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One Way Arbitrage, Foreign Exchange and Securities Markets: A Note

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BRANSON [1, CH. 2], GRUBEL [6], AND KENEN [8] have analyzed the interest parity theorem in the case of traders with exogenous needs to acquire foreign currency. Recently, Deardorff [3] coined the phrase "one way arbitrage" to describe the behavior of those traders and showed that it probably precludes "round trip" covered interest arbitrage from breaking even.

Deardorff's analysis focuses on the conditions under which, in the presence of transaction costs, there would be both spot and forward transactions on the foreign exchange markets at equilibrium. In this paper, we extend the analysis to derive the conditions under which transactions would exist in both the home and foreign securities markets and we generalize the results to the case of transactions in all markets.

I. Extension of the Analysis to the "Securities" Markets

Extending Deardorff's analysis to the "securities" markets is straightforward. In Table I, we complement Deardorff's list of conditions for direct transactions on the foreign exchange markets (Conditions (C1) to (C4)) with a list of the conditions for direct transactions on the markets for securities denominated in the two relevant currencies (Conditions (C5) to (C8)). The symbols F and F_0 represent the actual and the parity forward exchange rates, and the symbols t, t^*, t_F, and t_s represent the transaction costs on the home (dollar) and foreign (sterling) securities markets and on the forward and spot foreign exchange markets, respectively.[1] Transaction costs are assumed to be proportional to the amounts involved.

As was the case for the conditions for direct transactions on the foreign exchange markets, these conditions are not independent of each other. For example, if (C5) is violated, then (C6) must hold, with strict inequality, and vice versa.[2] The pair (C7)-(C8) exhibits the same property. Thus, equilibrium on both

[1] We use the standard approximations: expressions like $(1 + t)(1 + t^*)$ are approximated by $1 + t + t^*$, expressions like $\frac{1}{1+t}$ are approximated by $1 - t$, and expressions like $\frac{F - F_0}{F_0}$ and $\frac{F - F_0}{F}$ are seen as approximately equals.

Given the order of magnitude of the transaction costs and of the deviations of actual forward exchange rate from the interest-parity rate, those approximations are very accurate.

[2] The reasoning is similar to Deardorff [3, p. 359]. If (C5) is violated, then

$$\frac{F - F_0}{F_0} > t_F + t_S + (t - t^*)$$

$$-\frac{F - F_0}{F_0} < -(t_F + t_S) + (t^* - t) \leq (t_F + t_S) + (t^* - t)$$

Table I

Conditions for Direct Market Transactions

*A. Foreign Exchange Markets**

(C1) If demanders of current pounds – suppliers of current dollars use the spot market, then

$$-\frac{F - F_0}{F_0} \leq t + t^* + (t_F - t_S)$$

(C2) If demanders of future pounds – suppliers of future dollars use the forward market, then

$$\frac{F - F_0}{F_0} \leq t + t^* - (t_F - t_S)$$

(C3) If suppliers of current pounds – demanders of current dollars use the spot market, then

$$\frac{F - F_0}{F_0} \leq t + t^* + (t_F - t_S)$$

(C4) If suppliers of future pounds – demanders of future dollars use the forward market, then

$$-\frac{F - F_0}{F_0} \leq t + t^* - (t_F - t_S)$$

B. Securities Markets

(C5) If demanders of current pounds – suppliers of future pounds use the £ – securities market, then

$$\frac{F - F_0}{F_0} \leq t_S + t_F + (t - t^*)$$

(C6) If demanders of current dollar – suppliers of future dollars use the $ – securities market, then

$$-\frac{F - F_0}{F_0} \leq t_S + t_F - (t - t^*)$$

(C7) If suppliers of current pounds – demanders of future pounds use the £ – securities market, then

$$-\frac{F - F_0}{F_0} \leq t_S + t_F + (t - t^*)$$

(C8) If suppliers of current dollars – demanders of future dollars use the $ – securities market, then

$$\frac{F - F_0}{F_0} \leq t_S + t_F - (t - t^*)$$

* Note: Part A of this table is found in Deardorff [3, p. 359].

