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Session Topic: ISSUES IN CORPORATE FINANCE

Presiding: JOHN J. McCONNELL*

Optimal Managerial Contracts and Equilibrium Security Prices

DOUGLAS W. DIAMOND and ROBERT E. VERRECCHIA**

I. Introduction

RECENT DEVELOPMENTS IN ECONOMIC theory have focused attention on efficient methods for providing incentives within a corporation, especially to top management. Jensen-Meckling [12] have examined some implications of incentive (or agency) problems for corporate finance. In this paper we combine a simple model of security market equilibrium with that of an optimal managerial incentive contract, using results of Harris-Raviv [8] and Holmström [9]. We provide a rationale for including security prices in the managerial incentive contracts of publicly held firms. In this model we also show that non-systematic, diversifiable risks will be considered in the capital budgeting decisions of a managerial firm operated in the interests of its stockholders. This is in contrast to the implication of the Arbitrage Pricing Theory (a similar result follows from the Capital Asset Pricing Model) which abstracts from incentive problems and suggests that only systematic risks ought to be considered.

The optimal-contract model of the corporation takes the following view of the firm. Since decision makers within a firm are not sole owners of the business, but employed agents, the decisions which they make depend on the incentives which the organization provides. These incentives are embodied explicitly in employment contracts, and implicitly through the threat of dismissal or effect on reputation. The contract, whether explicit or not, describes the decision maker's anticipation of the consequences to him conditional on various observable outcomes. These outcomes are used to provide information useful in determining whether he did his job properly. In the simplest optimal contract scenario, there is a single principal and a single agent whose decision concerns the level of effort he expends. The level of effort affects the level of output of the firm, but not unambiguously because the output is also governed by other stochastic elements. As a result, some information about the agent's level of effort can be inferred

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from output; this implies that an optimal compensation scheme for the agent will depend on the realization of output. Furthermore, even in the polar case in which the principal is risk neutral and the agent is risk averse, some of the output risk is borne by the agent. Any piece of information (in addition to output) which allows a principal to distinguish random fluctuations in output from the agent's level of effort should be included in an optimal incentive contract. Critiques of the optimal contract model (e.g., Fama [5]) argue that there are many more precise measures of effort or ability, especially over time. These criticisms of the model are statements that some useful information has been omitted from the analysis. No one claims that agents work hard in the absence of incentives.

The analysis is kept simple; mathematical representations are used primarily to clarify and demonstrate our points. We proceed by modeling explicitly a problem in optimal contracting and deriving its solution. Explicit characterizations of optimal contracts are extremely rare in this literature; much of the discussion in Harris-Raviv [5] and Holmström [9], for example, involves contracts which cannot be explicitly solved. We obtain explicit solutions by introducing strong assumptions, but it should be clear that the economic intuition underlying our results is robust. The explicit characterizations allow us to make points involving comparative static analysis which would be cumbersome to obtain otherwise.

II. The Optimal Contract Model

Our starting point is a Harris-Raviv-Holmström model with a single agent and a single principal. The agent expends a level of effort, denoted by a , which affects the stochastic distribution of the output of the firm. Let $\tilde{u}$ represent the stochastic output, and $\tilde{u} = u$, its realization. The principal does not observe the level of effort directly, but only u , the realization of output, and, possibly, other information which allows him to imperfectly separate the agent's level of effort from the other random determinants of output. Apart from the agent's level of effort, which is observed only by the agent, there is symmetric information. The distribution of the random variable $\tilde{u}$ and a and the preferences of the principal and agent, including the agent's distaste for increased effort, are common knowledge. The objective of the principal is to choose an incentive contract; this contract can depend on anything which both he and the agent jointly observe. There is a minimum level of expected utility the agent must derive from the contract for him to accept the job (which represents his employment opportunities elsewhere), but apart from this constraint, the principal chooses the contract which maximizes his own expected utility.

To illustrate this, suppose that the realization of output $\tilde{u} = u$ is the only available information. Let the utility function of the agent be $U_A(W, a)$, where W is his end of period wealth and a is the level of effort he expends. U_A is increasing in W and decreasing in a . To accept the job with the principal, the agent must obtain expected utility of at least $\bar{H}$. The principal's utility function is given by $U_P(W)$.

