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Optimal Regulation Under Uncertainty

WILLIAM J. MARSHALL, JESS B. YAWITZ, and EDWARD GREENBERG*

ABSTRACT

This paper is concerned with the problem of price regulation when demand is uncertain. Uncertainty gives rise to substantial difficulties in determining both the return a firm's owners should be provided and a set of prices capable of producing that return. We argue that conventional approaches to price regulation are incapable of attaining the economically desirable objectives of efficiency and an equitable return to investors. The deficiencies in current practices are attributable to the separation of the risk measurement-return determination and price setting activities in the conventional approach. We present a model of the regulated firm that synthesizes contemporary financial market theory and the theory of the firm under uncertainty.¹ In our approach, the income stream produced by the firm is valued *ex ante* in the financial market according to investors' perceptions and preferences over risk-return characteristics. We portray the firm as producing risk and return by choosing among available production technologies to maximize its market value, given the prices set by regulators. Within this framework, it is shown that regulators can choose the lowest prices consistent with an equitable return to investors. We also show that prices so chosen induce the choice of the optimal technology by the firm.

IN THE TRADITIONAL REGULATORY framework, as thoroughly described and explained by Bonbright [3], the goal of regulators is to set rates in such a way that all costs are covered, including a return on invested capital commensurate with the risk of equity investment. Determination of the latter is difficult at best. It is generally agreed that: (1) the fair return depends on risk; (2) the return required by investors as compensation for risk bearing is most accurately measured by observing the returns to competitive firms in otherwise similar circumstances; and (3) estimation of required returns by observation of other regulated firms is generally inappropriate since the risk and regulation effects are confounded.

More recently, Myers [13] has argued for the application of finance theory to rate determination within the traditional regulatory framework. In particular, he argues that the capital asset pricing model (CAPM) developed by Sharpe [14], Lintner [9], and Mossin [12] is especially well suited for this purpose since a single quantitative measure of risk is relevant in that model. Further, since the CAPM offers an explicit equilibrium risk-return relationship, information about the risks and returns of all firms can be directly employed in determining the fair return to a regulated firm.

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¹ A recent paper by Baron [1] furnishes an excellent review of a host of diverse issues involving the behavior of the firm under uncertainty and the role of financial markets.

Although the arguments by Myers and others in support of the use of the CAPM are persuasive, the conventional method of implementation is improper. Typically, the financial risk of the regulated firm is measured by estimating the covariability of its return with that of the market, using historical stock price data. Then, the fair rate of return is determined from the risk-return relationship observed for a reference group of nonregulated firms. Finally, prices are set such that at expected levels of demand owners can anticipate earning the required rate of return on their investment.

The process described above is doomed to failure by the implicit and inappropriate assumption that a firm's financial risk is independent of the activities of regulators and managers. It will be evident from our analysis that risk is endogenous. The prices chosen by regulators serve to determine the impact of demand uncertainties on profits. Further, managers affect risk by their choice of a production technology. Put simply, the choice of technology determines the covariability of costs and revenues and, thereby, the sensitivity of profits to variations in demand.

The impact of the chosen technology on the financial market risk characteristics of a firm has been demonstrated by Greenberg, Marshall, and Yawitz [6]. Those authors also consider the pricing decision for a single product in the context of a financial valuation model. Meyer [11] has considered the pricing decision of a multi-product firm in a similar setting. Meyer's approach takes prices as "weights" on products in a portfolio and determines the optimal composition of a firm's product line. An optimal price-product set exists in Meyer's world since less-than-perfectly correlated disturbances to product demands contribute differentially to the firm's aggregate risk. The prices set by the firm for the group of products determine the level of demand in each market, and thereby weight the contributions of the individual submarkets to the uncertainty about aggregate firm profit and the expected level of profit.

