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An Examination of the Relationship between Pure Residual and Market Risk: A Note

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THE PROCESS OF DIVERSIFICATION has received considerable attention in the financial literature [6, 9, 11, 16, 17, 18]. Evans and Archer [6] found that through naive diversification one could effectively eliminate the majority of unsystematic risk in a portfolio after the selection of only ten securities. However, while naive diversification results in a swift reduction in the level of nonsystematic risk, a nonnaive selection of securities from a common industry or like beta class may handicap this risk reduction process [9, 11, 15, 16, 17]. Klemkosky and Martin [11] in their pioneering work on the relationship between market and residual risk used the ordinary least squares (OLS) method of regression to show that both the level of the beta coefficient of the portfolio and the portfolio size affect the process of diversification. They found that a high beta portfolio required a substantially larger number of securities to achieve the same level of diversification as a low beta portfolio.

In the present study we will re-examine these findings in the light of several studies which have questioned the stability of beta, and hence the appropriateness of the OLS method for measuring the relationship between the pure residual risk and the level of the portfolio beta. Studies by Altman, Jacquillat, and Levasseur [1], Baesel [2], Fabozzi and Francis [7], Levitz [12], Levy [13], and Roenfelt, Griepentrog, and Pflaum [22] all point to the fact that the beta coefficient is not stable over time. Fabozzi and Francis [7] for example, found that some individual security betas tend to be random coefficients fluctuating around their mean beta coefficients. It is this instability of beta that makes the use of OLS an inappropriate methodological tool with which to estimate systematic risk. Not only will the OLS method provide an inefficient estimate of systematic risk, but the estimate of unsystematic risk will also be influenced by the variability of beta. The purpose of this study is to allow for nonstationary beta coefficients and re-examine the relationship between market and pure residual risk. Specifically, we shall show that while the OLS method of regression will lead to the conclusion that larger residual risk tends to be associated with higher beta coefficients, if residual risk is estimated in such a way that it is not contaminated by the variability of the beta coefficient this relationship between beta and residual risk will essentially dissolve. That is to say, when nonstationarity of beta is allowed for, the beta effect on portfolio diversification, that is, the degree of diversification achieved for a high versus low beta portfolio of given size, is found to be nonexistent. In order to examine this proposition empirically, the effect of

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variability in the beta coefficient will be isolated from the estimate of residual risk through the use of Vasicek's Bayesian approach [24] for estimating beta as modified by Chen and Lee [5].

The first section of this paper explores the relationship between variability of beta and residual risk and the implications of this relationship for this study. The second section examines this relationship empirically focusing on individual securities while the final section provides the conclusions of the study.

I. Nonstationarity of Beta and Residual Risk

The capital asset pricing model as laid down by Sharpe [23], Lintner [14], and Mossin [20] can be expressed as

$$E(R_i) = RFR + \beta_i(E(Rm) - RFR) \tag{1}$$

where

$E(R_i)$ = the expected return on security i;
RFR = the expected risk free rate of interest;
$E(Rm)$ = the expected return on the market portfolio;
β_i = the measure of normalized systematic risk (Beta) for asset i

This states that the expected return on any security i, $[E(R_i)]$, is determined by the expected risk free rate of interest (RFR), plus a risk premium that is a function of the normalized systematic risk for asset $i(\beta_i)$, and the market risk premium ($E(Rm) - RFR$). This relationship is based upon the assumption that the measure of systematic risk, β_i, is stationary over time. However, if the beta coefficient is unstable over time as indicated by Blume [3], Levy [13], Fabozzi and Francis [7], and others [1, 2, 22], the time-series equation of the capital asset pricing model can be expressed as

$$R_{it} = RFR_t + \beta_{it}(R_{mt} - RFR_t) + \epsilon_{it} \tag{2a}$$

where

$$\beta_{it} = \beta_{io} + U_{it} \text{ [1]} \tag{2b}$$

and the subscript t denotes the time period;

$$\begin{aligned} U_{it} &= \beta_{it} - \beta_{io} \\ \beta_{io} &= E(\beta_{it}) \\ E(\epsilon_{it}) &= 0, \qquad E(\epsilon_{it}^2) = \sigma_{\epsilon i}^2; \\ E(U_{it}) &= 0, \qquad E(U_{it}^2) = \sigma_{ui}^2, \qquad \text{and} \quad E(U_{it}\epsilon_{it}) = 0 \quad \text{for all } t; \\ E(\epsilon_{it}\epsilon_{it'}) &= 0, \qquad \text{and} \quad E(U_{it}U_{it'}) = 0, \qquad t \neq t' \end{aligned}$$

