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The Pricing of Corporate Debt: A Note

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TO DEVELOP A THEORY of the risk structure of interest rates, R. C. Merton [2] defines the risk as "the possible gains or losses to bond holders as a result of (unanticipated) changes in the probability of default." (p. 449) He derives the pricing theory of a pure discount bond corporation debt along the line of Black-Scholes [1]. Comparative statics are used to develop graphs of the risk structure. Although his algebraic derivation is correct, his graphical presentation is inadvertently wrong.

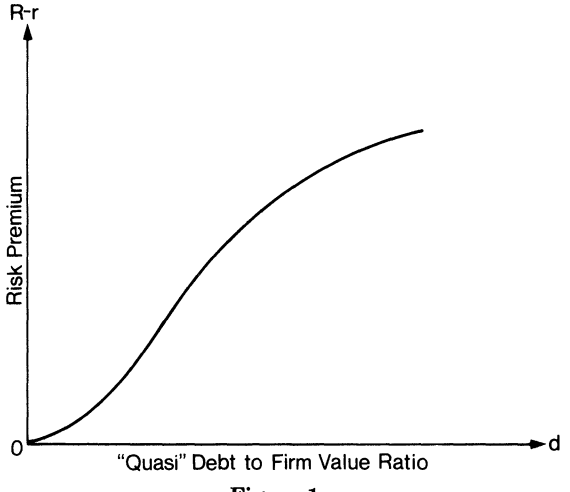

Figure 1.

Merton employes two diagrams to illustrate the effects of leverage and time until maturity on the risk premium of a discount bond. In Figure 1, d denotes the leverage ratio which is measured by the ratio of "quasi" debt to firm value.[1] The only risk considered in Merton's paper is the bankruptcy risk. When the leverage ratio of the firm increases, the bankruptcy risk rises accordingly. In Figure 2, τ denotes the time until maturity. When the maturity is imminent, any firm with a leverage ratio d greater than one is bound to go bankrupt. Any firm with a leverage ratio d smaller than one is solvent. When the maturity lies further ahead in the time horizon, the probability for a currently-insolvent ($d \geq 1$) firm to go bankrupt is declining. For a perpetual discount bond, the risk of bankruptcy is

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[1] The value of the "quasi" debt is equal to the present value of the promised payments on the corporate debt discounted at the riskless interest rate.

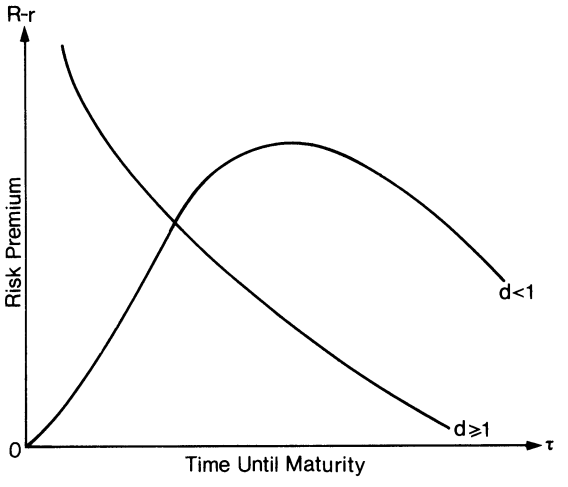

Figure 2.

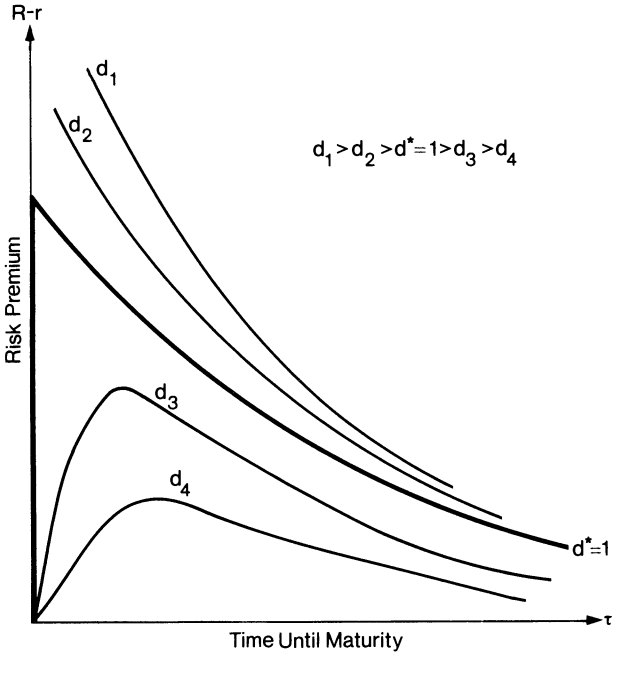

Figure 2[C].

asymptotically zero. For a currently-solvent ($d < 1$) firm, the probability of bankruptcy increases, when the maturity moves away from now. However, when the maturity of a quasi debt approaches perpetuity the risk of bankruptcy goes down gradually. Figure 2 illustrates these effects of maturity on the risk premium.

But Figures 1 and 2 are not consistent with each other. If the risk premium is

a monotonically increasing function of the leverage ratio, for any given maturity a discount bond with a higher leverage ratio pays a higher risk premium. Hence the $d < 1$ curve in Figure 2 should be no higher than the $d \geq 1$ curve. In fact, we have a two-parameter family of term structure of risk premium. The two parameters are leverage ratio d and the variance of the firm, σ^2. Since risk premium is monotonic in these two parameters, the family of term structure curves, in terms of either parameter, never intersect with each other.

Figure 2^c, the corrected version of Figure 2, illustrates the family of term structures in terms of the leverage ratio. The term structure curve with a unitary leverage ratio marks the borderline of two distinctive cases. When the maturity τ approaches zero, those risks premiums with $d > 1$ approach infinity; those with $d < 1$ approach zero. When the maturity τ approaches infinity, the whole family of curves approaches zero. For a leverage ratio close to zero, the term structure curve would asymptotically approach the horizontal axis.

REFERENCES

1. F. Black and M. Scholes. "The Pricing of Options and Corporate Liabilities." *Journal of Political Economy* 81 (May–June 1973), 637–54.
2. R. C. Merton. "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates." *Journal of Finance* (May 1974), 449–70.