An optimal incentive contract is a function $I(\tilde{u})$ which gives a payment I to the agent as a function of the observable quantity $\tilde{u} = u$. Furthermore, it maximizes

the principal's expected utility conditional on the requirement that the agent achieves a minimum level of expected utility and the realization that the agent will choose a level of effort to maximize his level of expected utility given the contract. That is, it solves the following:

$$\text{maximize } E[U_p(\tilde{u} - I(\tilde{u}))] \quad (1a)$$

$$\{I(u), a\}$$

$$\text{subject to } E[U_A(I(u), a)] \geq \bar{H} \quad (1b)$$

$$a \in \text{argmax } E[U_A(I(\tilde{u}), a')] \quad (1c)$$

$$a' \in [0, \infty)$$

where all the expectations are taken over the random variable $\tilde{u}$ (whose distribution is determined in part by the level of effort that the agent selects) and the notation "argmax" denotes the set of arguments that maximize the objective function that follows (which takes account of the effect of the contract $I(\cdot)$ on the agent's unobserved choice of effort, a).

We assume a preference structure and technology similar to that in an example of Holmström [10, p. 79]; this enables us to explicitly characterize an optimal contract. This avoids technical difficulties discussed in Holmström [10], Grossman-Hart [7] and Clark-Darrough [1].

There is an implicit cost associated with the principal's inability to observe the agent's level of effort. When the agent's level of effort can be observed, the agent can be directly instructed to select the optimal level of effort. This allows the risk of random fluctuations in $\tilde{u}$ to be shared optimally. In addition, the agent can then be paid a constant amount to reward him for his effort. The expected utility of the principal in the two cases of observability versus unobservability are quite different. The expected utility in the former case is, generally, higher than in the latter case. Furthermore, the level of effort expended by the agent when it is unobservable is normally less than the level of effort exercised when it is observed (but see Shavell [16] for a counterexample). This is because the agent knows that with unobservability the random noise in $\tilde{u}$ makes it impossible for the principal to exactly verify any claim about the level of effort: in particular, no claim that effort is greater than what he is induced to expend in (1c). Therefore, there will be lower expenditure of effort by the agent and/or inferior risk sharing as compared with the case of observability.

For the principal, the value of information which allows the agent's level of effort to be observed directly is the difference in his expected utility in the two cases. However, any information is of positive value if it reduces the ex post noise of direct estimates of an agent's level of effort. This is proven in Shavell [16] and Holmström [9]. It can be explained intuitively by noting that when the agent selects a level of effort, the more precisely he expects variations in his level of effort to be directly revealed to the principal, the harder he works for a given degree of risk sharing. We analyze the model to evaluate the usefulness in top management contracts of information found in capital markets and derive other implications of using this information.

III. Incentives in Corporations

Two important facts about public corporations need to be reconciled with the above model. First, if stockholders are the principals, then there are many principals. Second, if employees, including management, are the agents, there are many agents.

Beginning with the second point, it is probably inaccurate to model all of a firm's employees as agents directly for the stockholders. This is because information on which most employees' compensation is based is not observed by stockholders, but rather by their direct supervisors. That is, most firms have a hierarchial structure. Much of the information relevant for supervised employees is observed only within the firm. Incentive schemes for supervised employees should be designed using information produced internally by the firm. This, however, merely shifts the problem back one level, as someone within the firm must construct the incentives for its employees. How this someone makes these decisions depends on *his* incentives. The interpretation which appears most useful is to view the "top management" as a single agent for the stockholders; his, or her, job is to take a variety of actions, including devising incentives for other layers in the hierarchy. It is important that this top level have some implicit or explicit incentive contract with outside security holders, or there would be no reason to anticipate that the firm would be operated in their interest in any sense. The amount of information which outsiders have available for providing management with the necessary incentives is a determinant of, at the optimum, how clearly aligned are their goals with that of the corporation (supposing for now that stockholders' interests are identical, for example, as a result of trade in competitive markets). The agent we consider is interpreted to be the top management. The agent will be either top executives or the board of directors (in firms where the board of directors plays an active role).

