Optimal Bond Trading with Personal Tax: Implications for Bond Prices and Estimated Tax Brackets and Yield Curves: Discussion
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DISCUSSION

L. FISHER*: I think that the main contribution of the paper by Bierwag, Kaufman, and Toevs (BKT) is that it turns directly to the study of relationships among price changes of fixed-income securities. Previous studies were indirect. They analyzed the nature of the first and higher derivatives of price with respect to interest rate and the interactions between "duration" and such derivatives. Although some of the relationships among the derivatives with respect to interest rates are linear, others are not. Hence, it has been difficult to state interest-rate-generation processes that lead to interesting equilibrium conditions, especially for the portfolio immunization problem. By turning directly to consideration of relative price movements, BKT offer a class of processes that may be consistent with equilibrium conditions in which immunized portfolios are fully hedged.

Because of these qualities, I believe that the BKT additive process will provide a much more satisfactory framework for studying the relationships among fixed-income securities than its predecessors.

To put BKT'S accomplishment in better perspective, let me add to their review of the development of immunization procedures.

F. M. Redington stated three conditions for portfolio immunization:

1. That the present value of the assets be at least as large as the present value of liabilities
2. That the derivative of the present value of the assets with respect to the "force of interest," i.e., the discount rate compounded continuously, be equal to the derivative of the present value of the liabilities
3. That the second derivative for the assets be at least as large as the second derivative for the liabilities.

However, he considered the portfolio to be immunized even if only the first two conditions were met.

Since the negative of the derivative of the logarithm of price with respect to the force of interest is equal to Frederick R. Macaulay's *duration*, David Durand restated condition 2:

2. That the duration of the assets be equal to the duration of the liabilities.

In testing empirically the efficiency of immunizing a liability with a portfolio of coupon bonds, this discussant and Roman Weil saw that Redington's assumption of a level term structure of interest rates was needlessly strong. We could modify Redington's assumption to one of "parallel" shifts in the term structure. Then we stated a slightly more general formula for duration.

However, for the problem we studied, the second derivative of the assets is larger than the second derivative for the liabilities. Hence, large changes in interest rates appear to lead to positive profits. Either immunization is inconsist-

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ent with equilibrium (as it is under flat term structures), immunized portfolios are not perfectly hedged (as under parallel shifts), or the interrelationship among changes in short-, intermediate-, and long-term interest rates must be very complicated.

The BKT "immunizing durations" coupled with their direct consideration of changes in relative price may ovecome the objections. For example, when the immunizing durations of portfolios of zero-coupon bonds are the same as the Fisher-Weil (or Macaulay) durations, the BKT analysis is equivalent to assuming directly that

$$R_s(1, t) - R(0, 1) = (t - 1)S(s) \tag{1}$$

where $R(j, k)$ is the return period j of a zero-coupon bond due at time k and $S(s)$ is the "shock" associated with the state of the world s at time 1. This case would occur whenever BKT's $\lambda = 1$. Then, an immunized portfolio would have an immunizing duration equal to the Fisher-Weil duration; but there would be no convexity—no profit or loss from large changes in interest rates.[1] However, when $\lambda \leq 1$, the BKT assumptions require that, if they are finite, changes in short-term interest rates have larger magnitudes than changes in long-term rates and that the changes be functionally but nonlinearly related. I think that the single-factor analysis applies over a substantially wider range of relationships among price changes than BKT suggest. However, whenever the immunizing duration is not given by the Fisher-Weil formula, substantial empirical work will be required to estimate the form and parameters of the relationship among price changes and to determine whether any single-factor model was satisfactory.

In section II, BKT attempt to provide a general equilibrium characterization of their "nonconvex" [linear] process. That section is much less satisfactory than either section I or their basic innovation. In their analysis uncertainty vanishes after time 1. For some similar problems, violation of equilibrium conditions does not appear until uncertainty persists through at least time 2. In addition, perhaps inadvertently, the analysis applies only when there are at most two states of the world at time 1. When my equation 1 applies, the condition described by BKT's equation 2.3 cannot be satisfied if there are more than two states. BKT require that a portfolio of bonds be equivalent to a contingent claim that is valuable in only one state of the world and is worthless in all other states. However, if my equation 1 holds, any portfolio that can be valuable at all may be worth zero in one state at most. That unrealistic requirement may also be the reason that for $t \neq 1$ there must be no chance that $R_s(1, t) = R(1, 1)$.

[1] The implications of the assumptions in equation 1 can be far simpler to analyze than those of Redington's assumption that all term structures of interest rates are level. For example, suppose that conditions 1 and 2 were satisfied but that liabilities consisted of sums due at several dates. Then to attain "complete" (but not "riskless") immunization, satisfying Redington's condition 3 would be sufficient only if changes in interest rates were infinitesimal. Although well recognized, the problem of stating a third condition that was both necessary and sufficient when changes in the level of the term structure were large was not solved until 1980. Then Gifford Fong and Oldrich Vasicek showed that a separate inequality had to be satisfied for each date that a sum was due. Under equation 1, the solution is far simpler—the problem vanishes. Conditions 1 and 2 are necessary and sufficient for riskless, as well as for complete, immunization. Indeed, the problem vanishes under all of BKT's new single-factor models.

I hope that BKT find more realistic conditions in which their assumptions hold. Their idea is too good not to be applicable under rather general conditions. Indeed, their analysis shows why single-factor immunization can be far better than mere first-order approximations of optimal policy.

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DISCUSSION

J. V. JORDAN*: This discussion focuses on three aspects of the Vasicek-Fong (VF) paper: (1) the implicit assumption of the pure expectations hypothesis in sections I and II, (2) some technical points about spline functions and (3) the specification of the term structure estimating equation.

That the pure expectations hypothesis underlies their methodology of estimating market forecasts is obvious from several quotes, e.g., "Specifically, the market implicit forecast of the one-period rate is equal to the forward rate for that period . . .". Certainly there is much literature (e.g., [10]) which maintains that forward rates are not equal to expected rates, but that forward rates include a term premium which may be positive or negative depending on many factors, including the preferences of some investors for some maturities over others. If there are term premia of consistent sign, then VF's forecasts will be biased. To be sure, measurement of premia is not an exact science, but the potential problem in ignoring them should be acknowledged.

Spline functions have a long history of use particularly in the physical sciences where their greatest application probably comes in fitting curves to experimental data (e.g., see [1] and [12]). For example, in the context of this paper, if we had T observations on the discount function, D_t, a cubic spline $f(t)$, could be estimated by the regression (see [12, pp. 21–28])

$$\underset{\sim}{D}_{Tx1} = \underset{\sim}{W}_{Tx(m+1)}\,\underset{\sim}{Y}_{(m+1)x1} + \underset{\sim}{\epsilon}_{Tx1} \tag{1}$$

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