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Optimal Bond Trading with Personal Tax: Implications for Bond Prices and Estimated Tax Brackets and Yield Curves[†]

GEORGE M. CONSTANTINIDES and JONATHAN E. INGERSOLL, JR.*

BOND PRICES REFLECT investors' expectations of the after-tax stream of cash flows generated by the bond and of the path of interest rates, as embodied in the term structure. In the absence of taxes on capital gains income, the stream of cash flows generated by a default-free bond is deterministic. The valuation problem in this case consists of computing the expected present value of a stream of known cash flows, but with stochastically varying interest rates as a part of a stochastically varying investment opportunity set. Cox, Ingersoll and Ross [4, 5] addressed this problem in a general equilibrium context and presented a comprehensive theory of bond pricing and the term structure of interest rates. With personal taxes on accrued capital gains, a similar model applies. (See Skelton [7] and Torous [8].)

With capital gains taxes only on realized capital gains and losses, the after-tax stream of cash flows generated by a bond is stochastic and depends on the fluctuations of interest rates. A sophisticated bondholder should follow the optimal trading policy with the objective of minimizing the discounted value of his stream of tax payments, and consequently maximizing the value of his stream of after-tax cash flows. The corresponding problem for a portfolio of stocks and the equilibrium implications of personal tax on the pricing of stocks were discussed in Constantinides and Scholes [3] and Constantinides [1, 2].

The present paper unifies these two strands of research. We examine the optimal bond trading policies in the presence of personal taxes. We also explore the tax implications for the pricing of bonds. In particular we examine the bias in estimating tax brackets and yield curves, when the possibility that investors follow a sophisticated trading policy which differs for a buy-and-hold policy is ignored.

The methodology employed is similar to that used for solving intertemporal portfolio selection problems. At each point in time we need to determine two quantities, the price of the bond and the value that a given investor assigns to the bond.¹ In the absence of taxes (and other imperfections of the market), these quantities would be identical. With a capital gains tax, however, the value of a bond depends not only on the current state of the bond market, but also on the

[†] Because our work is incomplete, yet already lengthy, we have provided only a summary here. Copies of the paper are available from the authors through the Center for Research in Security Prices at the Graduate School of Business of the University of Chicago as working paper number 70.

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¹ By price we mean what a willing buyer would pay. Because of the "lock-in" effect of capital gains taxes, no owner may be willing to sell at this price.

individual's current and future net tax liabilities. These may vary across investors as their tax holding periods or bases differ. Indeed, a given investor typically will assign a different value to the same bond when considering buying or retaining it.

The valuation procedure of the paper is outlined below. For simplicity we have assumed: (1) the after-tax single-period interest rate is the sole stochastic state variable affecting bonds; (2) the expectations hypothesis² is assumed to hold on an after-tax basis; (3) all investors are in the same tax bracket paying the same rates on coupon income, the same rates on realized short term capital gains and losses, and the same rates on realized long term capital gains and losses.

When the bond matures its price is equal to its face value plus the after-tax value of the last coupon. Its value, however, is equal to its price less the capital gains tax due. This depends on the common capital gains tax rates, the owner's basis, and the holding period. Proceeding by induction, the price of the bond at any earlier point is equal to the discounted expectation of the value of the bond one period later to an investor with a basis equal to the current price and a tax holding period of one period. The current value to an investor with a given holding period and basis is the greater of: (1) the discounted expectation of the value one period later for the same basis³ and a holding period greater by one and (2) the current price less the capital gains tax due upon sale.

The optimal trading policy is governed by the following considerations: active realization of capital losses, short term if possible; deferment of the realization of capital gains, particularly if short term; changing the status of the holding period from long to short term, by sale and repurchase, so that future capital losses may be realized short term; trading to take advantage of the favorable amortization treatment; and recognition of the forced realization of capital gains at maturity.

Because of the interaction of these effects, no simple characterization of the optimal trading policy is possible, as in the case of equities (see Constantinides [1, 2]). We can say, however, that it differs substantially from the commonly assumed buy-and-hold policy. Furthermore, the optimal trading policy for municipal bonds differs from the policy for corporate and Treasury bonds because, unlike with corporate and Treasury bonds, the amortization of a premium municipal bond provides no tax shield.

