Bank Reserves and Financial Stability
Author(s): Jeremy J. Siegel
Source: *The Journal of Finance*, Vol. 36, No. 5 (Dec., 1981), pp. 1073-1084
Published by: Blackwell Publishing for the American Finance Association
Stable URL: http://www.jstor.org/stable/2327301
Accessed: 27/11/2009 08:38

THE JOURNAL OF FINANCE • VOL. XXXVI, NO. 5 • DECEMBER 1981

Bank Reserves and Financial Stability

JEREMY J. SIEGEL

ABSTRACT

A stochastic financial model is developed which derives the reserve levels on financial assets which minimize price level fluctuations. It is shown that these levels of reserves are a function of the structure of unanticipated shocks to asset demands and are, in general, quite different from the levels which minimize the fluctuations of either the nominal or real value of these assets. Application of the model to currency and demand deposits in the U.S.A. suggest that the price-stabilizing reserve ratio on demand deposits is approximately one-half of the 12% currently mandated by the Monetary Control Act of 1980.

REQUIRED RESERVES ON DEPOSITS at financial institutions have two major economic aspects. First, reserves may be regarded as a tax on bank liabilities, if the rate of return on reserves, measured either explicitly or implicitly, is less than the market rate of interest. Secondly, reserves may be considered an important monetary instrument for controlling the value of monetary aggregates and ultimately such variables as prices, interest rates, and income.[1] Unfortunately, there appears to be little agreement among economists either as to the optimal level of reserves or to the criteria for setting such reserves in order to achieve macroeconomic stability. Friedman [3] had formerly advocated 100 percent reserves on both time and demand deposits, but recently [4] has claimed that uniformity of reserves on transactions balances is far more important than the level at which they are imposed.[2] At the other end of the spectrum, Carson [1] has advocated abolishing required reserves on all deposits while Poole and Lieberman [11] have urged elimination of reserves on time deposits only if M_1 is accepted as the money stock definition.

This paper develops a macroeconomic model where it is demonstrated that the levels of reserves which minimize the fluctuations of such ultimate economic variables as prices are, in general, quite different from those which stabilize various monetary aggregates. These reserve levels which minimize price level fluctuations are determined primarily from the covariance structure of unanticipated disturbances influencing financial assets. Empirical estimation of asset disturbances for the United States suggests that these stabilizing levels of reserves on demand deposit are lower than those scheduled to prevail under the Monetary Control Act of 1980.

I would like to thank William Milne for providing computational assistance for this paper. Marcel Genet's help provided the mathematical structure for solving the N-asset case.

[1] Setting reserves to stabilize monetary aggregates has been the primary concern of most recent research, for example, see Kaminow [8] and Laufenberg [9]. Frequently it is implicitly assumed that reserve levels which minimize the disturbances to monetary aggregates will also reduce fluctuations in prices and income.

[2] This also appears to be the opinion of Alan Metzler [10] and Deane Carson [1, 2].

I. Structure of the Model

A. Asset Equilibrium

The economy is characterized by a simple one good production process with fixed inputs of labor and capital. There are four financial assets: high-powered money (H), currency (C), deposits (D), and equity (or bonds) (E).

High-powered money is issued solely by the central bank and is held as currency by the public and reserves by the banking system. Deposits, which along with currency will be frequently termed "monetary assets," are issued by the banking system and held by households. Bonds and equity, assumed to be perfect substitutes, are issued by firms against their capital and are held by both banks and households. The households' (h) real demand for currency, deposits, and equity are

$$\begin{aligned} C_h^d &= C^d(r_d, r_e, K, H/P) \\ D_h^d &= D^d(r_d, r_e, K, H/P) \\ E_h^d &= E^d(r_d, r_e, K, H/P) \end{aligned} \tag{1}$$

where r_d and r_e are the respective yields on deposits and equity, K is the fixed capital stock, and P is the price of output in terms of nominal high-powered money H, whose yield is assumed fixed at zero. It is assumed that the assets are all gross substitutes and nonnegatively dependent on capital and real high-powered money.