Equations (2a) and (2b) can be rewritten as

$$R_{it} = RFR_t + \beta_{io}(R_{mt} - RFR_t) + \epsilon_{it}^* \tag{3a}$$

[1] A reasonable approach of describing beta nonstationarity is to model the beta coefficient as if it undergoes independent random shifts (over time) around its mean.

where

$$\epsilon_{it}^{*} = (\beta_{it} - \beta_{io})(R_{mt} - RFR_t) + \epsilon_{it} \tag{3b}$$

As such, if the beta coefficient is nonstationary, the effect of its variability will surface in the residual error, ϵ_{it}^{*}, with the total effect equalling the variability compounded by the excess return on the market risk premium,

$$(\beta_{it} - \beta_{io})(R_{mt} - RFR_t) \tag{4}$$

The use of the OLS method to estimate this relationship has several problems associated with it. First, residual risk is heteroskedastic; hence the OLS estimate of the beta coefficient is inefficient. More importantly for this study, the variability in the beta coefficient results in a bias in the residual risk estimate. In effect, if the beta is nonstationary, part of the OLS estimate of residual risk is actually a nonzero contribution resulting from the OLS method's inability to deal with nonstationarity of beta.

In order to identify the effect of nonstationarity of beta, a Bayesian estimator for the time varying beta is determined using the method of Chen and Lee [5] as follows:

$$\hat{\beta}_{it} = \frac{\beta_{io} + \gamma(R_{mt} - RFR_t)(R_{it} - RFR_t)}{1 + \gamma(R_{mt} - RFR_t)^2}, \qquad t = 1, \cdots, T \tag{5}$$

where $\gamma = \sigma_{ui}^2/\sigma_{\epsilon i}^2$ and β_{io} and γ are maximum likelihood estimators. The γ can be obtained by numerically maximizing the likelihood function for γ:[2]

$$\begin{aligned} L(\gamma) = {} & \frac{-T}{2} \ln (2\pi) \\ & - \frac{T}{2} \ln \frac{1}{T} \left\{ \sum_{t=1}^{T} \frac{y_t^2}{1 + \gamma x_t^2} - \left(\sum_{t=1}^{T} \frac{x_t y_t}{1 + \gamma x_t^2} \right)^2 \Big/ \left(\sum_{t=1}^{T} \frac{x_t^2}{1 + \gamma x_t^2} \right) \right\} \\ & - \frac{1}{2} \ln \left[\prod_{t=1}^{T} (1 + \gamma x_t^2) \right] - \frac{T}{2}, \end{aligned} \tag{6}$$

where

$$y_t = R_{it} - RFR_t$$

and

$$x_t = R_{mt} - RFR_t$$

Then, prior β_{io} is estimated by the equation

$$\hat{\beta}_{io} = \left(\sum_{t=1}^{T} \frac{x_t y_t}{1 + \gamma x_t^2} \right) \Big/ \left(\sum_{t=1}^{T} \frac{x_t^2}{1 + \gamma x_t^2} \right) \tag{7}$$

[2] The likelihood function $L(\gamma)$ in (6) is a "smooth" function. The γ can be easily estimated with a desired degree of accuracy (error less than 10^{-5} or 10^{-6}) if single precision is used in the iterative algorithm. For a higher degree of accuracy, double precision can be used for the computation.

The estimate of the pure security residual variance can be obtained by using the equation

$$\hat{\sigma}_{\epsilon i}^2 = \frac{1}{T-1}\sum_{t=1}^{T}\left[\frac{(y_t - \hat{\beta}_{io}x_t)^2}{1+\hat{\gamma}x_t^2}\right] \tag{8}$$

Using Equation (8) it becomes possible to determine an estimate of the true or pure residual risk which is not influenced by variability of beta. This relationship between the pure estimate of residual risk and market risk will now be explored.