The other fact about corporations is that they have many security holders who can diversify across many firms. This suggests that circumstances can arise which are very different from that with a single principal. Imagine there is useful information which stockholders obtain costlessly, but which is expensive for them to directly communicate for use in managerial compensation. The externality problem associated with information transmission can be illustrated most simply by assuming there are N identical risk neutral stockholders who each own an equal fraction of the firm and who each observe an identical piece of information which is useful for managerial contracting. Suppose there is some cost $c > 0$ of making use of that information, e.g., the cost of a stockholder's time in effecting a change in the manager's pay. Suppose that the value of information, in terms of the increased utility to the stockholders as a group, is k and $k > c$. If there were a single principal, the information would be used because the return would more than cover his costs. For $N > 1$, the condition for an individual stockholder to make use of the information is $\frac{k}{N} \geq c$. If N is large, it is possible that $k > c$

while $\frac{k}{N} < c$: that is, even though the total return is large, the return per stockholder is small. In that case, each stockholder has the information, yet none make any use of it. A Nash equilibrium to the game between stockholders implicit

in the discussion is for no stockholder to spend c to transmit the information. The only direct incentive which stockholders have is to use the information for their own portfolio choice.

When stockholders use this information in their portfolio choice, competitive market prices become dependent on the information of investors. A simple model is suggested here in which writing a contract using the equilibrium market price of the firm's shares extracts the relevant information. This allows the stockholder's private information to be used for undelegated monitoring of the top management. In effect, the market is exploited as a low cost communication device. For clarity, the model continues to assume that investors obtain identical information. The models of Grossman [6] and Diamond-Verrecchia [4] show that market prices can aggregate the diverse information of various investors; so the assumption of identical information is used here solely for analytical convenience. In the more general case of diverse aggregation, the capital market would serve as a communication device in a decentralized information receipt system.

IV. A Model of Equilibrium Prices as Undelegated Monitors

A Description of the Economy

In the economy there are $N + 1$ consumers indexed by $i = 1, \dots, N + 1$; the first N of these are called stockholders and the one indexed by $N + 1$ is called a (top) manager. The economy has two assets, a riskless bond and claims to the output of a production process which are called shares; the bond pays out in the same single consumption good that the process produces as output. Stockholders are endowed with riskless bonds and shares.

Stockholders employ the manager to expend effort to increase the output of the production process. Call the manager augmented process a firm. The stockholders also exchange shares of the firm, along with bonds, amongst themselves in a competitive market. The manager is not endowed with assets but, rather, accepts a contract offered by the stockholders as compensation for expending effort.

We assume that the risk averse manager has a separable utility function of the form

$$U_A(W, a) = 2\sqrt{W} - a^2.$$

That is, the manager's utility function is defined over wealth and effort: in particular, it is concave and strictly increasing over wealth, and concave and strictly decreasing in effort. We assume that each stockholder is risk neutral; this implies that he has a utility function given by

$$U_P(W) = W.$$

The numeraire in the economy is the price of a bond which returns one unit of consumption at the end of the period; all stockholders know this return. Let P denote the equilibrium price of the firm: the aggregate price of all shares.

Five discrete intervals comprise a period in the economy. At the first interval stockholders are endowed with bonds and shares and contract with the manager to operate the firm. At the second interval the manager chooses his level of effort.

At the third interval stockholders receive identical information about the firm's output. At the fourth interval they exchange bonds and shares in a competitive market but do not consume; at this time an equilibrium price P for the exchange of shares of the firm is determined. Finally, in the last interval the firm's output is realized, and each stockholder consumes the return realized from his portfolio of riskless bonds and shares of the firm's output.

In the beginning of the period the output of the firm is not known: it is a random variable denoted by $\tilde{u}$. Let u denote the realized output of the firm. (Henceforth, random variables are denoted by a tilde, i.e., " $\sim$ "; realizations of a random variable are denoted without the tilde).