Banks are internally financed institutions which hold equity and high-powered money as assets against their deposit and currency liabilities. One way to simplify the ultimate demand for financial assets is to assume that the rate of return on deposits are set by government regulation at or below the rates which would exist under competition. In this case the quantities of deposits supplied will be determined by household demand at the regulated rates.[3] Hence the banks' (b) real demands for high-powered money (reserves) and equity is given by

$$\begin{aligned} H_b^d &= k_d D_h^d \\ E_b^d &= (1 - k_d) D_h^d. \end{aligned} \tag{2}$$

Equilibrium in the economy is determined by summing the households' and banks' real aggregate demand for high-powered money and equities and equating these with their respective real supply, i.e.,

$$C_h^d + k_d D_h^d = H/P \tag{3}$$

$$E_h^d + (1 - k_d) D_h^d = \bar{K} \tag{4}$$

$$W = C_h^d + D_h^d + E_h^d = \bar{K} + H/P. \tag{5}$$

Since the households' demands are constrained by total wealth, W, the above system (3) and (4) contains only one independent equation for the two endogenous

[3] This would also be true if the banking industry were competitive and subject to constant returns to scale. The competitive rate of return on deposits would then be a function of the equity rate and the reserve level. The deposit rate may be negative in equilibrium.

variables P and r_e. For the purpose of this analysis it is assumed that r_e is determined in the real sector at the marginal product of capital. Therefore the price level is the endogenous variable which clears the market simultaneously for both high-powered money and equity. Identical qualitative results could be obtained by assuming the price level is determined exogenously and the equity rate clears the financial markets.

Solving (3) for the price level yields the following implicit solution for the price level.

$$P = \frac{H^s}{C_h^d + k_d D_h^d}. \tag{6}$$

In equilibrium the price level is simply the nominal supply of high-powered money divided by the real demand for H. (6) is an explicit solution for the price level only in the case where real high-powered money does not affect currency and deposit demands. For analytical convenience in the subsequent analysis, the "real balance" effect in monetary assets demands will be assumed to be nil so that reserve changes, altering real balances, will not influence monetary demands.

B. Stochastic Form of the Model

In order to examine the stochastic behavior of the equilibrium, it is assumed that the demands for currency and demand deposits are subject to *proportional* real disturbances $\tilde{\epsilon}_C$ and $\tilde{\epsilon}_D$, which possess zero mean and a given variance structure designated by σ_C^2, σ_D^2, and σ_{CD}. It is assumed that these shocks are intertemporally uncorrelated so that there are no inflationary expectations in the economy. The nominal supply of high-powered money is assumed deterministic. Because of the households' wealth constraint, the following relationship among shocks must hold

$$C_h^d \tilde{\epsilon}_C + D_h^d \tilde{\epsilon}_D + E_h^d \tilde{\epsilon}_E = 0, \tag{7}$$

where $\tilde{\epsilon}_E$ is the proportional shock to equity demand. Taking the log of (6) and letting $p = \log(P)$ yields

$$\tilde{p} = \log H - \log(C(1 + \tilde{\epsilon}_C) + k_d D(1 + \tilde{\epsilon}_D)), \tag{8}$$

where C and D represent the demand for currency and deposits when $\tilde{\epsilon}_C = \tilde{\epsilon}_D = 0$. Linearizing (8) around the equilibrium $E(\tilde{\epsilon}_c) = E(\tilde{\epsilon}_D) = 0$ yields the following expression for disturbances to the price level, $\tilde{p}$,

$$-\tilde{p} = \frac{C\tilde{\epsilon}_C}{C + k_d D} + \frac{k_d D \tilde{\epsilon}_D}{C + k_d D} \tag{9}$$

where $\tilde{p}$ indicates deviations around the equilibrium. Note that the first term is the fraction of shock to real high-powered money demand due to shifts in currency demand and the second term is the fraction due to shifts in deposit demand. The variance of (9) is given by the following expression,

$$\sigma_p^2 = \frac{C^2\sigma_C^2 + k_d^2 D^2 \sigma_D^2 + 2k_d CD\sigma_{CD}}{(C + k_d D)^2}. \tag{10}$$

C. The Reserve Ratio which Minimizes Price Fluctuations

To evaluate the reserve level on deposits which minimizes the variance of prices, the derivative of (10) which respect to k_d is set equal to zero[4] which yields,

$$k_d^* = \frac{C[\sigma_C^2 - \sigma_{CD}]}{D[\sigma_D^2 - \sigma_{CD}]}. \tag{11}$$

In words, (11) states that the ratio of reserves behind deposits to currency is inversely related to the ratio of the variance of deposits to currency, each corrected by subtracting the covariance between the shocks to the assets. The following observations are made directly from this expression:

(1) The optimal reserve requirement on deposits is positively related to the size and proportional variability of currency and negatively related to the size and variability of deposits, holding the covariance constant.[5]

(2) An increase in the covariance between shocks to currency and deposits will decrease the optimal reserve ratio if and only if $\sigma_C^2 < \sigma_D^2$.