the securities markets requires that (C5) and (C8) are not both violated: indeed, if (C5) did not hold, (C6) would hold with strict inequality. It would follow that borrowers would always plan to borrow by issuing dollar-denominated securities, whatever the ultimate currency—dollar or pound—they ultimately want to obtain. If, simultaneously, (C8) were violated—implying (C7) with strict inequality—all lenders of funds would want to lend exclusively on the market for pound-denominated securities, possibly after converting their dollars into pounds on the spot market and hedging on the forward market. Violation of both (C5) and (C8) would thus clearly imply a disequilibrium on the securities markets. Similarly, simultaneous violation of (C6) and (C7) would also imply a disequilibrium, with all the demands for funds expressing themselves on the pound-denominated securities market and all the supplies of funds appearing on the dollar denominated securities market. Thus simultaneous equilibrium on both securities markets requires as a necessary condition that either (C5) or (C8) and either (C6) or (C7) hold simultaneously.[3] This result leads us to formulate Proposition 1:

PROPOSITION 1: *Equilibrium in both markets for securities is possible only if*

$$\left| \frac{F - F_0}{F_0} \right| \leq t_S + t_F + |t - t^*| \tag{1}$$

which can also be written:

$$\left| \frac{F - F_0}{F_0} \right| \leq (t_S + t_F + t + t^*) - 2\,\mathrm{MIN}(t, t^*) \tag{1'}$$

Proposition (1) implies that, in general equilibrium, covered interest arbitrage would never break even if there are transaction costs on both securities markets.

Inequality (1) may be satisfied, and equilibrium on both securities markets may prevail, even if no transactions are made in one of those markets as a result of the availability of one way arbitrage. If we want to rule out such no-trade equilibria, Conditions (C5) to (C8) must all be satisfied, leading us to the following proposition:

PROPOSITION 2: *Market equilibrium with transactions in both the domestic and the foreign securities markets is possible only if*

$$\left| \frac{F - F_0}{F_0} \right| \leq t_S + t_F - |t - t^*| \tag{2}$$

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which implies (C6) with strict inequality.

The appendix presents a table showing all the logical relationships of this nature existing between Conditions (C1) to (C8).

[3] As is clear from the appendix, more restrictions can be put as a necessary condition for equilibrium to prevail simultaneously on both markets. As simultaneous equilibrium on both markets requires all the necessary equilibrium conditions for each market be met simultaneously, simultaneous equilibrium on any two markets requires that at least one of the two conditions in each of the *10* different pairs of conditions listed in any of the two relevant columns be met. Our analysis in this section and Deardorff's are thus not efficient in the sense that we do not use all the information available. As shown in the appendix, those analyses focus in fact on the conditions under which a simultaneous disequilibrium on the relevant pair of markets can be ruled out (see Table II). In the next section however, we establish the most restrictive conditions for simultaneous equilibrium on all markets which can be derived from the available information.

which can also be written:

$$\left|\frac{F - F_0}{F_0}\right| \leqslant (t_S + t_F + t + t^*) - 2\,\mathrm{MAX}\,(t, t^*) \tag{2'}$$

Inequality (2) is of course more restrictive than Inequality (1). It indicates that one of the two markets for securities will never be used if the transaction costs on one of those markets accounts for more than 50% of the sum of the transaction costs on all four markets, and that, should both markets be used at equilibrium, the absolute value of the covered interest-arbitrage margin will be at most equal to the sum of the transaction costs on the foreign exchange markets minus the difference, in absolute value, between the transaction costs on the two securities markets.