Let $\tilde{\gamma}$ be a random variable which is the information observed by the principals at the second time interval. It has a uniform distribution between zero and one. Let $a \in (0, \infty)$ denote the unobserved level of effort selected by the manager. Implicitly define $\tilde{\gamma}$ as a random variable whose conditional distribution given $\tilde{\gamma} = \gamma$ is exponential with mean $a + \gamma + \alpha$ and variance $(a + \gamma + \alpha)^2$, where α is a known real valued parameter. Adjustments in the agent's effort thus influence the distribution of $\tilde{\gamma}$. Define output, $\tilde{u}$, as

$$\tilde{u} = \beta \tilde{x} + \tilde{\gamma} - \alpha$$

where $\tilde{x}$ is a random variable representing a systematic risk factor which is directly observable ex-post and has a uniform distribution on the set of reals $[-1, 1]$, and β is a known real valued constant. The random variable $\tilde{x}$ is distributed independently of $\tilde{\gamma}$ or γ . The principals' knowledge of γ gives them information about the mean and variance of output for any given level of a chosen by the manager.

To interpret the parameters α and β , notice that expected output $\tilde{u}$ is given by:

$$E[\tilde{u}] = E[\beta \tilde{x} + \tilde{\gamma} - \alpha] = \beta E[\tilde{x}] + E[\tilde{\gamma}] - \alpha = \beta \cdot 0 + (a + \gamma + \alpha) - \alpha = a + \gamma$$

and the variance of output is given by

$$\text{Var}[\tilde{u}] = \beta^2 \text{Var}[\tilde{x}] + \text{Var}[\tilde{\gamma}] = \beta^2 \left(\frac{1}{3} \right) + (a + \gamma + \alpha)^2.$$

Thus α and β influence the variance of output $\tilde{u}$ but have no effect on its mean. The random variables $\tilde{x}$ and $\tilde{u}$ are directly observable ex post: only by observing them can a value of $\tilde{\gamma}$ be inferred. Thus $\frac{1}{3} \beta^2$ is the variance of the part of $\tilde{u}$ known to be ex post observable and not under the manager's control, while α indicates the variance of the component of output influenced by the manager's action. It is useful to define $\tilde{v} = \beta \tilde{x} - \alpha$, which has uniform distribution between $-\beta - \alpha$ and $\beta - \alpha$; $\tilde{v}$ is simply the observable risk, minus a known constant. Any contract written over the observable variables $\tilde{u}$ and $\tilde{x}$ can equivalently be defined (by a simple change of variables) to implicitly be a contract written over $\tilde{\gamma}$ and $\tilde{v}$. This change of variables simplifies the exposition of our analysis.

The distribution of output $\tilde{u}$ has been described thus far for a given realization of $\tilde{\gamma}$, the random variable which represents the private information about $\tilde{u}$ which investors receive in the second time interval. Its realization is not known in the

first interval, when (for a given value of effort, a) $\tilde{v}$, $\tilde{\gamma}$, and $\tilde{y}$ have a joint density function which by common knowledge is given by

$$f(v, \gamma, y) = \frac{1}{2\beta} \cdot \frac{1}{a + \gamma + \alpha} \cdot e^{\frac{-y}{a+\gamma+\alpha}}.$$

An Optimal Contract

The stockholder's problem is to determine the optimal contract for the manager. Both the stockholders and manager observe the realization of output $\tilde{u} = u$, the realization of the component of the systematic risk $\tilde{x} = x$, and the equilibrium market price of the firm, P ; however, the manager does not observe $\tilde{\gamma} = \gamma$. Thus, a managerial contract between stockholders and manager can only be written on those things that are jointly observed, u , x , and P . Make the change of variables (described above) from u and x to v and y . Let $I(\cdot)$ be the manager's incentive contract. Explicitly, let $I(v, y, P) \in [0, u]$ denote that share of the output u that goes to the manager, as a function of the observations $\tilde{v} = v$, $\tilde{y} = y$, and P .

Let B_i and D_i , $i = 1, \dots, N$, denote the demand for bonds and fractional shares of the firm, respectively, of stockholder i in the (fourth) exchange interval. Let amount $u \cdot D_i$ denote the gross share of the firm's output that goes to stockholder i .¹ Stockholders are risk neutral and have homogeneous prior beliefs about $\tilde{u}$. Thus, given that principals anticipate a level of effort a^* , this implies that after observing $\tilde{\gamma} = \gamma$ (at the third interval of the period) P is equal to the mean of $\tilde{u}$, conditional on $\tilde{\gamma} = \gamma$:

$$P = E[u | \tilde{\gamma} = \gamma, a^*] = \int_0^\infty \int_{-\beta-\alpha}^{\beta-\alpha} (v + y) \frac{1}{2\beta} \cdot \frac{1}{a^* + \gamma + \alpha} e^{\frac{-y}{a^*+\gamma+\alpha}} dv dy = a^* + \gamma$$

