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Regulation and Corporate Investment Policy

MICHAEL J. BRENNAN and EDUARDO S. SCHWARTZ*

I. Introduction

IN A PREVIOUS PAPER¹ it was shown that the traditional rule that the allowed return on the equity of a regulated firm should be set equal to its cost of equity capital was conceptually deficient.² This deficiency arises because the cost of equity capital of a regulated firm depends upon its risk, which depends in turn upon the regulatory policy that will be followed in the future, including the allowed rate of return. It was argued therefore that this traditional rule should be replaced by the requirement that the regulatory policy be consistent. A consistent regulatory policy achieves what the traditional rule only attempts: it ensures that at the time of a regulatory hearing the market value of the firm equity is equal to the equity financed portion of the rate base. To devise a consistent regulatory policy, it is necessary to have a valuation model which takes explicit account of regulatory policy. Such a model was developed in the above-mentioned paper.

In this paper we generalize the earlier valuation model so that the investment policy of the regulated firm is determined by the value maximizing decisions of the management instead of being taken as exogenous. To the extent that investment policy is discretionary, its endogeneity must be taken into account in devising a consistent regulatory policy, and this is made possible by the generalized model. In reality it seems unclear to what extent the investment policies of regulated firms are predetermined by the requirement that they meet demand, and to what extent they are discretionary. Since the model presented here easily accommodates constraints on the investment policy it is the more appropriate model so long as there exists any element of discretionary investment behaviour, and permits a more adequate approach to the determination of consistent regulatory policies.

With discretionary investment policy, an important consequence of any particular consistent regulatory policy is the investment policy it induces. In Section 4 we show how the value maximizing investment policy is affected by the choice of regulatory policy. On the other hand, regulated firms are wont to argue that their investments are in the main predetermined by demand and must be undertaken regardless of their profitability. This putative lack of discretion, it is maintained, means that the appropriate allowed rate of return is higher than would otherwise be the case. In Section 4 also we evaluate the strength of this consideration by comparing the allowed rate of return under consistent regulatory policies when investment policy is and is not discretionary. Section 5 shows briefly how the analysis can be extended to account for the effect of debt in the capital structure.

* The University of British Columbia. The authors thank Bruce Dietrich Campbell for computer programming assistance.

¹ Brennan and Schwartz (1981).

² This rule was referred to there as "the modern consensus approach".

Section 2 develops briefly a theory of the valuation of the optimally managed firm, which is specialized in Section 3 to apply to the regulated firm. It will be apparent that the approach followed in this paper, which is developed from the classic papers of Merton (1973) and Cox, Ingersoll and Ross (1978), may also be applied to the analysis of the optimal financial strategies and valuation of unregulated firms.

2. Valuation of the Optimally Managed Firm

Consider an economy in which

1. All investors have time-additive Von Neumann Morgenstern utility functions of the logarithmic type defined over the rate of consumption of a single consumption good.

2. There are no taxes or transactions costs, trading takes place continuously, and the capital market is always in equilibrium.

3. The state of the economy is completely described by aggregate wealth, W , time, t , and an m -dimensional vector of state variables, X . The state variables follow a controlled mixed stochastic process of the general type

$$dX_j = \mu_j(X, u, t) dt + \eta_j(X, u, t) dz_j + \Delta X_j(X, u, t) dq_j \quad (j = 1, \dots, m) \quad (1)$$

dz_j is a standard Gauss-Wiener process with $dz_i dz_j = \rho_{ij} dt$; $q_j(t)$ is an independent Poisson process with intensity $\pi_j(X, t)$ and ΔX_j is the change in the state variable if the Poisson event occurs; jumps in the state variables are assumed to be uncorrelated with the return on aggregate wealth. $u(X, t)$ is a vector of controls from the admissible set available to the individuals in the economy.

It follows then that for a given vector u , the equilibrium rates of return on individual assets will satisfy the specialized version of the intertemporal capital asset pricing model³:

$$\alpha_i - r = \sigma_{iw} \quad (2)$$

Since our concern is with the policy of an individual firm, it will be convenient to interpret $u(X, t)$ as the vector of controls available to the management of the firm in question, and to take as given the characteristics of the distribution of the rate of return on aggregate wealth.⁴ Then define $K_i \equiv K(X, u, t)$ as the market value of firm i when management follows the known policy represented by the control $u(X, t)$.