(3) The optimal reserve requirement will be zero as advocated by Carson [2] and others if and only if the covariance between shocks to currency and deposits is equal to the variance of currency demand. A special case of this occurs when currency demand is totally predictable, subject to no unanticipated shocks.

(4) The optimal reserve ratio on demand deposits is unity, as formerly advocated by Friedman, if shocks to the demand for currency are perfectly negatively correlated to those of demand deposits, so that $D\tilde{\epsilon}_D = -C\tilde{\epsilon}_C$. This is the case if total money demand defined as currency plus deposits is stable and there are no shocks to the equity market.[6]

By substituting (11) into (10), the minimized variance of the price level, σ_p^{2*}, is

$$\sigma_p^{2*} = \frac{\sigma_C^2\sigma_D^2(1-\rho^2)}{\sigma_C^2 - 2\rho\sigma_C\sigma_D + \sigma_D^2}, \tag{12}$$

where ρ is the correlation coefficient between currency and deposit disturbances. The minimized variability of prices is independent of the quantity of currency and deposit demanded. Note that as ρ approaches ± 1, the minimized variability of prices approaches zero.

[4] It is assumed that the shock structure is independent of the reserve ratio. However, it can be shown that if k_d affects currency and deposit demands without influencing disturbances, Condition (11) holds.

[5] If the variability of currency goes to zero, then $k_d = 0$, since one wants all high-powered money to be currency and thus insulated from disturbances. As the variability of deposits goes to zero, k_d approaches infinity since one wants the proportion of high-powered money in deposits to be as high as possible.

[6] It is possible for the optimal reserve requirement, k_d, to be negative or greater than one. A negative reserve ratio would exist if banks are allowed to issue currency as a given fraction of deposits, holding no reserves, and this bank currency is viewed by the public as a perfect substitute for high-powered money. If the reserve ratio fell below $-C/D$, then the model is unstable since by (8) the price level would respond positively to an increase in the demand for high-powered money. A reserve ratio greater than unity can prevail if banks issue equity to obtain a quantity of reserves greater than the size of their deposits.

D. Behavior of Monetary Aggregates

It is important to relate the disturbances affecting asset demands to the disturbances affecting the money supply defined as currency plus deposits. Since the real quantities of currency and deposits demanded by the public are always supplied by the banking system, one can write the expression for the log of the real money supply, m, as

$$\tilde{m} = \log[C(1 + \tilde{\epsilon}_C) + D(1 + \tilde{\epsilon}_D)]. \tag{13}$$

Linearizing around the equilibrium yields the following expressions for the disturbances and variance of the real money supply,

$$\tilde{m} = \frac{C\tilde{\epsilon}_C + D\tilde{\epsilon}_D}{C + D}, \tag{14}$$

$$\sigma_m^2 = \frac{C^2\sigma_C^2 + 2CD\sigma_{CD} + D^2\sigma_D^2}{(C + D)^2}. \tag{15}$$

Note that the variance of the real money supply is independent of the reserve ratio on deposits.

The behavior of the nominal money supply is more important since information on such nominal quantities as currency and deposits can be gathered before knowledge of the general price level, used to calculate the real money supply, is known. For this reason, the nominal money supply is frequently used as the intermediate target of the Central Bank for controlling such targets as prices and income. However, as is easily demonstrated below, the reserve level which stabilizes the nominal money supply does not, in general, stabilize the price level in the economy.