II. Generalization and General Equilibrium

Focusing on conditions for simultaneous equilibrium on the two forward markets, as in [3], or on the two securities markets, as in the previous section, is of course arbitrary. As there are four markets involved in this analysis, it is possible to form six different pairs of markets and to present the conditions of simultaneous equilibrium for any pair. For example, following reasoning analogous to that presented above, it can be shown that a simultaneous violation of both (C1) and (C6) or of both (C3) and (C8) would be inconsistent with simultaneous equilibrium on both the spot foreign exchange market and the market for dollar denominated securities.[4] Simultaneous equilibrium on both those markets would require the following inequality:

$$\left|\frac{F_0 - F}{F_0}\right| \leqslant t_F + t^* + |t - t_S| \tag{3}$$

which can be written also as:

$$\left|\frac{F_0 - F}{F_0}\right| \leqslant t + t^* + t_S + t_F - 2\,\mathrm{MIN}(t, t_S) \tag{3'}$$

Simultaneous equilibrium with actual transactions on each of those two markets would occur only if all four conditions C1, C3, C6, and C8 are met simultaneously, i.e. if the following inequality holds:

$$\left|\frac{F_0 - F}{F_0}\right| \leqslant t_F + t^* - |t - t_S| \tag{4}$$

which can be written also:

$$\left|\frac{F_0 - F}{F_0}\right| \leqslant t + t^* + t_S + t_F - 2\,\mathrm{MAX}(t, t_S) \tag{4'}$$

When the analysis along those lines is done for each of the 6 pairs of markets, we obtain six equilibrium conditions of the same form. Representing each of the

[4] The analysis would be similar for each of the six pairs of markets which can be formed. To derive more results by himself, the reader can refer to the synthetic presentation of the necessary information in the Appendix.

relevant transaction costs by t_i, those six conditions can be written:

$$\left|\frac{F_0 - F}{F_0}\right| \leqslant \sum_{i=1}^{4} t_i - 2\, MIN\, (t_a, t_b) \tag{5}$$

where (t_a, t_b) represents for each condition one of the six different pairs of transaction costs which can be formed.

Similarly, the six conditions for equilibrium with transactions occurring on each of the two relevant markets of each pair are

$$\left|\frac{F_0 - F}{F_0}\right| \leqslant \sum_{i=1}^{4} t_i - 2\, \mathrm{MAX}\, (t_a, t_b) \tag{6}$$

General equilibrium on all four markets requires of course that all six conditions of simultaneous equilibrium for each pair of markets (Inequalities (5)) be met simultaneously. However, because of the direction of those inequalities and because the left-hand side is identical for all six expressions, it follows that if the inequality with the smallest term on the right-hand side holds, the five other inequalities will hold as well. The strongest result that can be derived, therefore, is the following proposition:

PROPOSITION 3. *General equilibrium on all four markets is possible only if*

$$\left|\frac{F_0 - F}{F_0}\right| \leqslant \sum_{i=1}^{4} t_i - 2\, \mathrm{MAX}[\overline{\mathrm{MIN}}\, (t_a, t_b)] \tag{7}$$

where $\overline{\mathrm{MIN}}$ (t_a, t_b) *represents the set of the smallest terms in each of the six possible pairs of transaction costs.*

This set of smallest terms of each pair includes all the transaction costs but the highest one. The larger element of the set is thus the second highest transaction cost. Therefore, we can write Condition (7) simply as

$$\left|\frac{F_0 - F}{F_0}\right| \leqslant \sum_{i=1}^{4} t_i - 2\, t_3 \tag{7'}$$

where the subscript indicates the rank of each transaction cost in an ordering according to size from the smallest (t_1) to the highest (t_4).

Now consider the six conditions for equilibrium with trade occurring in all markets (Inequalities (6)). Again, when the inequality with the smallest term at the right-hand side holds, all the other ones hold as well. The strongest condition can thus be stated as follows:

PROPOSITION 4. *General equilibrium with transactions occurring in all four markets is possible only if:*

$$\left|\frac{F_0 - F}{F_0}\right| \leqslant \sum_{i=1}^{4} t_i - 2\, \mathrm{MAX}[\overline{\mathrm{MAX}}\, (t_a, t_b)] \tag{8}$$

where $\overline{\mathrm{MAX}}$ (t_a, t_b) *represents the set of the largest terms in each of the six possible pairs of transaction costs.*