As shown in Brennan and Schwartz (1981), the value of the firm under this known policy satisfies the partial differential equation:

$$\begin{aligned} & \sum_{j=1}^m K_{x_j} (\mu_j - \eta_j \rho_{jw} \sigma_w) + K_w (rW - C) + K_t \\ & + \frac{1}{2} \sum_{j=1}^m \sum_{k=1}^m K_{x_j x_k} \rho_{jk} \eta_j \eta_k + \sum_{j=1}^m K_{x_j w} \rho_{jw} \eta_j \sigma_w W \\ & + \frac{1}{2} K_{ww} \sigma_w^2 W^2 - rK + D \\ & + \sum_{j=1}^m [K(W, X + \Delta X_j, u, t) - K(W, X, u, t)] \pi_j = 0 \end{aligned} \quad (3)$$

³ Merton (1973); the specialization arises from the assumption of logarithmic utility.

⁴ This is the standard competitive assumption.

In equation (3) the firm subscript has been dropped and subscripts now denote partial derivatives. $\underline{\Delta X}_j$ is an m -dimensional vector all of whose elements are zero except element j which is equal to ΔX_j . C is the rate of aggregate consumption; ρ_{jw} is the correlation between changes in X_j and in W ; σ_w is the instantaneous standard deviation of the rate of return on aggregate wealth. $D \equiv D(X, u, t)$ is the instantaneous aggregate payout rate to investors in the firm net of any security issues.

If the interest rate depends only on the state variables and time, and the covariance of the state variables with the rate of return on aggregate wealth is wealth independent, then it follows from Lemma 4 of Cox, Ingersoll and Ross (1978)⁵ that the value of the firm can be written as

$$K(X, u, t) = \hat{E} \int_t^\infty \exp \left[- \int_t^\tau r(X, s) ds \right] D(X, u, \tau) d\tau \quad (4)$$

where $\hat{E}$ denotes that the expectation is taken with respect to the risk adjusted process for the state variables:

$$dX_j = \hat{\mu}_j dt + \eta_j dz_j + \Delta X_j dq_j \quad (5)$$

and

$$\hat{\mu}_j(X, u, t) = \mu_j(X, u, t) - \eta_j \rho_{jw} \sigma_w (j = 1, \dots, m) \quad (6)$$

The value of the firm under the value maximizing policy, $F(X, t)$, is defined by

$$F(X, t) = \max_{u \in U} K(X, u, t) \quad (7)$$

where U is the set of admissible controls.

Let $L^u K$ be the differential operator of K associated with the control u :

$$\begin{aligned} L^u K = & K_t + \sum_{j=1}^m \hat{\mu}_j K_{x_j} + \frac{1}{2} \sum_{j=1}^m \sum_{k=1}^m \rho_{jk} \eta_j \eta_k K_{x_j x_k} \\ & + \sum_{j=1}^m \pi_j(X, t) [K(X + \underline{\Delta X}_j, u, t) - K(X, u, t)] \end{aligned}$$

Then the value-maximizing vector of controls, $\hat{u}(X, t)$, and the value of the firm under the value maximizing strategy, $F(X, t)$, satisfy⁶,

$$L^{\hat{u}} F(X, t) + D(X, \hat{u}, t) - rF = 0 \quad (8)$$

$$L^{\hat{u}} F(X, t) + D(X, \hat{u}, t) = \max_{u \in U} [L^u F(X, t) + D(X, u, t)]. \quad (9)$$

Equation (9) may be solved for $\hat{u}$ in terms of $F(\cdot)$ and its partial derivatives. Substitution for $\hat{u}$ in (8) then yields a partial differential equation for $F(\cdot)$ which may be solved given the appropriate boundary conditions.

3. The Regulated Firm

In this section we specialize the foregoing general model and apply it to the valuation of the regulated firm and the determination of the value maximizing

⁵ See also Theorem 5.2 of Friedman (1975).