II. Individual Securities: The Relationship between Pure Residual Risk and Market Risk

The relationship between pure residual risk and the time-varying beta coefficient or market risk for individual securities was tested over the 110-month period February 1966 to March 1975, as well as two nonoverlapping 55-month subperiods, February 1966 to August 1970 and September 1970 to March 1975. Over these periods, a sample of monthly prices on 360 NYSE stocks adjusted for cash dividends, stock dividends, and stock splits was obtained from the Compustat PDE tapes and used to calculate the logarithmic price relatives.

To determine the relationship between pure residual and market risk, the individual time-varying beta coefficients and pure residual risks were estimated using Equations (5) and (8) with the yield on the monthly treasury bill rate and Standard and Poor's 500 stock price index used to calculate the monthly logarithmic returns for risk free rate and market respectively. First, the relationship between the OLS estimate ($\hat{\beta}_{io}^*$) and the Bayesian estimate ($\hat{\hat{\beta}}_{io}$) of the mean of the time varying beta was examined. As shown in Equation (3), when a beta is not stationary over time, the OLS estimator of beta is an unbiased estimate of the mean (β_{io}) of a nonstationary beta, β_{it}. However, the OLS estimate is not efficient when beta is not stationary over time. Moreover, the OLS estimate ($\hat{\beta}_{io}^*$) of the mean of a time varying beta can be shown to be highly correlated (cross-sectionally) to the Bayesian estimate ($\hat{\hat{\beta}}_{io}$) of the β_{io}. Specifically, if the OLS estimate ($\hat{\beta}_{io}^*$) is regressed cross-sectionally on the Bayesian estimate, $\hat{\hat{\beta}}_{io} = \sum_{t=1}^{T}\hat{\beta}_{it}/T$, of the mean of the time varying beta, the $\hat{\beta}_{io}^*$ is highly and positively correlated with the $\hat{\hat{\beta}}_{io}$. The result is given as follows:

$$\begin{aligned} \hat{\beta}_{io}^* &= .0023 + \underset{(89.25)}{.9553}\hat{\hat{\beta}}_{io}, \qquad i = 1, 2, \cdots, 360 \\ R - \text{square} &= .9571 \end{aligned} \tag{10}$$

Thus, there appears to be little difference between the OLS estimate ($\hat{\beta}_{io}^*$) and the Bayesian estimate ($\hat{\hat{\beta}}_{io}$) of the mean of the time varying beta. Then, linear regression analysis was used to test the cross-sectional relationship between the pure security residual variance and the mean of each time varying security beta with the results given in Table I. The results in Table I demonstrate that, by allowing for nonstationarity of beta through the use of a Bayesian estimate of beta, the significant positive relationship between market and residual risk found both here and in earlier empirical studies [11, 19] when an OLS estimate of beta