From these two studies, one would infer that both technology and pricing would affect the riskiness of the firm. Though a utility sells, in a sense, one product, different market segments are typically charged different prices; thus Meyer's findings seem relevant. The assumption that the demands for a utility's product are not perfectly correlated across segments of the firm's market is intuitively appealing.²

In an important paper dealing with uncertainty, Leland [8] developed a model in which investors are assumed to be expected utility maximizers. Using his "unanimity theorem," Leland shows that firms will employ inputs in a technologically efficient manner if the firm is allowed to maximize its value given the prices set by the regulator. As part of his argument, Leland asserts the existence of a

² Some suggestive empirical evidence to support the hypothesis that demand is not perfectly correlated across segments of the firm's market is presented in the following table where we report the correlations between use of electric power and GNP for four regions in the U.S.A. Note that these regions vary substantially in manufacturing concentration, with direct correspondence in the power use-GNP correlations. The inference is that one would expect demand from different sectors of the economy to differ in its sensitivity to GNP and in the risk measure appropriate to the CAPM, the covariability between a firm's profit and general economic activity. One effect of setting different prices for different groups of consumers of a product is to establish weights on their respective

function which relates the market's valuation of the firm to the output price. Optimal behavior for a regulator is to choose the lowest price for which the market's valuation of the firm equals the cost of its capital equipment.

Leland's work represents our point of departure. In what follows we show that the CAPM can be used to derive an explicit relationship between prices and market value. Of course, completing the argument in this way results in some loss of generality in the assumptions concerning attitudes toward risk and/or probability distribution of returns.³

Uncertainty in the demand for a public utility's output has also been studied extensively in the context of peak load pricing problems, e.g. Littlechild [10] and the references cited therein. These studies differ from ours in two ways: (1) they assume risk neutrality rather than risk aversion; and (2) they are concerned with uncertainty that results from random variations in weather rather than from variations in economic activity.

The remainder of the paper is organized around models which allow us to examine more subtle aspects of the basic problem. We sacrifice some generality by imposing assumptions about functional forms of demand and production relationships. The marginal loss seems minor since the assumption that the CAPM adequately represents the capital markets is already strong. We recognize the desirability of general results, but weigh that against the fact that some functional forms must ultimately be chosen for any approach to be implemented. The functional forms chosen are meant to be reasonable and to permit explicit analysis of those aspects of the problems which seem to be most important. We attempt

demands in determining the aggregate risk of the firm. Therefore, risk is dependent on the prices chosen, and is not exogenous as it is conventionally treated.

Region	% of Workers Employed in Manufacturing ^a	Correlation Coefficient of Detrended KWH with Real GNP ^b
Detroit	40.9	+0.836
Pittsburgh	34.8	+0.957
San Diego	22.0	+0.402
Tampa	15.2	+0.020

^a Source: County Business Patterns, 1974.

^b Annual observations are detrended to remove the effect of differential growth rates. Period used is 1954-75.

³ The generality of our approach is limited in other respects. We ignore several aspects of the price regulation problem which are of obvious importance. We have no contribution to make here on the issues of rate base measurement or regulatory lag. We ignore inflation and attribute to the firm more than realistic flexibility in varying its choice over labor-capital inputs. Issues of property rights and equity arise when we consider a case in which customer groups differ in important ways and prices may vary. In that case, we identify a set of prices consistent with the basic objectives of regulation, but do not make the value judgments necessary to choose one set of prices. We ignore the effects that the prices charged one group (say, producers) may have on the welfare and demands of another (say, homeowners). We ignore the agency problem by assuming that managers seek to maximize the value of the firm. Finally, we do not consider the importance of the firm's "conjectural response" to regulatory policies and the resultant implications for production efficiency [2]. While these are important problems, they are beyond the scope of this paper.

to identify conclusions which appear to be qualitatively dependent on the structure of the models.

We begin Section I with a brief discussion of the CAPM and then proceed to analyze in Section I-A a regulated monopolist within the context of a one market demand model. With the introduction of a simple cost relationship, that scenario is adequate to demonstrate (1) the determination of a fair rate of return; (2) the choice of a price that induces efficient production and maximum output consistent with a fair rate of return; and (3) the adverse effects of regulation *via* a rate of return constraint.

In Section I-B we consider the case in which the firm's output is sold in two markets which differ in price sensitivity and demand uncertainty, but not in real costs of production. The primary result obtained is the finding that no optimal set of prices can be determined without imposing further assumptions about regulatory or social preferences; however, we are able to derive an optimal relationship between the prices in the two markets. The price-pairs are optimal in the sense that given the price charged in either market, the price in the other is the lowest consistent with a fair return. A simulated example is used to demonstrate the derived relationship, and the additional assumptions that might be used to determine a unique solution are briefly explored.