However, because the realization of $\tilde{\gamma}$ is unknown at the beginning of the period, stockholders and the manager regard the equilibrium price as a random variable $\tilde{P} = a^* + \tilde{\gamma}$; that is, when a contract is initially written, $\tilde{P}$ has a uniform distribution between a^* and $a^* + 1$. Finally, note that because stockholders are identical in every relevant economic characteristic, a symmetric competitive equilibrium is for each to demand an equal fraction $1/N$ of ownership of the firm. Therefore, $D_i = 1/N$ for $i = 1, \dots, N$.

Let $\bar{H}$ be the exogenously specified amount of expected utility the manager can achieve elsewhere on the labor market. Principals choose a contract to maximize their expected return net of payment to the agent (see footnote 1). The (constrained) optimal incentive contract $I(v, y, P)$ which solves the generalization of

¹ Our analysis proceeds by assuming that the payment to each shareholder is directly proportional to $\tilde{u}$ (the gross return) and each shareholder then makes his proportional payment to the manager from the shareholder's personal account. This serves to simplify the analysis. Conventionally, one buys a share of the net output of the firm. In our model this implies that price will be the expected value of $\tilde{u}$ net of the expected compensation. In this case there exists an invertible function from price into γ , and hence γ can always be inferred from price. (Invertibility follows from the fact that $E[\tilde{u} - I(\cdot) | \gamma]$ is a monotone function of γ .) Therefore, the contract discussed in this paper can be implemented by substituting the invertible function in place of price wherever price appears in our analysis.

equation (1) described here is the solution to:

$$\begin{aligned} \text{maximize}_{I(v, y, P) \in [0, u], a \in [0, \infty]} & \int_{-\beta-\alpha}^{\beta-\alpha} \int_0^\infty \int_a^{1+a} \{B_i + [v \\ & + y - I(v, y, P)]D_i\} f(v, y, P) dv dy dP \quad (2a) \end{aligned}$$

$$\text{subject to} \int_{-\beta-\alpha}^{\beta-\alpha} \int_0^\infty \int_a^{1+a} \{2\sqrt{I(v, y, P)} - a^2\} f(v, y, P) dv dy dP \geq \bar{H}, \quad (2b)$$

$$\text{and} \int_{-\beta-\alpha}^{\beta-\alpha} \int_0^\infty \int_a^{1+a} \{2\sqrt{I(v, y, P)}\} f_a(v, y, P) dv dy dP = 2a. \quad (2c)$$

where f is the joint density function of v , y , and P , and f_a is the partial derivative of f with respect to a .²

It is shown in Appendix A that the optimal contract is given by

$$I(u, x, P) = \left[\frac{1}{2}(a^* + \bar{H}) + a^*(a^* + \alpha)(1 + a^* + \alpha) \left\{ \frac{u - \beta x - P}{(P + \alpha)^2} \right\} \right]^2 \quad (3)$$

where $a^* \geq 0$ is a constant, the unique real-valued solution to the polynomial expression

$$1 - a^{*3} - a^*\bar{H} - (a^* + \alpha)(2a^* + 3a^{*2} + 2a^*\alpha) - a^{*2}(1 + a^* + \alpha) = 0. \quad (4)$$

The expected utility of the principal given the optimal contract is

$$E[\tilde{u} - I(\tilde{u}, \tilde{x}, \tilde{P})] = a^* + \frac{1}{2} - \{ \frac{1}{4}(a^{*2} + \bar{H})^2 + (a^{*2} + \alpha)(1 + a^* + \alpha) \}. \quad (5)$$

Several features of the optimal contract are illuminating. The output, u and observable risk, βx , enter the contract in the form $u - \beta x$. This means that all of the observable risk, βx , is filtered out of the agent's compensation and the risk is borne by the risk neutral principals. This has strong implications for the optimal capital budgeting decision. This is analyzed below, where we equate observable risk with systematic risks.