The log of the nominal money supply, denoted M, is simply the sum of the log of the price level and log of real money balances. Summing (9) and (14) yields the expressions for disturbances to the log of nominal money,

$$\tilde{M} = \tilde{p} + \tilde{m} = a(\tilde{\epsilon}_D - \tilde{\epsilon}_C), \tag{16}$$

$$\sigma_M^2 = a^2(\sigma_D^2 - 2\sigma_{CD} + \sigma_C^2), \quad \text{where} \tag{17}$$

$$a = \left[\frac{CD}{C + D}\right]\left[\frac{1 - k_d}{C + k_d D}\right].$$

The reserve level which minimizes disturbances to nominal money is not, in general, the level which minimizes disturbances to the price level, since $\tilde{p}$ and $\tilde{m}$ are correlated. The variability of nominal money is a monotonically decreasing function of the reserve level and is clearly minimized at zero when k_d equals unity. When this occurs, nominal money is perfectly stable, since the nominal money supply is identical to the quantity of high-powered money. However, as shown in Section C above, only in some very special cases such as when disturbances to currency are perfectly negatively correlated to deposits will such 100% reserves also minimize price level fluctuations. In fact, as will be shown in Section II below, there is a wide range of reserve ratios where price level variability is increasing although nominal money variability is decreasing.

This important result demonstrates that the stabilization of monetary aggregates should not necessarily be the primary criterion for the monetary authority. A low reserve requirement, while increasing the instability of nominal money, actually may shield the demand for high-powered money from deposits shocks and hence stabilize the price level. Since the monetary aggregates are strictly intermediate targets for the monetary authorities, little or no weight should be placed on their own variability *per se* as long as a deterministic rule is followed for the supply of nominal high-powered money.

E. Extension to N Assets

It is possible to extend the model developed in the previous section to the case of N monetary assets. Let $A_1, \cdots\cdots, A_n$ be the real demands for monetary assets, $1, \cdots\cdots, n; k_1, \cdots\cdots\cdots, k_n$ their respective reserve ratios in terms of high-powered money; and $[\sigma_{ij}]$ the covariance matrix of contemporaneous proportional shocks, $\tilde{\epsilon}_i$, among the monetary assets. Analogous to (9), disturbance to the price level can be expressed.

$$-\tilde{p} = \frac{\sum_{i=1}^{n} k_i A_i \tilde{\epsilon}_i}{\sum k_i A_i} \tag{18}$$

and the variance of the price disturbances as

$$\sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{A_i A_j k_i k_j \sigma_{ij}}{(\sum A_i k_i)^2}. \tag{19}$$

It is shown in the appendix that the set of k_i which minimize (19) can be expressed in the compact form,

$$k_i^* = \frac{c \sum_{j=1} \sigma_{ij}^{-1}}{A_i}, \qquad i = 1, \cdots\cdots n, \tag{20}$$

where σ_{ij}^{-1} is the ijth coefficient of the *inverse* of $[\sigma_{ij}]$, and c is an arbitrary normalization constant. The minimized variance of prices at the optimal set of k_i^* above is simply

$$\sigma_p^2(k_i^*) = \sum_{i,j} \sigma_{ij}^{-1}, \tag{21}$$

the sum of all the elements of the inverse of the variance-covariance matrix.

Since (19) is homogeneous of degree zero k_i, one can set the reserve ratio on currency at unity so that (20) can be written

$$k_i = \frac{a_1}{\sum_j \sigma_{ij}} \frac{\sum \sigma_{ij}^{-1}}{A_i}, \qquad i = 2, \cdots n. \tag{22}$$

If n equals two, (22) reduces to (11). If n is greater than two, (22) becomes a complex function of the variance and covariance among disturbances. For a third monetary asset, say time deposits denoted by T, it can be shown that if all cross disturbances with currency are zero, i.e., $\sigma_{CD} = \sigma_{CT} = 0$, then the optimal reserve ratios on demand the time deposits are

$$k_d^* = (C/D) \frac{\sigma_C^2(\sigma_T^2 - \sigma_{DT})}{\sigma_D^2 \sigma_T^2 - \sigma_{DT}^2} \tag{23}$$

$$k_t^* = (C/T)\frac{\sigma_C^2(\sigma_D^2 - \sigma_{DT})}{\sigma_D^2\sigma_T^2 - \sigma_{DT}^2} \tag{24}$$

or expressed as a ratio,

$$\frac{k_t^*}{k_d^*} = \frac{D(\sigma_D^2 - \sigma_{DT})}{T(\sigma_T^2 - \sigma_{DT})}. \tag{25}$$

Note that this condition is analogous to (11) in Section I.C. above. Hence the criterion for setting the reserve level on time deposits (relative to that on demand deposits) is identical to that of demand deposits discussed above if and only if cross disturbances with currency are zero. The quantity of reserves behind demand deposits and time deposits will be equal, at the optimum, only if the proportional variability of disturbances to demand deposits equals that of time deposits.