I. Regulation, Production, and Valuation under Uncertainty

To establish the notation used throughout the paper, we first present our basic valuation model. As a condition for capital market equilibrium, the Sharpe-Lintner-Mossin [14, 9, 12] capital asset pricing model provides an explicit expression for the value of the firm (V):

$$V = \frac{1}{1+r} [E(X) - \lambda \text{Cov}(X, X_m)] \quad (1)$$

where

X is the uncertain income of the firm;

X_m is the income on all assets (denoted in the finance literature as the market portfolio);

r is the risk-free rate of interest;

$\lambda = \frac{[E(X_m) - rV_m]}{\text{Var}(X_m)}$ is the market price of risk;

$E(X)$ is the expected value of X ;

$\text{Cov}(X, Y)$ is the covariance between X and Y ; and

$\text{Var}(X)$ is the variance of X ;

The random variable X is measured at the end of one time period, and the other variables are defined accordingly. In Equation (1), the value of the firm is found by discounting a "certainty equivalent" income, $E(X) - \lambda \text{Cov}(X, X_m)$, at the riskless rate, r . The certainty equivalent is the expected income to the firm less an amount proportionate to the riskiness of that income. The latter is the compensation required by the firm's owners for bearing risk.

We now consider the question of optimal regulation under two different specifications of the monopolist's sales environment.

A. One output market

We first consider the case in which the monopolist provides a single product at the same price to all customers.⁴ In addition, we assume initially that the quantity demanded (Q) is uncertain and *unaffected* by price (P). The monopolist must meet demand and in so doing incurs a unit cost of C , which depends on the amount of his capital equipment, K .

At the end of the period, the market value of the equipment will be $\Pi(1 - d)K$, where Π is the uncertain end-of-period unit price of equipment and d is the rate of physical depreciation. Adding the random net revenue from operations to the value of equipment, we obtain the random end-of-period value of the firm, X :

$$X = (P - C)Q + \Pi(1 - d)K \quad (2)$$

From the CAPM, the market value of the random variable X is given by

$$V = \frac{1}{1 + r} [(P - C)(\bar{Q} - \lambda \text{Cov}(Q, X_m)) + (1 - d)K(\bar{\Pi} - \lambda \text{Cov}(\Pi, X_m))] \quad (3)$$

where $\bar{Q} = E(Q)$ and $\bar{\Pi} = E(\Pi)$. Equation (3) clearly demonstrates the impact of P and K on the value of the firm. Since Q is assumed to be independent of P , an increase in P will always increase the value of the firm, but the positive effect on the expected value of cash flows is attenuated by the effect of the higher P on the riskiness of the firm (we assume $\text{Cov}(Q, X_m) > 0$). To see this, rewrite the first part of the bracketed term in (3) as

$$(P - C)\bar{Q} - \lambda(P - C) \text{Cov}(Q, X_m)$$

For a given C , an increase in P will increase $(P - C)\bar{Q}$, the expected return. Also increased, however, is $\lambda(P - C) \text{Cov}(Q, X_m)$, the required compensation for risk. The benefit from a decline in C is similarly reduced by an increase in risk. The greater the difference between P and C , the more risky are net operating revenues.⁵ This feature of the model has interesting implications for cases, discussed below, in which there are several customers with different covariances.

For the monopolist to receive a fair return on his investment, the capital market value of the uncertain income must exactly equal the purchase price of his beginning-of-period investment in physical capital, $\Pi_0 K$. If the firm has no debt, a fair return requires that V must equal $\Pi_0 K$.⁶ When this condition holds,

⁴ A simplification we adopt in what follows is that there is one unit price to each category of user. In practice, unit electric rates decline with kWh usage. Moreover, rate schedules of electric companies often include a demand charge based on peak usage as well as a minimum charge based on usage in the past year. Although these charges may help smooth out revenues over a business cycle, they are not explicitly addressed in the formal modeling below.

⁵ We use the term "more risky" to denote an increase in the risk in the firm's income stream. This should not be confused with riskiness in terms of rate of return.