This set of largest terms of each pair includes all the transaction costs but the smallest one. The highest element of the set is thus simply the largest transaction cost and Inequality (8) can be rewritten

$$\left|\frac{F_0 - F}{F_0}\right| \leqslant \sum_{i=1}^{4} t_i - 2t_4 \tag{8'}$$

Condition (8′) is in fact identical to the condition for existence of direct market transactions on the market characterized by the highest transaction costs: should Inequality (8′) be violated, one way arbitrage would prevent any direct transaction on this market, as the roundabout method via the other three markets would enable both suppliers and demanders to attain this goal at a lower cost. Similarly, Condition (7′) is identical to the condition of utilization of direct market transactions on the market with second highest transaction costs: indeed, should one-way arbitrage preclude utilization of the most expensive market, equilibrium requires that at least all other three markets—including the next most expensive one—be used for direct transactions.

An important implication of Condition (7′) is that a sufficient condition to preclude covered interest arbitrage from breaking even at equilibrium is the existence of transaction costs on *any two* of the four relevant markets.

III. Estimates of the Threshold Value of Covered Interest Differential Needed to Induce Covered Interest Arbitrage

Branson [2] interpreted deviations from covered interest parity during nonspeculative periods as evidence of transaction costs and proposed to estimate the threshold value of the covered interest rate differential needed to offset the transaction costs and to induce covered interest arbitrage by computing the mean of the absolute values of the difference between the interest rate differential and the forward premium or discount. Using data covering the period 1959–64 for arbitrage between UK and US Treasury bills, he concluded that transaction costs involved by covered interest arbitrage amount to 0.18% annually.

The analysis initiated by Deardorff [3] suggests that deviations from covered interest parity systematically underestimate the transaction costs involved in covered interest arbitrage. Furthermore, our inequality (8′) allows us to assess quantitatively the order of magnitude of that underestimation. If, as it is reasonable to assume in the case of the pound and the dollar, some transactions occur regularly on all four markets, the threshold is at least the double of the observed discrepancies: indeed, (8′) indicates that the observed discrepancies will be no greater than the difference between the sum of the transaction costs on all four markets and twice the highest transaction cost; as this highest transaction cost accounts for at least one fourth of the sum, the difference amounts to at most one half of the sum.

Branson's evidence should thus be reinterpreted as showing that the minimum covered arbitrage margin necessary to generate the activity of the interest arbitragers would be at least .36%. This figure can be compared to the subjective estimate of .25% proposed, for the same period, by Holmes and Schott [7] and to the estimates of .58% implicit in Frenkel and Levich [5].[5]

[5] Holmes and Schott [7, p. 55] and Frenkel and Levich [5, Tables 1 and 2]. Frenkel and Levich's lowest estimates for the period 1962–67 are .145 per set of transactions and must be multiplied by four to measure the transactions costs *per year* of covered interest arbitrage on 90 days investments. Note that the failure to standardize the estimates on a per year basis makes invalid the comparison presented in [4, p. 331]. For a critique of [4, 5], see [9].

IV. Concluding Remarks and Qualifications

The central conclusion of this paper is identical to Deardorff's. In the analytical framework developed here, covered interest arbitrage is irrelevant, at equilibrium, as one way arbitrage will preclude the deviations from covered interest parity being large enough to even match the transaction costs involved in covered interest arbitrage. Our analysis has generalized Deardorff's results by taking into account the equilibrium conditions on all four markets, adding the securities markets to the two foreign exchange markets discussed by Deardorff, and has established additional restrictions on the size of the covered interest arbitrage margin consistent with equilibrium.

The analysis has of course abstracted from several considerations of portfolio management which have been suggested in the literature as possible explanations for departure from interest parity. After having listed those additional considerations, Deardorff concludes that, although they "weaken our quantitative conclusion as to the range of deviation from interest parity that is consistent with equilibrium, they do not interfere with our result that covered interest arbitrage is unlikely to occur": indeed, "if accumulation of assets and liabilities makes further one-way arbitrage less attractive, that same accumulation also decreases the attractiveness of covered interest arbitrage" [3, p. 363].