⁶ See Cox, Ingersoll and Ross (1978) Lemma 1; Fleming and Rishel, Ch. VI (1975); Merton (1971) Theorem 1.

investment strategy. We assume for convenience that the interest rate r is known and constant and we maintain until Section 5 the assumption that the firm is financed entirely by equity capital. Then $K(\cdot)$ is defined as the market value of the equity of the firm and, under the assumptions described below, it may be written as a function of time, the (equity financed portion of the) rate base, B , the instantaneous rate of return on the rate base, x , and the rate of growth of the rate base, g : $K(x, B, g, t)$. The growth rate of the rate base is the investment policy control which the management is assumed to choose to maximize $K(\cdot)$.

As in Brennan and Schwartz (1981), the rate of return on the rate base is defined as the ratio of the earnings rate to the current value of the rate base, where the earnings rate is the sum of the rate of distributions to investors and additions to the rate base.

The instantaneous aggregate earnings rate, xB , is locally riskless in the sense that it follows a continuous sample path between regulatory hearings. The holding of regulatory hearings is assumed to be determined by a Poisson process whose intensity may depend upon the current rate of return on the rate base: $\pi(x)$. When a hearing is held, output prices are assumed to be adjusted instantaneously so that the rate of return jumps from its current value to the allowed rate of return, x^* . Between regulatory hearings changes in the rate of return are determined by a purely stochastic element which is represented by a Gauss-Wiener process, and by a drift term: the direction of the drift depends on both the current rate of return and the growth rate in the rate base, except in the singular case in which the rate of return on increments to the rate base is equal to the rate of return on the existing rate base. Thus the stochastic process for the return on the rate base is:

$$dx = \mu(x, g) dt + \sigma(x) dz + (x^* - x) dq. \quad (10)$$

The investment policy control, g , determines the growth in the rate base so that

$$dB = g B dt. \quad (11)$$

According to the rule for determining the rate base, the net payout to equity investors is the difference between earnings and the change in the rate base, so that

$$D(x, B, g, t) = B(x - g). \quad (12)$$

Then, corresponding to (7), the value of the firm under the value maximizing investment strategy is

$$F(x, B, t) = \max_{g \in G} \hat{E} \int_{\tau}^{\infty} e^{-r(\tau-t)} B(\tau)(x - g) d\tau, \quad (13)$$

where again expectations are with respect to the risk adjusted process for x in which $\mu(x, g)$ in (10) is replaced by $\hat{\mu}(x, g) = \mu(x, g) - \rho_{xw}\sigma(x)\sigma_w$. G is the set of feasible growth rates available to management. It is natural to take this as the interval $(g_{\min}, g_{\max})$. If investment policy were purely discretionary but there were no secondary market for physical assets then $g_{\min}$ would be determined by the

rate of depreciation of the rate base: in other circumstances it might be the minimum investment required to meet demand. The case for an upper bound on the range of feasible growth rates is less strong and is not necessary for the model so long as the return on additions to the rate base is constrained appropriately: an upper bound could be based upon physical constraints.

Using (8) and (9) the optimal investment strategy, $\hat{g}(x, t)$ and the value of the firm under that strategy are determined by

$$\begin{aligned} \max_{g \in G} [F_t + \hat{\mu}(x, g)F_x + g B F_B + \frac{1}{2} \sigma^2(x)F_{xx} \\ + \pi(x)[F(x^*, B, t) - F(x, B, t)] + B(x - g) - rF] = 0. \end{aligned} \quad (14)$$

Maximization of the expression in braces yields $\hat{g}$ in terms of $F(\cdot)$ and its derivatives; substitution for $\hat{g}$ in the same expression yields a partial differential equation for $F(\cdot)$ which, given the appropriate boundary conditions, may be solved. Once $F(\cdot)$ has been determined, $\hat{g}$ is known. The boundary conditions will depend upon the precise specification of regulatory and management policy. We shall assume that they are time independent so that $F_t = 0$. Assuming in addition that the boundary conditions for $F(\cdot)$ are homogeneous of degree one in B we may make the substitution $y(x) \equiv F(x, B)/B$ to obtain the equivalent expression in terms of y , the "normalized" equity value:

$$\begin{aligned} \max_{g \in G} [\frac{1}{2} \sigma^2(x)y_{xx} + \hat{\mu}(x, g)y_x + (g - r)y \\ + x - g + \pi(x)[y(x^*) - y(x)]] = 0. \end{aligned} \quad (15)$$