⁶ In fact, the condition that $V = \Pi_0 K$ will result in a fair return on equity even when K is partly financed by debt. Under such conditions, the value of the equity investment in K equals V less the

the firm will earn profits which are just sufficient to attract capital to its current use. As Leland points out, $V = \Pi_0 K$ corresponds to the long run equilibrium with freedom of entry and exit:⁷

If $V > \Pi_0 K$, entrepreneurs could make a sure profit by buying $\Pi_0 K$ worth of assets, then selling stock for V . If $V < \Pi_0 K$, the entrepreneur could make a profit by buying the firm for V and selling its assets for $\Pi_0 K$. [8, pp. 6-7]

For a given K , P^* , the price which results in $V = \Pi_0 K$, can be determined from Equation (3):

$$P^* = \frac{[(1+r)\Pi_0 - (1-d)\Theta]K}{\Gamma} + C \quad (4)$$

where

$$\begin{aligned}\Gamma &= \bar{Q} - \lambda \text{Cov}(Q, X_m) \\ \Theta &= \bar{\Pi} - \lambda \text{Cov}(\Pi, X_m)\end{aligned}$$

Therefore, for given K , the price required to provide a fair return is unique and depends on characteristics of the firm's product market, the technology of its production process, the price of a unit of capital, and the risk-return preferences of investors in the capital market. The price includes the direct cost of production plus a share of the decline in the value of physical capital which results from its use for one period in this risky project. Within this setting, the regulator's objective is to minimize price consistent with a fair return (income) objective. The price-minimizing value of K is given by the solution to

$$\frac{\partial P}{\partial K} = \frac{(1+r)\Pi_0 - (1-d)\Theta}{\Gamma} + \frac{\partial C}{\partial K} = 0 \quad (5)$$

or, equivalently,

$$(1+r)\Pi_0 - (1-d)\Theta = -\Gamma \frac{\partial C}{\partial K} \quad (5a)$$

The left-hand side is the (risk-adjusted) cost of adding a unit of capacity; $(1+r)\Pi_0$ is the certain end-of-period value of Π_0 dollars invested in the riskless security, and $(1-d)\Theta$ is the value of the uncertain investment of Π_0 dollars in physical capital. The right-hand side is the change (saving) in unit cost resulting from the additional K times the risk-adjusted quantity of output.

To proceed further it is necessary to specify the cost function. We analyze in detail the case where $C = \alpha/K$. Given K , unit costs are constant. In the long run, costs are inversely proportional to K . Since $\frac{\partial C}{\partial K} = -\alpha/K^2$, the optimal K is

value of debt. If the equity owners must pay creditors a "fair" return, then $V = \Pi_0 K$ is required to provide a fair return to equity. Because of the tax-deductibility of interest payments, an increase in leverage, *ceteris paribus*, would increase the value of the firm. In order to maintain the $V = \Pi_0 K$ criterion when leverage increases, the regulator must decrease the price charged by the utility. Because of the complication introduced by leverage changes, we consider an all equity firm.

⁷ We have changed several of Leland's symbols to conform to our notation.

determined as

$$K^* = \left(\frac{\alpha \Gamma}{(1+r)\Pi_0 - (1-d)\Theta} \right)^{1/2} \quad (6)$$

Substituting for K in Equation (4), one obtains the minimal P as

$$P^* = 2 \left(\frac{\alpha[(1+r)\Pi_0 - (1-d)\Theta]}{\Gamma} \right)^{1/2} \quad (7)$$

An alternative procedure is to compute the value maximizing level of K for a given P , and then to select P such that $V = \Pi_0 K$. Of course, Equation (7) would again result.

In Equations (6) and (7), $[(1+r)\Pi_0 - (1-d)\Theta]$ can be thought of as the risk-adjusted opportunity cost (net) of holding an investment in the form of physical capital. The larger P^* is, the greater this cost is. Γ is the "certainty equivalent" level of demand. The smaller P^* is, the greater Γ is, since the relevant costs are apportioned over a larger base. The constant α reflects the relative productivity of capital. The larger α is, the less productive capital is and the greater P^* must be. The partial effects of r and λ are both positive: the higher the riskless rate of return and the greater the market premium required for risk, the higher P^* is.