The contract also depends directly on P , the market equilibrium price of the firm. P provides access to principals' private information. If there were no market price, then the information would be unavailable and principals would be worse off. This illustrates an additional allocational benefit of informationally efficient security markets. The optimal contract will depend on P in more general settings

² Note that

$$f(v, y, P) = \frac{1}{2\beta} \frac{1}{P + \alpha} e^{\frac{-y}{P + \alpha}},$$

and

$$f_a(v, y, P) = \frac{1}{2\beta} \left\{ \frac{y - (P + \alpha)}{(P + \alpha)^2} \right\} \frac{1}{P + \alpha} e^{\frac{-y}{P + \alpha}}.$$

(as long as P contains some information about $\tilde{\gamma}$) since this allows a more precise direct inference about the agent's action, a .

The model illustrates the idea that equilibrium prices can be useful in contracts because they provide access to information which is not otherwise available publicly. The example presents a case in which the private information $\tilde{\gamma}$ can be perfectly inferred from the price. This is not a general result. Informational efficiency of the security market is not sufficient for prices to be useful in contracts, even if stockholders have private information which would be useful for contracting. This is because only information about the value of the firm will be contained in the price, and some information useful for contracting may not be useful for valuation. A simple example will illustrate this. Suppose that stockholders privately observe the realizations of $\tilde{\gamma}$ and $\tilde{u}$ before they trade, and, in addition, they privately observe a , the actual action selected by the agent. If a were public information, or, equivalently, if there were only one principal, there would be no incentive problem because a is observed. In our many principal setting, however, there is not costless access to principals' private information: only P can be observed freely. But because $\tilde{u} = u$ is observed (and interest rates are zero), the equilibrium price will be $P = u$, and it will contain no more information about the action than does output, u , by itself. Note that it no longer even contains any additional information about $\tilde{\gamma}$. This illustrates that information efficiency (even in a strong form) does not imply that using the market price is tantamount to reading the minds of investors. In the situation described here, the many principals ought to delegate the monitoring of the agent's action (and enforcement of the contract) to a single individual. A theory of delegated monitoring (interpreted as financial intermediation) is presented in Diamond [3].

V. Capital Budgeting Implications

The optimal contract derived above shows the general result that the observable risks $\tilde{x}$ ought to be filtered out of the manager's compensation. In Diamond [3] and Holmström [10] the point is made that it is reasonable to view systematic risks as directly observable risks and unsystematic risks as not directly observable. The systematic factors which influence the returns of many firms are often directly observable, viz., GNP. Furthermore, if the returns of firms in the economy have the factor form assumed by Ross [15], Connor [2], and Huberman [11], then the systematic risks can be directly estimated.

For example, suppose that there are N firms in the economy and firm j 's return is given by

$$\tilde{u}_j = \beta_{1j}\tilde{x}_1 + \beta_{2j}\tilde{x}_2 + \cdots + \beta_{mj}\tilde{x}_m + \tilde{y}_j(a_j),$$

where the β coefficients are non-zero, the $\tilde{x}_i$'s are the same factors across firms, and the $\tilde{y}_j(a_j)$ for given action a_j are uncorrelated with the $\tilde{x}_i$. If the number of firms in the economy (i.e., N) is large, as a good approximation only the systematic risks $\tilde{x}_1, \dots, \tilde{x}_m$ will be priced (i.e., influence equilibrium expected returns)

because the nonsystematic risks $\tilde{y}_j$ ($j = 1, \dots, N$) will be diversified away by investors. Thus, these models imply that when risk averse investors select among investment projects, the systematic risks ought to be taken into account, but not the unsystematic.

Our model demonstrates this result does not hold when firm managers are viewed as agents who must be given effort incentives by stockholders. A simple scenario is to assume that the firm in our model has available two possible technologies ("projects") with different parameter values, and that the one selected is freely observable to all. Given that the agent will be employed for either and provided with an optimal contract, what criteria would stockholders (principals) use in selecting which project to undertake; that is, what capital budgeting rule would the firm use if operated in the interest of the stockholders? When stockholders are risk neutral, no risks are "priced;" thus the traditional prescription would be to select the project with the highest expected return.