Under the assumption that investors follow intelligent trading policies, equilibrium bond prices differ from those implied by a buy-and-hold policy. This difference is the value of the "timing option" and can amount to a substantial portion of a bond's value. The timing option may also be interpreted as follows. Consider a bond that would sell at par if it were priced by investors following a buy-and-hold policy; these investors pay zero capital gains tax over the term to the bond's maturity. The same bond would sell at a higher price if it were priced by investors following the optimal trading policy. This price difference, the timing option, is the present value of the tax subsidy which the government provides

² By "expectations hypothesis" we mean that the expected (after-tax) rates of return on all bonds are the same and equal to the (after-tax) short term rate. See Cox, Ingersoll, and Ross [4] for a discussion of the different forms of the expectations hypothesis. In another paper [5] they show how this assumption may be weakened.

³ When the basis is greater than par, this procedure must be altered to account for amortization. The paper discusses this issue in detail.

under the current tax laws. Our numerical results indicate that the government offers substantial subsidies to bondholders who follow the optimal trading policy.

The magnitudes of these differences depend on the exact modeling of the tax code employed. For example, if all net short-term losses and half of all net long-term losses were deductible against ordinary income, the timing option would represent approximately 20% of the value of a 25-year par bond. Actually, there are two limitations on deductions. First, no more than \$3,000 may be deducted from income in this way each year. Second, if an investor has both long term gains and short term losses, they must be offset dollar for dollar prior to any deductions.

The question of whether or not the first bias materially affects the calculated prices depends upon the average size of capital losses for the marginal investor. Even if there is only a negligible effect on prices, some investors still get the tax subsidy mentioned earlier.

The importance of the second bias can be bounded by assuming that only one half of all capital losses are usable deductions. When this is done, the timing option still represents close to 6% of the value of bonds with maturities in excess of 15 years.

The potential price "errors" can lead to other errors as well. For example, if the bonds are priced at the margin by investors following the optimal policy, but the tax-adjusted yield curve is estimated with the standard assumption that bonds will be held to maturity (see McCulloch [6]), both interest rates and marginal tax brackets may be seriously misestimated. For example, the assumption that short term losses are fully deductible leads to an estimated tax bracket of 48% to 50% when all investors are in the 50% bracket, while long term yields are in error by as much as 350 basis points.

Our paper focuses on the case when the tax bracket of the "marginal investor" holding the bond remains unchanged. That is, an investor may buy and sell the bond in the course of the optimal policy, but the bond is held by investors in the same tax bracket throughout its term to maturity. The next step in this research effort is to explicitly recognize the existence of tax clienteles and explore the implications of optimal trading policies where the bond is passed on from one tax bracket investor to another as its maturity shortens or as it changes from a discount bond to a premium bond.

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DISCUSSION

L. FISHER*: I think that the main contribution of the paper by Bierwag, Kaufman, and Toevs (BKT) is that it turns directly to the study of relationships among price changes of fixed-income securities. Previous studies were indirect. They analyzed the nature of the first and higher derivatives of price with respect to interest rate and the interactions between "duration" and such derivatives. Although some of the relationships among the derivatives with respect to interest rates are linear, others are not. Hence, it has been difficult to state interest-rate-generation processes that lead to interesting equilibrium conditions, especially for the portfolio immunization problem. By turning directly to consideration of relative price movements, BKT offer a class of processes that may be consistent with equilibrium conditions in which immunized portfolios are fully hedged.

Because of these qualities, I believe that the BKT additive process will provide a much more satisfactory framework for studying the relationships among fixed-income securities than its predecessors.

To put BKT'S accomplishment in better perspective, let me add to their review of the development of immunization procedures.

F. M. Redington stated three conditions for portfolio immunization:

1. That the present value of the assets be at least as large as the present value of liabilities
2. That the derivative of the present value of the assets with respect to the "force of interest," i.e., the discount rate compounded continuously, be equal to the derivative of the present value of the liabilities
3. That the second derivative for the assets be at least as large as the second derivative for the liabilities.

However, he considered the portfolio to be immunized even if only the first two conditions were met.

Since the negative of the derivative of the logarithm of price with respect to the force of interest is equal to Frederick R. Macaulay's *duration*, David Durand restated condition 2:

2. That the duration of the assets be equal to the duration of the liabilities.

In testing empirically the efficiency of immunizing a liability with a portfolio of coupon bonds, this discussant and Roman Weil saw that Redington's assumption of a level term structure of interest rates was needlessly strong. We could modify Redington's assumption to one of "parallel" shifts in the term structure. Then we stated a slightly more general formula for duration.

However, for the problem we studied, the second derivative of the assets is larger than the second derivative for the liabilities. Hence, large changes in interest rates appear to lead to positive profits. Either immunization is inconsis-

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