We are also interested in the determinants of K^* . Since additional investment in K reduces the variable cost, C , the riskiness as measured by $(P - C) \text{Cov}(Q, X_m)$ increases with K . Therefore, the greater the price of risk, the lower will K^* be. For reasons mentioned earlier, K^* varies inversely with $[(1+r)\Pi_0 - (1-d)\Theta]$ and directly with α .

It is interesting to contrast the above results to a situation in which Π and Q are known with certainty to equal their respective expected values. In that case, Equation (4) for price may be used by setting both $\text{Cov}(Q, X_m)$ and $\text{Cov}(\Pi, X_m)$ equal to zero. On the assumption that these had been positive, P^* would be smaller under certainty, since $\Gamma < \bar{Q}$ and $\Theta < \bar{\Pi}$. A similar analysis indicates that K^* would be larger under certainty on the above assumptions.

Our analysis has demonstrated that the regulator can accomplish his objective if allowed to set both P and K . In fact, the regulator need only control P , because the optimal level of K minimizes the sum of the capital market value of the uncertain costs of meeting demand and the opportunity cost of holding an investment in the form of physical capital. This level will therefore be chosen by a value-maximizing firm.

Figure 1 contains, for several different prices, the firm's attainable combinations of V and K (from Equation (2), with $C = \alpha/K$). If the regulator sets price at P^* , the firm is able to earn a fair return by employing K^* in physical capital. For $K \neq K^*$, the firm's income stream will be valued at less than the value of its capital. For $P < P^*$, no value of K will permit the expectation of a fair return. On the other hand, for $P > P^*$, the firm can achieve a capital gain ($V > K$) when $K_1 < K < K_2$. $V - K$ is maximized where the VK relationship has a slope of one.

Using Figure 2, one can analyze the impact of a rate-of-return ceiling quite easily. From Equation (2), express the expected rate of return, $\bar{R}$, to the firm's

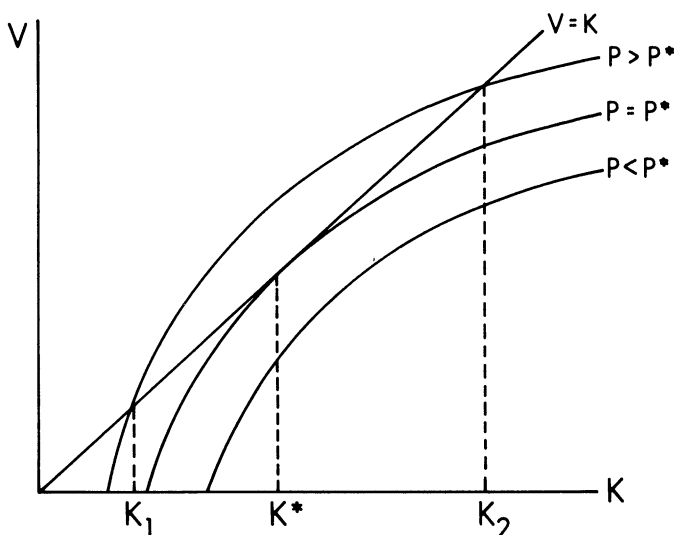


Figure 1. Effect of Price on Attainable Combinations of Firm's Valuation and Physical Capital equity holders as

$$\bar{R} = \frac{(P - C)\bar{Q} + \bar{\Pi}(1 - d)K - \Pi_0 K}{\Pi_0 K} \quad (8)$$

where Π_0 is the beginning-of-period unit price of physical capital. For $\Pi_0 = \bar{\Pi}$, Equation (8) can be expressed

$$\bar{R} = \frac{(P - C)}{\Pi_0 K} \bar{Q} - d \quad (8a)$$

Note that the relationship between profit and K is concave since $\frac{\partial^2 C}{\partial K^2} > 0$. Hence,

$\bar{R}$ and K must be inversely related. Define $\bar{R}^*$ as the expected rate of return when $P = P^*$ and $K = K^*$. Now suppose the regulator places a ceiling of $\bar{R}_{\max}$ on the expected return on equity, where $\bar{R}_{\max} < \bar{R}^*$. In order to earn $\bar{R}_{\max}$, assume that the firm must employ capital of K_4 for $P = P^*$ (See Figure 2). Although K_4 is the best of the allowable levels of K , it results in $K > V$ and the firm is not viable. If the rate of return ceiling is retained, the regulator must increase P^* to P' so that the firm is able to earn a fair return. For $P = P'$, K_5 is chosen and $\Pi_0 K_5 = V$. The binding return ceiling has resulted in a higher price and a greater value for K than if the producer were free to determine K so as to maximize value.⁸