Using the fact that $\hat{\mu}_g(\cdot) \equiv \mu_g(\cdot)$, and assuming the requisite concavity, the optimal investment policy, $\hat{g}(x)$ is given by

$$\begin{aligned} 1) \hat{g}(x) &= g_{\min}; & \mu_g(x, g_{\min})y_x + y - 1 &\leq 0 \\ 2) g_{\min} &\leq \hat{g}(x) \leq g_{\max}; & \mu_g(x, \hat{g})y_x + y - 1 &= 0 \\ 3) \hat{g}(x) &= g_{\max}; & \mu_g(x, g_{\max})y_x + y - 1 &\geq 0 \end{aligned} \quad (16)$$

Further analysis of the optimal investment policy requires assumptions about the way in which the control g affects the return on the rate base. We assume that new investment is included in the rate base instantaneously and that the rate of return on the rate base addition, ρ , follows the same stochastic process as, and is perfectly correlated with, the rate of return on the pre-existing rate base. Then, if the rate of return on the rate base in the absence of any net change in the rate base follows the stochastic process

$$dx = \mu(x) dt + \sigma dz \quad (17)$$

it is simply shown that the effect of growth in the rate base is to change the stochastic process to

$$dx = (\mu(x) + (\rho(x, g) - x)g) dt + \sigma dz \quad (18)$$

In equation (18) we have allowed for the possibility that the rate of return on additions to the rate base $\rho(x, g)$, may depend on the current return on the rate base as well as on the rate of growth in the rate base. With the stochastic process

(18) the conditions for the optimal investment policy (16) become

$$\begin{aligned} 1) \hat{g}(x) &= g_{\min}; [\rho(x, g_{\min}) - x + g_{\min}\rho_g(x, g_{\min})]y_x + y - 1 \leq 0 \\ 2) g_{\min} &\leq \hat{g}(x) \leq g_{\max}; [\rho(x, g) - x + \hat{g}\rho_g(x, \hat{g})]y_x + y - 1 = 0 \\ 3) \hat{g}(x) &= g_{\max}; [\rho(x, g_{\max}) - x + g_{\max}\rho_g(x, g_{\max})]y_x + y - 1 \geq 0 \end{aligned} \quad (16')$$

The optimal investment policy is particularly simple if the rate of return on new investment is independent of the growth rate, for $\rho_g = 0$ implies the bang-bang policy

$$\begin{aligned} \hat{g}(x) &= g_{\min}; (\rho(x) - x)y_x + y - 1 \leq 0 \\ \hat{g}(x) &= g_{\max}; (\rho(x) - x)y_x + y - 1 \geq 0 \end{aligned}$$

When the return on new investment is also equal to the current rate of return on the rate base, $\rho = x$, the bang-bang rule is even simpler. The optimal policy is to invest at the maximum (minimum) rate whenever the market to book value ratio, y , exceeds (falls short of) unity. It appears to be this case which the managers of regulated firms have in mind when they argue that it is costly for the owners of the firm if the firm has to issue stock when the stock price is less than the book value.

If the regulator follows a consistent policy so that $y(x^*) = 1$, and the probability rate of hearings is a constant π , then the normalized value of the firm under the value maximizing investment strategy for $\rho = x$ satisfies the two partial differential equations obtained by substituting $\hat{g}$ in the maximand in equation (15)

$$\begin{aligned} \frac{1}{2}\sigma^2 y_{xx} + (\mu - \lambda\sigma)y_x + (g_i - r)y + x - g_i + \pi(1 - y(x)) &= 0 \\ i &= 1, 2 \end{aligned} \quad (19)$$

where $g_1 = g_{\min}$, $g_2 = g_{\max}$ and $\lambda = \rho_{xw}\sigma_w$. Equation (19) holds for $i = 1$ over the region $x \leq x^*$, and for $i = 2$ over the region $x \geq x^*$. The solution to the equations (19) for constant μ is of the form

$$\begin{aligned} y(x) &= C_1 e^{\gamma_1 x} + C_2 e^{\delta_1 x} + a_1 + b_1 x \quad x \leq x^* \\ y(x) &= C_3 e^{\delta_2 x} + a_2 + b_2 x \quad x \geq x^* \end{aligned} \quad (20)$$

where

$$\begin{aligned} a_i &= \frac{-g_i + \pi}{r - g_i + \pi} + \frac{\mu - \lambda\sigma}{(r - g_i + \pi)^2} \\ b_i &= \frac{1}{r - g_i + \pi} \\ \gamma_1 &= h + g_1 \\ \delta_i &= h - g_i \\ h &= -(\mu - \lambda\sigma)/\sigma^2 \\ g_i &= [(\mu - \lambda\sigma)^2 + 2\sigma^2(r - g_i + \pi)]^{1/2}/\sigma^2 \\ g_1 &= g_{\min}, g_2 = g_{\max} \end{aligned}$$

