulated as if each ratio is exogenous.

How close do the demand shifts caused by income and interest rate movements conform to condition (23) above? It is generally thought that currency and demand deposits (high reserve assets) have much lower market interest rate sensitivity than lower reserve assets, such as time deposits, due to the latter's greater substitutability with bonds. However, since low reserve assets have yields which are frequently linked to the bond rate, their opportunity cost, or the difference between their yield and the bond rate, fluctuates less than the opportunity cost of currency and transactions deposits. Therefore, it is altogether possible that the relative demands among monetary assets would remain approximately unchanged in response to bond rate changes. On balance, however, it is not unreasonable to expect that an increase in the bond rate would favor a relative shift to low reserve assets, so that a rise in the bond rate would result in an induced rise in the multiplier. Note that this result is critically dependent on the state of regulation of deposit rates. The more deposit rates are linked to bond rates, the less likely there will be a relative shift to low reserve assets, and the closer condition (23) would prevail. Rigidly fixed deposit rates, as essentially prevailed until the last several years, result in a greater interest sensitive multiplier.

The fall in income caused by the rise in the reserve ratio will also bring about a fall in monetary asset demands. If this shift in demand is uniform across monetary assets, as would be the case if all monetary assets demands were proportional to income, then income effects would not alter the results obtained from the simple multiplier analysis. However, it is generally observed that higher order (low reserve) monetary assets include demands for both transaction and wealth motives, while currency and transactions deposits contain primarily an income motive. Accordingly, the shift across monetary aggregates accompanying an income change may not be proportional. A decline in income may reduce the demand for high reserve monetary assets to a greater extent than for low reserve assets. This will have the effect of decreasing the total demand for high-powered money and increasing the multiplier. However, this reasoning is also dependent on the state of deposit market regulation. If near market rates are paid on deposits, then the wealth motive will be important for transaction accounts also, increasing the possibility of proportional shifting, and lowering the bias in the multiplier.

The third and last source of bias arises from the possible endogeneity of the reserve ratios. For large banks the assumption of a fixed reserve ratio is quite plausible, since prudent reserve management dictates keeping zero excess reserves. However, for some small banks, desired vault case exceeds the required level of high-powered money. In these cases it is reasonable to assume that these reserves will be sensitive to the market rate of interest, so overall there might be a slight decline in the actual reserve ratio when interest rates rise. To the extent that this occurs, a rise in the bond rate will also result in an induced increase in the multiplier.

For the reasons given above, it is empirically likely that an exogenous shift which causes a rise in the bond rate and/or a fall in income will cause the multiplier to be greater than predicted by simple multiplier analysis. This upward bias will probably be reduced if there is greater flexibility of deposit rates to the market rate of interest and a greater uniformity in binding reserve constraints.

C. Applications to Other Exogenous Shifts

The analysis that was applied to reserve ratio changes can be adapted to changes in any exogenous variable. By linearizing the equilibrium system Equations (13), one may determine the effect on income and the rate of interest of any shift. This will yield the direction on the bias in general equilibrium analysis compared to standard multiplier analysis.

An open-market purchase (exchange of government bonds for high-powered money) has no direct impact on the multiplier but will cause interest rates to decline and income to rise. As outlined in Section IIIB above, this will likely result in a relative shift to higher reserve assets and hence reduce the multipler, leading to a *less* than proportional increase of the monetary aggregates to high-powered money.

An increase in the supply of government debt will also have no direct impact on the multipler, but in general equilibrium will raise interest rates and income. In this case, since interest rates and income are moving in the same direction, the effect on the multiplier is ambiguous. The higher market rates induce individuals to switch to lower reserve assets, but the higher income has a tendency to do the opposite. The net effect on the multiplier is ambiguous unless one effect can be shown empirically to dominate the other.

When analyzing the effect of a shift in asset preferences, it is important to remember that the portfolio fractions (the a_i's) are constrained (unlike the reserve ratios k_i) to sum to unity. Any exogenous shift in the demand for the j^{th} asset caused, for example, by a rise in the deposit rate (such as the institution of interest bearing NOW accounts) must be matched by a decline in the demand for another asset or set of assets. Therefore the direct impact effect on the multiplier must be calculated as

$$dM^J = \sum \frac{\partial M^J}{\partial a_i} da_i, \text{ where } \textstyle\sum_0^n da_i + da_E = 0. \tag{24}$$

If the shift is from a lower reserve asset to a higher reserve asset, the result is a net increase in the demand for high-powered money and, like an increase in reserve ratios, will cause interest rates to rise and income to fall. Therefore it is likely an induced increase in the multiplier will result, partially offsetting the contractionary direct effect of such an asset shift on the multiplier. If the demand shift occurs between assets with the same reserve ratio, there will be no impact on interest rates or income, and the direct impact on the multiplier is simply calculated from the change in desired asset holdings.

IV. Summary and Conclusions

The result of this analysis is two-fold. First a "multiplier" relationship is formulated from a general equilibrium model in terms of assets demands relative to wealth, rather than relative to some arbitrary asset as demand deposits. Such a general multiplier can accommodate any number of assets and is based on the inverse of the weighted average of reserve ratios, the weights being the relative portfolio demand fractions. Secondly, it is shown that theory can specify the bias, if any, in taking the partial derivative of such a general multiplier holding all other asset demand and reserve ratios constant.

If changes in income and interest rates caused by the exogenous shifts result in the same relative demand for monetary assets, then there is no bias in manipulating the multiplier in a partial equilibrium framework. However it is empirically likely that a rise in the market interest rates or a fall in income causes a relative shift to low reserve assets and increases the value of the multiplier. Therefore the expansionary effect on monetary aggregates of an open-market purchase or a decline in a reserve ratio will often be overstated by direct multiplier analysis. The extent of this bias is directly dependent on the level of regulation of deposit rates. Deregulation of deposit rates and binding reserve ratios implies that standard multiplier analysis will more closely predict the movements in monetary aggregates.

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