The rate of return expected by investors at the optimal value of K can be inferred from the CAPM. Assuming that $\Pi = \Pi_0$ with certainty and that $V = \Pi_0 K^*$, we have

$$(1 + r)\Pi_0 K^* = (P - C)[\bar{Q} - \lambda \text{Cov}(Q, X_m)] - d\Pi_0 K^* \quad (9)$$

⁸ Leland [7, p. 123] also points out that neither technical efficiency nor maximal output is achieved under a rate of return constraint. Of course, if the regulation specifies $R_{\max} = \bar{R}^*$ and $P = P^*$, rate of return regulation will result in the same solution as simply specifying $P = P^*$. On the other hand, setting $R_{\max} = \bar{R}^*$ without specifying price will not, in general, result in an optimal solution.

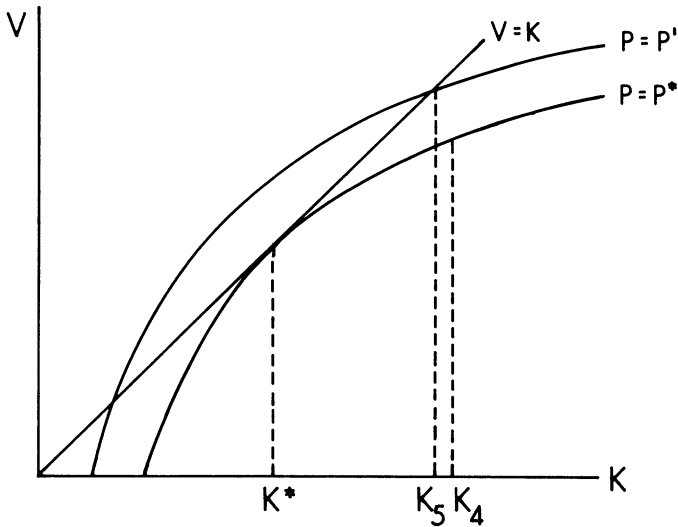


Figure 2. Effect of a Rate of Return Constraint on the Choice of Physical Capital

so that

$$\begin{aligned}\bar{R} &= \frac{(P - C)\bar{Q}}{\Pi_0 K^*} - d \\ &= r + \frac{\lambda(P - C) \text{Cov}(Q, X_m)}{\Pi_0 K^*}\end{aligned}\quad (10)$$

This expression is the risk-adjusted rate of return that will be anticipated by investors. It consists of a risk premium added to the risk free interest rate. This rate of return depends on K^* and P^* ; it is thus inappropriate to determine a “fair” rate of return independent of the optimal values of K and P , as is attempted in the traditional regulatory literature.

It is straightforward to relax our assumptions to analyze alternative specifications of costs with inelastic demand and to extend the approach to constant elasticity demand functions, under various assumptions concerning risk.⁹

B. Two markets for output

In practice, regulators usually subdivide markets and impose different prices for the same good. Issues arising from this possibility are discussed next, in the context of the following simplifying assumptions regarding costs and demand functions.

$$\text{Average (and Marginal) Cost:} \quad C = \alpha/K$$

$$\text{Demand in market } i \ (i = 1, 2): \quad Q_i = AP_i^{\eta_i} Y_i$$

where Y is random, $Y_i = Y^{\delta_i}$, and δ_i is the income elasticity of demand for the i th

⁹ Details are in a working paper available from the authors.

market. The value of the firm is given by

$$V = \frac{1}{1+r} \left[AP_1^{\eta_1} \left(P_1 - \frac{\alpha}{K} \right) \Gamma_1 + AP_2^{\eta_2} \left(P_2 - \frac{\alpha}{K} \right) \Gamma_2 + (1-d)K \right]$$

where $\Gamma_i = E(Y_i) - \lambda \text{Cov}(Y_i, Y)$. Given P_1 and P_2 , the firm is assumed to maximize $V - K$. The optimal value of K is given by

$$K^* = \left[\frac{A\alpha P_1^{\eta_1} \Gamma_1 + A\alpha P_2^{\eta_2} \Gamma_2}{r+d} \right]^{1/2} \quad (11)$$