In (20), C_1 , C_2 , C_3 are constants to be determined by the boundary conditions. These come from the bankruptcy condition which determines the value of x such that $y(x) = 0$, and from the requirements that both expressions (20) are equal to unity and have the same slope for $x = x^*$, where x^* is the allowed rate of return and is to be determined. Calculation of the constants requires the solution of a set of four non-linear equations.

For arbitrary forms of $\rho(\cdot)$ the differential equation can only be solved numerically. To illustrate, we assume that $\rho(x, g)$ is of the form

$$\rho = a + bg + cx; \quad b < 0, \quad c > 0 \quad (21)$$

Under (21) the return on additions to the rate base is assumed to be positively related to the rate of return on the existing rate base, and to decline as the rate of investment increases. Using equations (18) and (21) in the optimizing condition (15), the value maximizing growth strategy is

$$\hat{g} = \frac{1}{2b} \left[\frac{1-y}{y_x} + x(1-c) - a \right] \quad (22)$$

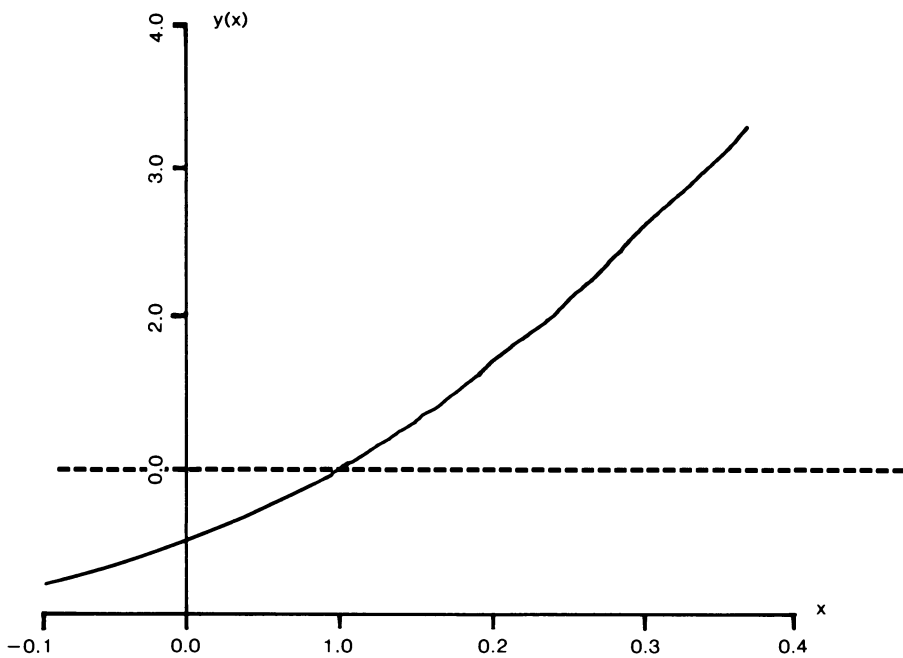
subject to the constraint that $\hat{g}$ be in the admissible range. Substitution for g from (22) in the maximand in (15) yields a non-linear ordinary differential equation which is solved numerically. Table 1 presents the underlying assumptions of a standard example, the results of which are shown in Figure 1. The firm value is a monotonic increasing function of the rate of return. The optimal investment rate also increases with the rate of return within the admissible range, reflecting the fact that the profitability of investment increases with x .