Suppose the two projects, project 1 and project 2, have identical parameters, except that the first project has a higher variance of nonsystematic risk ($\alpha_1 > \alpha_2$). The asset pricing theories suggest that the two projects are equally desirable when stockholders are risk neutral. Our agency model demonstrates that the project with the lower nonsystematic risk (project 2) is preferred by principals. Principals are concerned with their expected return net of payments to the agent: this value is

$$\begin{aligned} E[\{\tilde{u} - I(\tilde{v}, \tilde{y}, \tilde{P})\} \cdot D_i] &= E\left[\{\tilde{u} - I(\tilde{v}, \tilde{y}, \tilde{P})\} \frac{1}{N}\right] \\ &= \frac{1}{N} \{E[\tilde{u}] - E[I(\tilde{v}, \tilde{y}, \tilde{P})]\}. \end{aligned}$$

It can be shown that this expression is strictly decreasing in α , the variance of nonsystematic risk (see appendix B). This implies that in choosing among investment projects, nonsystematic risk is important because an increase in the variance of the unobservable nonsystematic risk of a project causes a corresponding increase in the incentive costs associated with that project.

The general result which will apply with risk averse stockholders is as follows. The agent must bear some nonsystematic risks precisely because they are nonsystematic and hence not directly observable from sources other than the particular firm's returns and security prices. The agent must bear some of the risk of the firm's output for incentive reasons, and there is no independent way to observe the nonsystematic risks. That is, the only source of information about the realization of non-systematic risk is the firm's realized return, u . The nonsystematic risks are important to the stockholders because a risk averse manager must bear some of them directly for incentive purposes and requires increased compensation for bearing them.³ The systematic risks are important to the

³ The discussion in the text deals with the Arbitrage Pricing Theory, where systematic and unsystematic risk are well defined. In the Capital Asset Pricing Model with a general covariance matrix, "residual risk" (i.e., that part of a project's variance which cannot be attributed to covariance with the market portfolio) is not the best measure of those unobservable risks which are relevant for incentive costs. This is because the best independent prediction of any particular project's return is not generally the market portfolio, but depends on the particular covariance matrix. For example, the returns to two projects might be almost perfectly correlated while neither was highly correlated with the market portfolio.

stockholders because they cannot be diversified away and, therefore, must be borne by someone. In brief, the capital budgeting rule desired by the stockholders will involve management consideration of nonsystematic risks (as well as systematic risks when stockholders are risk averse).

VI. Conclusions and Extensions

The combination of an optimal incentive contract model with a simple asset pricing model provides a useful perspective for many issues in corporate finance. It provides an informational role for using security prices in managerial incentive contracts, and produces interesting conclusions for the positive and normative theories of capital budgeting. The model used here contains only the most basic elements necessary to address these issues. Several interesting extensions of the analysis come to mind. The first is to use an asset pricing model with risk-averse principals. Another is to examine optimal incentive contracts for managers when both the effort and the capital projects are unobservable. In this case, the levels of systematic and nonsystematic risks of the selected project will not be observed directly and it is probable that systematic risks would not then be filtered away perfectly. Finally, there are some difficult issues involved in formulating empirical tests of the theory. An important issue is whether to use an explicit contract which prevails at a given time, or try to infer an implicit contract from a time series of explicit contracts and actions such as dismissal. Tax considerations also cause empirical problems, as discussed in Miller-Scholes [13].

While many questions remain to be answered, we hope that this very basic model serves to clarify at least a few of them.

Appendix

A

A brief sketch of the mechanical steps required to solve for an optimal contract and level of effort in our model is provided here. For a more detailed discussion see Holmström [9] and especially his example on page 79. Let λ be the multiplier for equation (2) and μ the multiplier for equation (3). Pointwise optimization of the Lagrangian with respect to $I(\cdot)$ yields

$$\frac{1}{N} f(v, y, P) = \lambda \{I(v, y, P)\}^{-1/2} f(v, y, P) + \mu \{I(v, y, P)\}^{-1/2} f_a(v, y, P).$$

This expression can be rearranged to produce

$$I(v, y, P) = N^2 \left\{ \lambda + \mu \left[\frac{f_a(v, y, P)}{f(v, y, P)} \right] \right\}^2 = N^2 \left\{ \lambda + \mu \left[\frac{y - (P + \alpha)}{(P + \alpha)^2} \right] \right\}^2. \quad (A1)$$