Imposing the fair return requirement, one obtains an implicit function relating P_1 and P_2 (after replacing K^* with (11))

$$AP_1^{\eta_1} \left(P_1 - \frac{\alpha}{K^*} \right) \Gamma_1 + AP_2^{\eta_2} \left(P_2 - \frac{\alpha}{K^*} \right) \Gamma_2 - (r+d)K^* = 0 \quad (12)$$

This equation provides the regulator with a choice of combinations of P_1 and P_2 that provide a fair return and minimize costs, given output levels. To illustrate the price combinations generated by Equation (12), we have specified values for the parameters of interest. Of primary interest is the variation of price as a function of relative riskiness, so we assume that one market is risk free, i.e. $\delta_1 = 0$, so $\Gamma_1 = 1$ (recall $Y_1 = Y^{\delta_1}$). As δ_2 increases, more of the variation in Y is passed through to Q_2 , increasing $\text{Cov}(Y_2, Y)$, and decreasing Γ_2 . Accordingly, we set $\Gamma_2 = .95$ and $\Gamma_2 = .5$ alternatively to generate two loci of points which satisfy the no-excess profit condition. The larger Γ_2 implies less risk relative to market 1 than does the smaller value. To complete the specification, we set

$$\eta_1 = \eta_2 = 0.7, \quad A = 1000, \quad \alpha = 500, \quad r = .1, \quad \text{and} \quad d = .02.$$

For given values of P_1 and Γ_2 , we compute the lowest value of P_2 which is consistent with the fair return requirement.

The resulting combinations of P_1 and P_2 , which yield zero excess expected profits, appear in Figure 3. To interpret the figure, first assume that the regulator requires equal prices in the two markets. If the second market is characterized by $\Gamma_2 = .95$, this will require a price of about .60; if $\Gamma_2 = .5$, the required price is over .70. Now assume that the regulator wants to reduce P_1 and compensate the firm with an increase in P_2 . For a constant K^* (and therefore constant costs), the more risky is the second market, the greater is the increase in P_2 that is required because of the greater uncertainty about the revenue yield from the market. This revenue is needed to satisfy the fair return constraint because the inelastic demand for Q_1 results in a reduction in revenues from the first market. There is a partial offset because K^* will rise as a result of a decrease in P_1 , but the extent of this rise is an increasing function of Γ_2 (a decreasing function of risk). Therefore, at the 45° line, we find that $|dP_2/dP_1|$ at $\Gamma_2 = .5$ is greater than the same expression at $\Gamma_2 = .95$. Since the constant elasticity demand function assures that the curves will be asymptotic to the P_1 and P_2 axes, the difference in slope means that the two figures will cross. Thus, although a relatively more risky market pays a higher price than a relatively less risky market for values of P_1 up to about 1.20, the reverse is true for greater values.