4. Investment Policy and Regulation

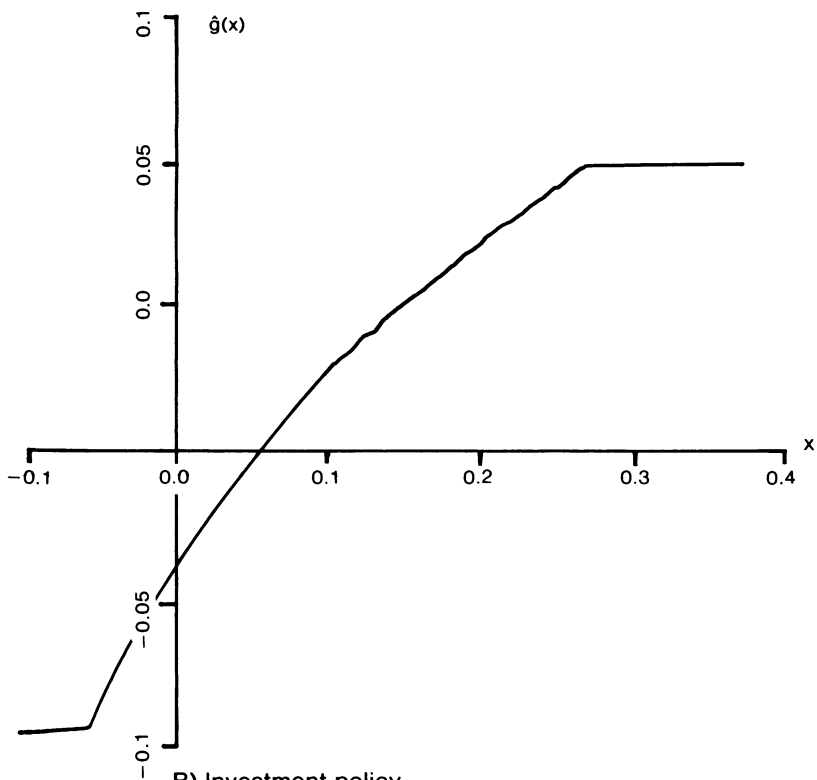
The valuation model presented in the previous section explicitly accounts for the effects of regulatory policy on the value of the firm. A change in the policy governing the holding of hearings, represented by $\pi(x)$, or a change in the allowed rate of return, x^* , will change either the differential equation governing $y(x)$; or its boundary conditions: in either case $y(x)$ will be affected. If the investment policy is discretionary then it is apparent from (16) that the value maximizing

Table 1
The Standard Example

Firm Characteristics
$dx = (\rho - x)g \, dt + .05 \, dz$
$\rho = .05 - g + x$
$g \in (-.10, .10)$
Regulatory Policy
$x^* : y(x^*) = 1$
$\pi(x) = .10$
Market Parameters
$\lambda \sigma_w = .14$
$r = .08$



A) Value



B) Investment policy

Figure 1. Firm Value and the Optimal Investment Policy

strategy will also be affected. This of course is taken into account by the model in relating $y(x)$ to the regulatory policy.