Substituting this expression for $I(\cdot)$ into equation (2b) yields $\lambda = \frac{1}{2N} (a^2 + \bar{H})$;

substituting this expression for $I(\cdot)$ into equation (2c) yields $\mu = \frac{1}{2N} a(a + \alpha)$

$(1 + a + \alpha)$. Substituting these expressions back into equation (A1) results in

$$I(v, y, P) = \left[\frac{1}{2} (a^2 + \bar{H}) + a(a + \alpha)(1 + a + \alpha) \left\{ \frac{y - (P + \alpha)}{(P + \alpha)^2} \right\} \right]^2. \quad (\text{A2})$$

Making a change of variables from v and y to x and u gives the expression in the text. Finally, the optimal $a \in [0, \infty)$ is found by substituting the expression for $I(\cdot)$ in equation (A2) into the equation

$$\begin{aligned} & \int_{-\beta-\alpha}^{\beta-\alpha} \int_0^\infty \int_a^{1+a} \{B_i + [v + y - I(v, y, P)]D_i\} f_a(v, y, P) dv dy dP \\ & + \mu \left\{ \int_{-\beta-\alpha}^{\beta-\alpha} \int_0^\infty \int_a^{1+a} \{2 \sqrt{I(v, y, P)}\} f_{aa}(v, y, P) dv dy dP - 2 \right\} = 0, \quad (\text{A3}) \end{aligned}$$

where

$$f_{aa}(v, y, P) = \frac{1}{2\beta} \left\{ \frac{y^2 - 4y(P + \alpha) + 2(P + \alpha)^2}{(P + \alpha)^5} \right\} e^{\frac{-y}{P+\alpha}}.$$

Simplifying the expression in equation (A3) yields equation (4): the value $a \in [0, \infty)$ which solves the resulting polynomial expression (i.e., equation (4)) is the agent's optimal level of effort).

B

Let a^* be the level of effort expanded by the manager at the optimum for a particular α and $\bar{H}$: that is, a^* is the a which satisfies

$$1 - \{a(a^2 + \bar{H}) + (a + \alpha)(2a + 3a^2 + 2a\alpha) + a^2(1 + a + \alpha)\} = 0, \quad (\text{B1})$$

and is implicitly a function of α . Then, ignoring the constant term $1/N$, a stockholder's expected return net of incentive costs is

$$E[\tilde{u}] - E[I(\cdot)] = a + \frac{1}{2} - E[I(\cdot)], \quad (\text{B2})$$

where

$$E[I(\cdot)] = \frac{1}{4} (a^2 + \bar{H})^2 + a^2(a + \alpha)(1 + a + \alpha). \quad (\text{B3})$$

Differentiating (B2) with respect to α yields

$$\begin{aligned} \frac{\partial}{\partial \alpha} \left\{ a + \frac{1}{2} - E[I(\cdot)] \right\} &= \frac{\partial a^*}{\partial \alpha} \left\{ \frac{\partial}{\partial a^*} \left(a + \frac{1}{2} - E[I(\cdot)] \right) \right\} \\ &= \frac{\partial a^*}{\partial \alpha} \left\{ 1 - \frac{\partial E[I(\cdot)]}{\partial a^*} \right\} - \frac{\partial E[I(\cdot)]}{\partial \alpha}. \quad (\text{B4}) \end{aligned}$$

Observe that

$$\frac{\partial E[I(\cdot)]}{\partial \alpha} = a^2(a + \alpha) + a^2(1 + a + \alpha) > 0, \quad (\text{B5})$$

and that

$$\frac{\partial E[I(\cdot)]}{\partial \alpha^*} = a(a^2 + \bar{H}) + (a + \alpha)(2a + 3a^2 + 2a\alpha) + a^2(1 + a + \alpha). \quad (\text{B6})$$

However, (B6) combined with (B1) implies that $\frac{\partial E[I(\cdot)]}{\partial \alpha^*} = 1$, and therefore (B4) reduces to $\frac{-\partial E[I(\cdot)]}{\partial \alpha}$, which is clearly negative from (B5).

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