Similar analysis can be used to establish the optimal reserve requirement on such monetary assets as money market mutual funds, which have experienced such sharp growth in recent years. In general, money funds and demand deposits should have identical reserve ratios only if each are regarded as perfect substitutes for the other. The fact that checks can be drawn the money funds is not sufficient to subject them to the same reserves as checking accounts (on stability grounds alone) since it is quite likely that the households' demands for such money funds will be subject to shocks that demand deposits in banks might be insulated from. As indicated earlier, higher unexplained variability will in general means a lower reserve requirement although the covariance of shocks between all monetary assets will influence the optimal reserve level, as indicated in (22) above.

II. Empirical Results

In order to calculate the reserve ratio on demand deposits which minimizes price level instability, it is necessary to determine the variance-covariance matrix of unanticipated shocks to monetary asset demands. Since in the United States over 90% of the high-powered money backs currency and demand deposits, empirical estimates center around these assets. A time period and set of independent variables similar to those estimated by Goldfeld [6] has been chosen. The following equations for real demand deposits (D) and real currency (C) are estimated from quarterly data from February 1952 to April 1973.

$$\begin{aligned}\log(D) = \underset{(.43)}{0.087105} + \underset{(3.49)}{0.10327}\log \text{GNP} - \underset{(-5.00)}{0.0045529}\,\text{CPR}\\ - \underset{(-2.19)}{0.0094301}\,\text{TDEP} + \underset{(13.22)}{0.85822}\log(D)_{-1}\\ \bar{R}^2 = .9627 \text{ SEE} = .0081262, \text{ Durbin } h = 1.1467.\end{aligned} \tag{26}$$

$$\begin{aligned}\log(C) = -\underset{(-2.51)}{.09821} + \underset{(4.49)}{.025859}\log \text{GNP} - \underset{(-2.72)}{0.0015135}\,\text{CPR}\\ + \underset{(12.76)}{1.2815}\log(C)_{-1} - \underset{(-3.02)}{0.30053}(C)_{-2}\\ \bar{R}^2 = .9969, \text{ SEE} = .004961, \text{ Durbin } h = -1.99, N = 84.\end{aligned} \tag{27}$$

where GNP = real gross national product, CPR = rate on 90-day prime commercial paper, and TDEP = average rate on time deposits. t-statistics are noted in parentheses under the coefficient.

The variance-covariance estimates of the residuals[7] of (26) and (27) are

$$\sigma_C = 0.00482, \quad \sigma_D = 0.00789, \quad \rho = \sigma_{CD}/(\sigma_C \sigma_D) = 0.256 \tag{28}$$

The ratio of currency to deposits (C/D) in the latest quarter is 0.27429.