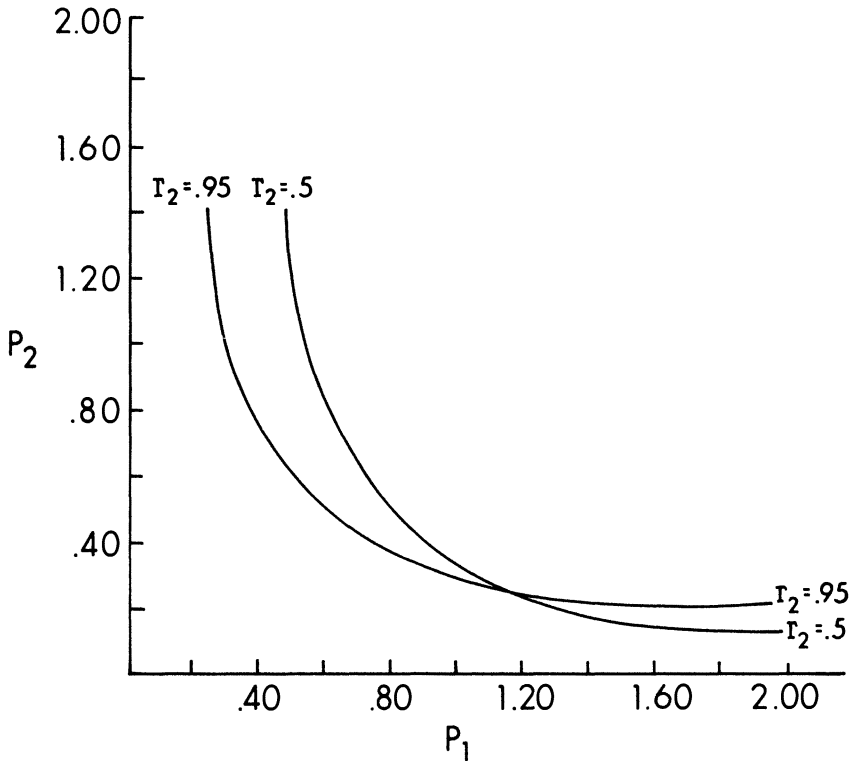


Figure 3. Sets of "Fair" Prices Corresponding to Different Level of Relative Risk

Figure 3 indicates a wide range of possible price combinations which satisfy the zero-excess return condition. The price combination chosen by the regulator will affect the outputs and incomes of the submarkets. Students of regulation may be interested in studying regulators' price choices in the context of a model of regulatory behavior.

The range of possible prices may be narrowed somewhat by injecting additional conditions. For example, it might be required that the price in each submarket be set high enough to cover average costs. In our simulated case, this restriction imposes the limits $P_1 > .3$, $P_2 > .275$ for the $\Gamma_2 = .95$ case; and $P_1 > .31$, $P_2 > .36$ for the $\Gamma_2 = .5$ case. As another possible consideration, we note that K^* is approximately minimized for $P_1 = .60$, $P_2 = .56$ in the $\Gamma_2 = .95$ case; and at $P_1 = .70$, $P_2 = .73$ in the $\Gamma_2 = .5$ case. K^* does not reach a maximum in this example.

II. Conclusions

Our primary aim has been to show that it is possible to define an optimal solution to the problem of regulating public utilities under uncertainty. By basing the analysis directly on the production relationships and on the riskiness of the utilities' customers, one is able to utilize the CAPM to cut through the circularity that characterizes the conventional approach to the problem. The synthesis of financial and economic models of the regulated firm enables us to structure the

problem so that the endogeneity of risk is made explicit, and prices can be chosen simultaneously to provide a fair return and to motivate the use of the efficient technology. Implementation of the model requires no knowledge beyond that required in the use of the CAPM in conventional price regulation. Indeed, less information is required, since the risk of the firm is modeled as a function of production technology and demand uncertainty and thus need not be measured directly. Admittedly, the problems in estimation of key relationships which plague conventional approaches will remain important concerns. The advantage of an internally consistent theoretical basis may, however, be substantial.

Though our interest is primarily in producing normative results, the explicit models we construct should be of use in explaining observed behavior. Although the use of functional forms for the financial market, production technology, and demand relationships results in some loss of generality, it provides explicit expressions for the optimal values of firm decision variables and for the market value and financial risk of the firm. On the presumption that regulated firms optimize within the confines of regulatory constraints (that is, given the prices set by regulators), these expressions are appropriate for use in estimating the financial and economic parameters which characterize the firm's environment. The measurement problems likely to be encountered are no more severe than those encountered in estimating supply-demand systems. The paper has little to say empirically about the present actions of regulators, since these are based on the conventional approach to price setting rather than the approach described above.

Our analysis suggests an empirical study that may be of some interest. Assume that the utility chooses a price structure which is then approved by the regulators. Other things equal, one might expect to find that the rate structure is chosen with risk minimization as a goal; thus, a utility with a relatively large proportion of industrial customers (whose demands are highly correlated with the market) might tend to charge them lower rates than their residential customers (whose demands are not very sensitive to business conditions).

Finally, it might be possible to reduce the information requirements of regulators by employing a price-setting procedure that converges to the optimum. One such procedure has been developed by Vogelsang and Finsinger [15] for a multiproduct monopolist under certainty. In their approach, the regulator is assumed to know only ex post costs, prices, and quantities; the firm is assumed to maximize profits and have knowledge of relevant price elasticities. It may be possible to develop a similar approach for the value-maximizing firm under uncertainty.

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