The argument developed so far may suggest that the concept of covered interest arbitrage has no empirical relevance. However, our results depend crucially on the assumption that transaction costs are proportional to the amounts of the transactions and that "fixed" transaction costs are negligible. Such an assumption is valid in the case of covered interest arbitrage as this activity, if profitable, would involve unlimited amounts. On the other hand, if covered interest arbitrage is not profitable, one way arbitrage typically involves a large number of specific transactions, undertaken for various purposes. One-way arbitrage ensures that those transactions will be done at the least cost but is mostly irrelevant to the decision to engage in those transactions in the first place. For those transactions, each of which involves, by nature, a limited amount, fixed transaction costs—for example, the very cost of inquiring about the transaction costs on the various markets—may deter one way arbitrage if the deviations from covered interest parity are small. Should this be the case, covered interest arbitrage may well be, after all, the main factor responsible for maintaining the fluctuations around covered interest parity within the narrow band determined by the proportional transaction costs. The relevance of this qualification of our theoretical results is an empirical question.

APPENDIX

This appendix is composed of two tables. The upper half of Table II shows the logical implications for each market of the violation of the conditions (C1) to (C8). Thus, for example, violation of (C1)—the condition at which economic actors who want to exchange dollars for pounds will use the spot foreign exchange market rather than the roundabout method through the other three markets—implies that there will be no demand for pounds nor supply of dollars on the spot

Table II

A. Effects of Violation of the Conditions for Direct Market Transactions*

	Spot Foreign Exchange Market	Forward Foreign Exchange Market	$ Securities Market	£ Securities Market	Implications: the following conditions hold
Violation of (C1)	No Demand for £ No Supply of $	Demand for £ Supply of $	Lending $	Borrowing £	(C2), (C8), (C5)
Violation of (C2)	Supply of $ Demand of £	No Demand for £ No Supply of $	Borrowing $	Lending £	(C1), (C6), (C7)
Violation of (C3)	No Supply of £ No Demand of $	Demand for $ Supply of £	Borrowing $	Lending £	(C4), (C6), (C7)
Violation of (C4)	Supply of £ Demand for $	No Supply of £ No Demand of $	Lending $	Borrowing £	(C3), (C8), (C5)
Violation of (C5)	Supply of $ Demand of £	Demand of $ Supply of £	Borrowing $	No £ Borrowing	(C1), (C4), (C6)
Violation of (C6)	Supply of £ Demand of $	Demand of £ Supply of $	No $ Borrowing	Borrowing £	(C3), (C2), (C5)
Violation of (C7)	Supply of £ Demand of $	Demand of £ Supply of $	Lending $	No £ Lending	(C3), (C2), (C8)
Violation of (C8)	Supply of $ Demand of £	Demand of $ Supply of £	No $ Lending	Lending £	(C1), (C4), (C7)

B. Equilibrium Conditions

Equilibrium on each market requires that the two conditions in each pair listed below are not both violated simultaneously

Spot Foreign Exchange Market	Forward Foreign Exchange Market	$ Securities Market	£ Securities Market
(C1) – (C4)	(C2) – (C3)	(C6) – (C1)	(C5) – (C2)
(C1) – (C6)	(C2) – (C5)	(C6) – (C4)	(C5) – (C3)
(C1) – (C7)	(C2) – (C8)	(C6) – (C7)	(C5) – (C8)
(C3) – (C2)	(C4) – (C1)	(C8) – (C2)	(C7) – (C1)
(C3) – (C5)	(C4) – (C6)	(C8) – (C3)	(C7) – (C4)
(C3) – (C8)	(C4) – (C7)	(C8) – (C5)	(C7) – (C6)

* Note: Conditions (C1) to (C8) are defined in Table I.

market; rather, there will be plans to borrow pounds, lend dollars, and exchange dollars for pounds on the forward foreign exchange market. As a result, we know that violation of (C1) implies that (C2) must hold—for (C2) is the necessary condition for direct use of the forward market by those actors wanting to exchange future dollars for future pounds for whatever reason—as well as (C8) and (C5)—the necessary conditions for direct use of the securities markets by lenders of dollars and by borrowers of pounds. The last column of the table summarizes those implications.