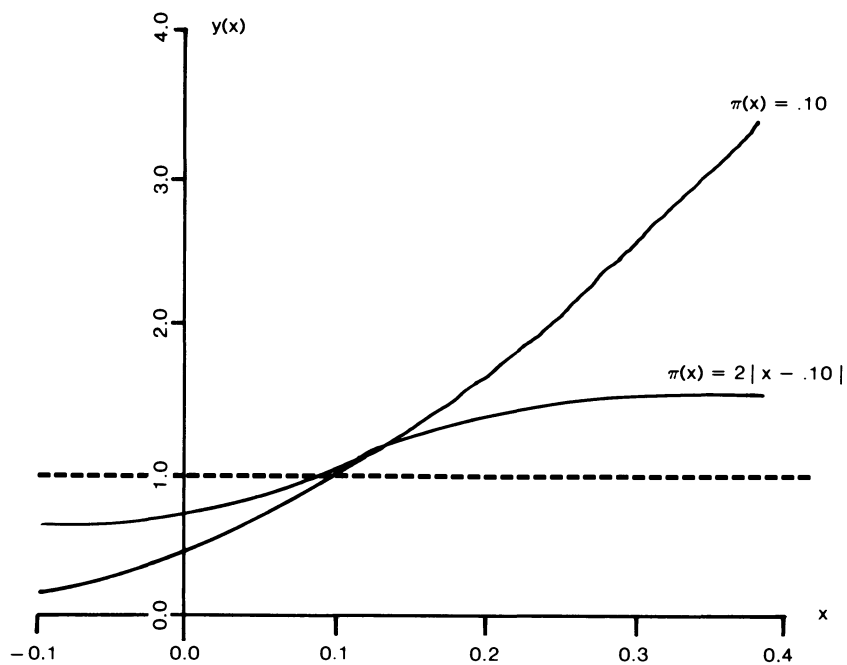
To illustrate how regulatory policy affects investment policy, Figure 2 compares the firm value and investment policy resulting from the assumptions of Table 1 with those that obtain if the probability rate of a hearing is changed from $\pi(x) = 0.10$ to $\pi(x) = 2|x - .10|$. We note first that the increasing probability of a hearing as the rate of return diverges from 10% under the new policy prevents $y(x)$ departing far from the value of unity which it will take on if there is a hearing. Turning to the investment policy, we observe that the optimal growth rate rises above its constrained minimum and even above zero long before the rate of return is sufficient to make $y > 1$. This gives the lie to those who have naively argued that a market to book ratio less than unity is a signal that a regulated firm should cease investment. As predicted, the value maximizing investment strategies are quite different under the two different regulatory policies: for example, when the rate of return is 20% the optimal growth rate under the original regulatory policy was only 6.69%; under the new policy it is 10%. The reason for this can be seen from equation (22) which shows that $\hat{g}$ depends upon both the level of y and its first derivative: the top half of the figure shows that these are quite different under the two policies. It is clear from this example that, to the extent the investment policies of regulated firms are discretionary, regulators in choosing between alternative regulatory policies should give thought to the consequences of their choice for the investment policy of the utility.

In discussing unregulated firms Myers (1977) has suggested that their growth opportunities may be regarded as options. The same is true for regulated firms only to the extent that their investment policy is discretionary and, as we have already remarked, it is unclear how much discretion in its investments a regulated firm possesses. To evaluate the importance of the element of discretion⁷, a firm was valued under the assumption that its growth rate was fixed at 10%, and again under the assumption that the growth rate in the rate base could be varied between plus and minus 10%. The rate of return on new investment was assumed to be equal to the current rate of return: $\rho = x$; the other assumptions are those of Table 1. The two firm value schedules are given in Figure 3. The investment strategy is not shown since in the one case it is predetermined and in the other case it is of the bang-bang variety according as $y \leq 1$. It is apparent from this example that the value of discretion in investment policy may be considerable. The appropriate allowed rate of return for the firm with discretion is 7.07%; for the firm without discretion it is 11.07%. If the allowed rate for the firm with discretion is set at 11.07%, the appropriate rate if it had no discretion, the firm would sell at a premium of 25.4% over book value. Conversely, if it had no discretion, but the allowed rate were set as if it had, then it would sell at a discount of 31.9% from book value.

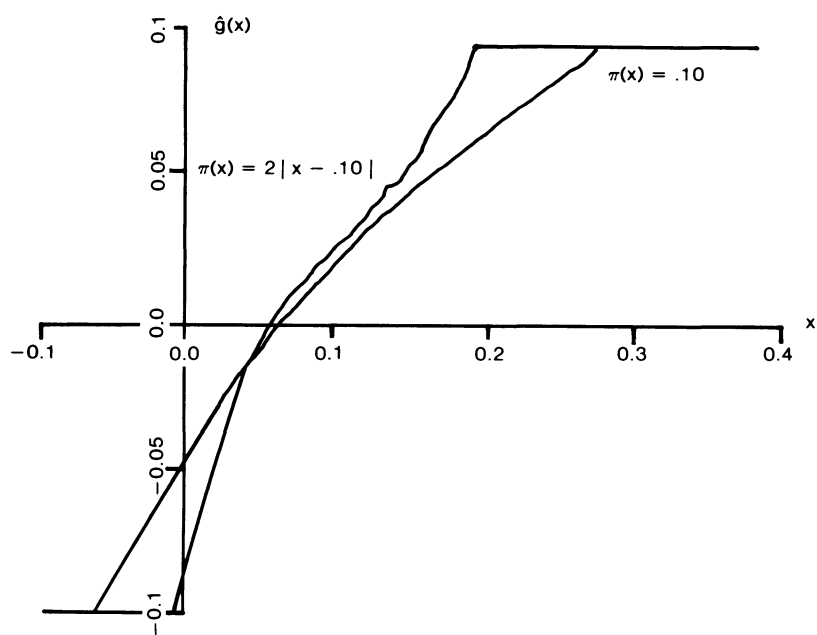
5. Investment Policy and Capital Structure

To this point we have assumed that the regulated firm has no debt in its capital structure and have been able to ignore therefore the problem of adverse invest-

⁷ i.e., to value the *optional* component of future growth opportunities.