Substituting these values into (1) yields

$$k_d^* = 7.0311\%. \tag{29}$$

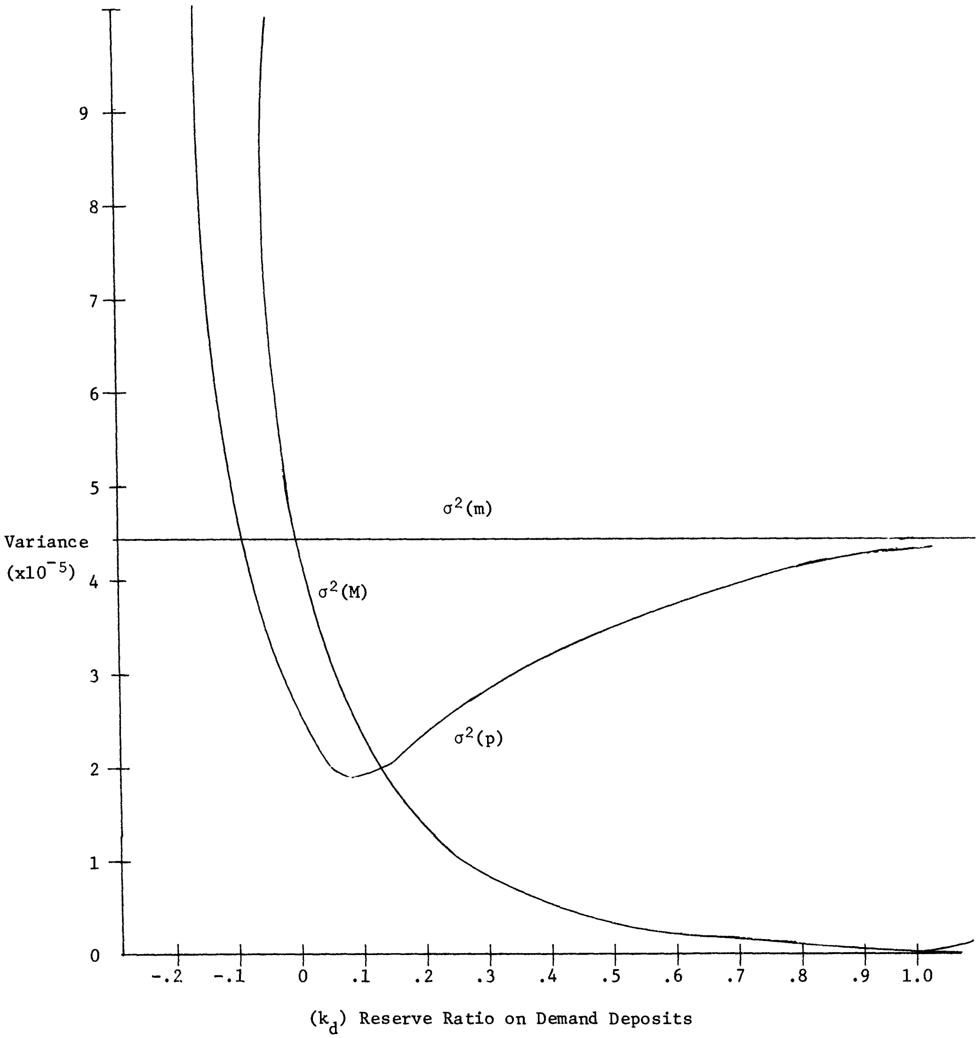
Therefore, the optimal reserve ratio for stabilizing prices is approximately 7 percent, below the nationwide average of 11.5 percent and approximately one-half the average ratio found for Federal Reserve member banks. It is also significantly below the 12 percent mandated by the Monetary Control Act of 1980 for all depository institutions (over $25 million in size) on transactions accounts. The positive covariance between deposits and currency decreases the optimal ratio from 10.22 percent which would be the value of k_d^* of σ_{CD} were zero. It is clear that the rather low value of k_d^* is due to the fact that (1) the per-dollar variance of deposits is approximately 2½ times the per dollar variance of currency; and (2) the value of deposits is 3½ times that of currency. Substituting (28) into (12) indicates that the minimized standard deviation of the price level is

$$\sigma_p^* = 0.00452. \tag{30}$$

This indicates that if reserve ratios were set optimally and high-powered money were deterministic, unexpected asset shocks would cause almost one-half percent standard error in quarterly price level estimates. If reserves on demand deposits were zero, it can be seen from Equation (10) that the standard deviation of the price level would be identical to that of currency, equalling .00482 or 6.5 percent higher than (30). As k_d approaches infinity, the standard deviation of prices approaches σ_D^2 or .00789. The standard deviation of the price level at the current level of reserves is .00485 or 1.4 percent higher than (30). At a reserve level of 100 percent, the variance is more than double the minimized value, although nominal money is perfectly stable. Figure 1 displays the variance of the price level and the variance of nominal and real money for differing levels of reserves on demand deposits. As can be seen from the asymmetry of the price variability curve, reserve levels below the optimal level cause a greater increase in price variation than an equivalent increment above the optimal level.

From Equation (17) it is also possible to determine the variance of the nominal money supply. The level of reserves indicated in (29) which minimized price level variability results in a standard deviation of nominal money of .00472. This value steadily declines until nominal monetary variability is zero when $k_d = 1$. When $k_d = 11.5\%$, the current nationwide average, the nominal monetary variability is .00339 which is 28% *below* that found at the reserve level which minimized price

[7] The unanticipated shocks are used to estimate the residuals since the monetary authority can offset any anticipated and determinate changes in monetary demands by direct manipulation of H. If the authority keeps high-powered money constant, then the variance-covariance structure of the currency and deposit series alone would be implemented.



Variance of selected variables for U.S. postwar data.