The bottom half of Table II uses the information provided in the upper half to derive necessary conditions of equilibrium on each market. Of each pair of conditions listed in the column assigned to a specific market, at least one *must* hold for that market to be possibly in equilibrium. Thus, for example, either (C1) or (C4) must hold for the spot market to be in equilibrium or, alternatively, if both (C1) and (C4) are violated, then the spot market cannot be in equilibrium.

The list is easily constructed form the information provided in each column. Consider, for example, the spot market. Table II indicates that violation of (C1) implies that there will be no demand for pounds and no supply of dollars on the spot market. Therefore, if equilibrium is to prevail on that market despite the violation of (C1), all the conditions whose violation would imply a supply of pounds or a demand of dollars on the spot market—(C4), (C6), and (C7)—*must* hold. If they didn't hold, there would be an excess supply of pounds and an excess demand of dollars on the spot market. Similarly, violation of both (C3) and (C2), (C5), or (C8) would imply an excess demand of pounds and an excess supply of dollars.

Any disequilibrium on one of those four markets implies a disequilibrium on at least another one. From the equilibrium conditions listed in the bottom half of Table II, it is possible to construct Table III which shows sufficient conditions for

Table III

Simultaneous Disequilibrium on Market Pairs

Simultaneous violations of the conditions in any pair shown on this table implies a disequilibrium on both the relevant markets

	Spot Foreign Exchange Market	Forward Foreign Exchange Market	\$ Securities Market	£ Securities Market
Spot Foreign Exchange Market		(C1) – (C4) or (C2) – (C3)	(C1) – (C6) or (C3) – (C8)	(C1) – (C7) or (C3) – (C5)
Forward Foreign Exchange Market			(C2) – (C8) or (C4) – (C6)	(C2) – (C5) or (C4) – (C7)
\$ Securities Market				(C6) – (C7) or (C5) – (C8)
£ Securities Market				

simultaneous disequilibrium in any two pairs of markets. Thus, for example, Table III shows that simultaneous disequilibrium in both foreign exchange markets will occur if either (C1) and (C4) or (C2) and (C3) are violated simultaneously.

General equilibrium requires of course that none of the 12 pairs of conditions listed in Tables II or III be violated simultaneously.

REFERENCES

1. W. H. Branson. *Financial Capital Flows in the U.S. Balance of Payments.* Amsterdam: North-Holland, 1968.
2. ———. "The Minimum Covered Interest Differential Needed for International Arbitrage Activity." *Journal of Political Economy* (December 1969).
3. A. V. Deardorff. "One-Way Arbitrage and its Implications for the Foreign Exchange Markets." *Journal of Political Economy* (April 1979).
4. J. A. Frenkel and R. M. Levich. "Covered Interest Arbitrage: Unexploited Profit?" *Journal of Political Economy* (April 1975).
5. ———. and R. M. Levich. "Transaction Costs and Interest Arbitrage: Tranquil versus Turbulent Periods." *Journal of Political Economy* (December 1977).
6. H. G. Grubel. *Forward Exchange, Speculation and the International Flow of Capital.* Stanford: Stanford University Press, 1966.
7. A. Holmes and F. Schott. *The New York Foreign Exchange Market.* New York: Federal Reserve Bank of New York, 1965.
8. P. B. Kenen. "Trade, Speculation, and the Forward Exchange Rate." In *Trade, Growth, and the Balance of Payments*, Essays in Honor of Gottfried Haberler. Chicago: Rand-McNally, 1965.
9. F. McCormick. "Covered Interest Arbitrage: Unexploited Profits? Comment." *Journal of Political Economy* (April 1979).