Table I

The Cross-Sectional Regression of Pure Residual Variances and Beta Coefficients

Residual Variance $\hat{\sigma}^2_{\epsilon i}$		Intercept + α_0	+	α_1	Beta Coefficient $\hat{\beta}_{io}$	Product Moment Correlation γ_p	Spearman Rank Correlation r_s
February 1966–March 1975							
OLS:	$\hat{\sigma}^{*2}_{\epsilon i}$	.0034	+	.0044 (4.416)**	$\hat{\beta}^*_i$	$\gamma_p =$ .2543 (4.416)**	$r_s =$ .4193 (7.755)**
Bayesian:	$\hat{\sigma}^2_{\epsilon i}$	.0051	+	.0005 (1.167)	$\hat{\bar{\beta}}_{io}$	$\gamma_p =$ .0674 (1.167)	$r_s =$ .0780 (1.378)
	$\hat{\sigma}^2_{\beta m}$	−.0016	+	.0024 (4.213)**	$\hat{\bar{\beta}}_{io}$	$\gamma_p =$ .2371 (4.213)**	$r_s =$ .2951 (5.332)**
February 1966–August 1970							
OLS:	$\hat{\sigma}^{*2}_{\epsilon i}$	.0015	+	.0035 (6.240)**	$\hat{\beta}^*_i$	$\gamma_p =$ .3798 (6.240)**	$r_s =$ .4712 (8.120)**
Bayesian	$\hat{\sigma}^2_{\epsilon i}$	.0050	+	.0003 (0.693)	$\hat{\bar{\beta}}_{io}$	$\gamma_p =$ .0433 (0.693)	$r_s =$ .0801 (1.286)
	$\hat{\sigma}^2_{\beta m}$	.0001	+	.0005 (2.060)*	$\hat{\bar{\beta}}_{io}$	$\gamma_p =$ .1277 (2.060)*	$r_s =$ .1552 (2.514)*
September 1970–March 1975							
OLS:	$\hat{\sigma}^{*2}_{\epsilon i}$	.0027	+	.0061 (5.409)**	$\hat{\beta}^*_i$	$\gamma_p =$ .3661 (5.409)**	$r_s =$ .2656 (3.7876)**
Bayesian:	$\hat{\sigma}^2_{\epsilon i}$	.0052	+	.0002 (0.387)	$\hat{\bar{\beta}}_{io}$	$\gamma_p =$ .0257 (0.387)	$r_s =$ .0069 (0.105)
	$\hat{\sigma}^2_{\beta m}$	.0004	+	.0007 (2.154)*	$\hat{\bar{\beta}}_{io}$	$\gamma_p =$ .1976 (2.154)*	$r_s =$ .2042 (3.136)**

$\hat{\beta}_{io}$ = the mean of the estimates of time-varying beta, $\hat{\beta}_{it}$, of security i.
$\hat{\beta}^*_i$ = the OLS estimate of the beta coefficient.
$\hat{\sigma}^{*2}_{\epsilon i}$ = the OLS estimate of residual variance.
$\hat{\sigma}^2_{\epsilon i}$ = the Bayesian estimate of pure residual variance.
$\hat{\sigma}_{\beta m}$ = the estimated variance of the combined effect of beta nonstationarity and the market return
$= \hat{V}\text{ar}[(\beta_{it} - \beta_{io})(R_{mt} - RFR_t)]$.
* Significant at the 5% level (t-ratios in parentheses).
** Significant at the 1% level.

is used can be explained and eliminated. This relationship was examined further using the Spearman rank correlation coefficient to test the degree of association between rankings of beta coefficients and residual variances in each period. This test provided similar results indicating the absence of a positive relationship between market and pure residual risk when variability of beta was allowed for. In addition, the product moment correlation test further confirms the absence of this relationship. On the other hand, the OLS estimate of the beta coefficient and the OLS estimate of residual variance consistently displayed a positive and significant relationship at one percent level across all sample periods. Thus, by allowing for a time-varying beta, the positive relationship between market and pure residual risk can effectively be eliminated.

These findings were further confirmed by an examination of the relationship between a portfolio's beta and pure residual risk. Here it was found that much of the relationship between market and pure residual risk dissolves when individual betas are allowed to be nonstationary.[3] Moreover, an examination of the degree of diversification attained in high versus low beta portfolios indicated that it is not necessary to diversify among a larger number of securities in a high beta portfolio in order to achieve the same level of diversification in a low beta portfolio.

III. Conclusions

In the light of recent evidence as to the stability of betas, the relationship between market and pure residual risk was re-examined. As the existence of variability in beta introduces an adjustment factor into the OLS estimate of residual risk, a Bayesian model allowing for nonstationarity of beta was used to estimate pure residual risk. Examining the relationship between individual security betas and pure residual risk revealed that, by allowing for nonstationarity of beta, the highly positive and significant correlation found by Klemkosky and Martin [11] largely dissolved. Finally, in examining the process of diversification it was shown that there is little difference in diversification properties of high versus low beta portfolios. Thus, while past studies have indicated that betas and residual risk are highly correlated and that this results in the process of diversification taking longer in high beta portfolios, these findings were largely a product of the nonstationarity of beta resulting in a nonzero contribution to the residual risk component.

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