σ^2p = variance of log of price level
σ^2m = variance of log of real money supply (C + D)
σ^2M = variance of log of nominal money supply

Figure 1. Variance of Selected Variables for U.S. Postwar Data

level variability. In fact as k_d increases from 7%, price level variability rises although the variability of the money supply falls. This indicates the pitfalls of concentrating on minimizing the variability of some nominal aggregate as has been done in most studies published thus far.

The standard deviation of the real money supply is given by Equation (15) at .00653 and is unaffected by reserve ratios. When the reserve level on deposits is unity so that there is no variability to nominal money, then the variability of real money and prices must be equal since these two variables are perfectly negatively correlated.

When the currency and deposit equations are estimated through 1977, the optimal reserve ratio on deposits which minimizes price variability declines by approximately 2 percentage points to 5 percent. This is due primarily to the increased instability of the demand for deposits relative to currency which has been described by Goldfeld [7] and Garcia and Pak [5]. It should be noted that the greater unpredictability of the deposit ratio leads to a *reduction* of the reserve ratio on deposits although this unambiguously increases nominal monetary instability.

The above empirical results are subject to a number of qualifications. First, no attempt has been made to correct for the potential simultaneity problem involved in Equations (26) and (27). If the price level does not move quickly enough to continually equilibrate the demand and supply of high-powered money, then the effects of asset supplies should be taken into account. However, Goldfeld [6, 7] has not found a significant simultaneity bias in similar demand equations. Secondly, time deposits are ignored in the empirical work, although the optimal ratio on such deposits has been derived theoretically in Section I.E. As mentioned earlier, over 90 percent of all high-powered money is held behind currency and demand deposits, so that any recommendation calling for a substantial increase in reserves on time deposits would clearly amount to a major institutional change. However, because of these qualifications, a more sophisticated model would be needed in order to calculate a reserve ratio that would warrant significant policy recommendation. The estimates in this section are typical, nonetheless, of the sensitivity of price and monetary variability to the reserve ratio.

III. Summary

This research develops a stochastic financial model from which one can derive the reserve levels on financial assets which minimize price level fluctuations. It is shown that these levels of reserves are a function of the structure of unanticipated shocks to asset demands and are, in general, quite different from the levels which minimize the fluctuation of either the nominal or real value of monetary assets. This suggests that the use of policy instruments, such as reserve levels, to stabilize the money supply may lead to a systematic destabilization of such ultimate economic variables as prices or income.

Application of the model to the demands for currency and demand deposits in the United States suggests that the optimal reserve ratio on demand deposits which minimizes price variability is approximately 7 percent, or slightly lower if the estimation period is extended to include the recent period of demand deposit instability. At this ratio the variance of nominal money would be over 20% higher than exists under current reserve levels, although the variance of the price level is lower. These figures suggest the monetary authority would thus have to accept a greater level of nominal monetary aggregate instability in order to reduce the variability of prices.

APPENDIX

The formal problem of finding the optimal reserve ratios in the N-asset case is

$$\min_k f(K) = \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{A_i A_j k_i k_j \sigma_{ij}}{(\sum_{i=1}^{n} A_i k_i)^2} \tag{A.1}$$

where $K = (k_1, k_2, \cdots, k_n)$ and, since f is homogeneous of degree zero in K, K is defined up to an arbitrary constant. In matrix form, (A.1) can be written

$$f(K) = \frac{Q'\sum Q}{Q'\,UQ} \tag{A.2}$$

where

$$Q = (Q_1, Q_2, \cdots, Q_n),\; Q_i = A_i k_i$$

$$\sum = [\sigma_{ij}]_{n\times n}$$

$$U = \begin{vmatrix} 1 & \cdots\cdots & 1 \\ \vdots & & \\ 1 & \cdots\cdots & 1 \end{vmatrix}_{n\times n} = \text{the unity matrix} \tag{A.3}$$

Minimizing $f(K)$ is equivalent to maximizing $\lambda = 1/f(K)$, or,

$$\max_k \lambda = \frac{Q'UQ}{Q'\sum Q}. \tag{A.4}$$

This problem is formally equivalent to the simultaneous diagonalization of the two matrices U and $\sum$ encountered in discriminant analysis. The first order condition for maximization is the vector condition

$$\frac{\partial\lambda}{\partial Q} = \frac{2[UQ(Q'\sum Q) - \sum Q(Q'UQ)]}{(Q'\sum Q)^2} = 0. \tag{A.5}$$