A) Value



B) Investment policy

Figure 2. Value and Optimal Investment Policies Under Alternative Regulatory Policies

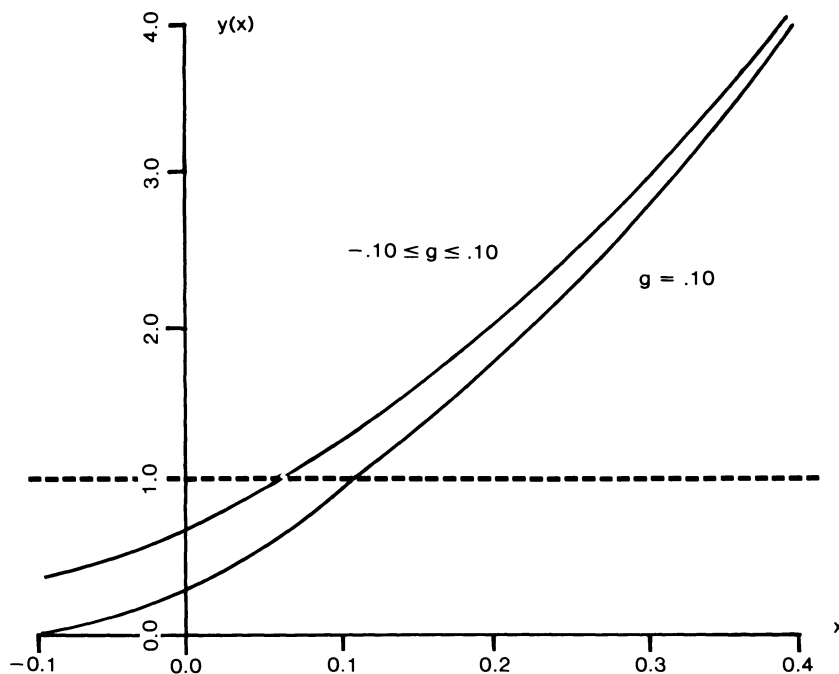


Figure 3. The Value of a Discretionary Investment Policy

ment incentives created by the divergent interests of bondholders and stockholders in certain situations.⁸ In this section we sketch the development of the model to account for outstanding debt liabilities.

Define A as the total rate base of the regulated firm⁹, and x as the rate of return on the rate base before interest payments. Under a given investment and financing policy¹⁰ the total market value of the firm, $V(\cdot)$, the value of the equity, $E(\cdot)$, and the value of the outstanding debt $H(\cdot)$ can all be written as functions of x , A and t , so that

$$V(x, A, t) = E(x, A, t) + H(x, A, t) \quad (23)$$

If the face value of the debt is M and the continuous coupon rate is c , then the payout rate to bondholders is cM . From the rate base definition the aggregate net payout rate to bondholders and stockholders together is $(x - g)A$ where g is the growth rate of the rate base. Therefore the net payout rate to shareholders is $(x - g)A - cM$.

For a given investment policy g , the value of the firm is determined as the solution to

$$L^g V + (x - g) - rV = 0 \quad (24)$$

Assuming, as in Brennan (1973) and Myers (1976), that the investment policy is chosen to maximize the value of the equity, $\hat{g}$, and the value of the equity

⁸ See Brennan (1973) and Myers (1976).

⁹ Including that portion which is debt financed.

¹⁰ It is beyond the scope of this paper to consider the determination of an optimal financing policy.

under the optimal investment strategy are determined by the solution to

$$\max_{g \in G} [L^s E + (x - g)A] - cM - rE = 0 \quad (25)$$

Having determined the value of the equity and $\hat{g}$ from (25), (24) may be solved for the total value of the firm. The value of the debt is determined residually from (23).

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