This can be simplified by substituting $\lambda = (Q'UQ)/(Q'\sum Q)$ to obtain

$$UQ - \lambda\sum Q = 0. \tag{A.6}$$

Assuming $\sum$ is nonsingular, (A.6) can be written

$$(\sum\nolimits^{-1} U - \lambda I)Q = 0. \tag{A.7}$$

The solution to this problem is to find the largest eigenvalue of $\sum^{-1}$ and its associated eigenvector. Let $\sum^{-1}$ be indicated by

$$\sum\nolimits^{-1} = \begin{vmatrix} \sigma_{11}^{-1} & \sigma_{1n}^{-1} \\ \sigma_{n1}^{-1} & \sigma_{nn}^{-1} \end{vmatrix} \tag{A.8}$$

where σ_{ij}^{-1} is the ijth element of $\sum^{-1}$ (not $1/\sigma_{ij}$). Note that $\sum^{-1}$ is also symmetrical if $\sum$ is. $\sum^{-1} U$ can then be written

$$\sum\nolimits^{-1} U = \begin{vmatrix} a_1, a_1, & \cdots\cdots & a_1 \\ a_i, a_i, & \cdots\cdots & a_i \\ a_n, a_n, & \cdots\cdots & a_n \end{vmatrix} \tag{A.9}$$

where $a_i = \sum_{j=1}^{n} \sigma_{ij}^{-1}$. The rank of $\sum^{-1} U$ is obviously unity, which means that the matrix has $n - 1$ eigenvalues equal to zero and one nonzero value. Since the sum of all the eigenvalues is equal to the sum of the diagonal elements, and $n - 1$ are null, the largest eigenvalue is therefore

$$\lambda_1 = \sum\nolimits_{i=1}^{n} a_i = \sum\nolimits_i \sum\nolimits_j \sigma_{ij}^{-1}, \tag{A.10}$$

Thus the eigenvalue is equal to the sum of all the elements of $\sum^{-1}$. The associated eigenvector is found by solving

$$(\Sigma^{-1} U - \lambda_1 I)Q = 0 \tag{A.11}$$

whose solution is

$$Q_i = ca_i, \tag{A.12}$$

where c is an arbitrary constant.

Finally the vector K is found as

$$k_i = \frac{c \sum_{j=1}^{n} \sigma_{ij}^{-1}}{A_i} \quad i = 1, \cdots\cdots n, \tag{A.13}$$

the constant determined by the normalization constraint.

REFERENCES

1. Deane Carson. "Is the Federal Reserve System Necessary?" *Journal of Finance* 19 (December 1964), 652–61.
2. ———. "Should Reserve Requirements be Abolished?" *The Bankers Magazine* 156 (Winter 1973), 12–17.
3. Milton Friedman. *A Program for Monetary Stability.* Fordham University Press, New York City, 1959.
4. ———. "Statement of 21 August 1978," Submitted to the Committee on Banking, Housing, and Urban Affairs.
5. G. Garcia and S. Pak. "The Ratio of Currency to Demand Deposits in the United States." *Journal of Finance* 34 (June 1979), 703–15.
6. Stephen M. Goldfeld. "The Demand for Money Revisited." *Brookings Papers on Economic Activity* (3: 1973), 577–638.
7. ———. "The Case of the Missing Money." *Brookings Papers on Economic Activity* (3: 1976), 683–730.
8. Ira Kaminow. "Required Reserve Ratios, Policy Instruments, and Money Supply Control." *Journal of Monetary Economics* 3 (October 1977), 389–408.
9. Daniel E. Laufenberg. "Optimal Reserve Requirement Ratios Against Bank Deposits for Short-run Monetary Control" *Journal of Money Credit and Banking* 11 (February 1979), 91–105.
10. Allan H. Meltzer. "Discussion." *Controlling Monetary Aggregates.* Federal Reserve Bank of Boston. (June 1969), 96–103.
11. William Poole and Charles Lieberman. "Improving Monetary Control." *Brookings Papers on Economic Activity* (2: 1972), 293–342.
12. A. Santomero and Jeremy Siegel. "Bank Regulation and Macroeconomic Stability." *American Economic Review* 44 (March 1981), 39–53.