THE JOURNAL OF FINANCE • VOL. XXXVI, NO. 3 • JUNE 1981

A Note on Exchange-Rate Expectations and Nominal Interest Differentials: A Test of the Fisher Hypothesis

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THIS NOTE TESTS THE hypothesis that nominal interest differentials between similar assets denominated in different currencies can be explained entirely by the expected change in the exchange rate over the holding period. This proposition, often called the "Fisher open" hypothesis or the hypothesis of perfect asset substitutability, has been a major component of recent theories of exchange-rate determination, and has important implications for monetary policy.[1]

The Fisher hypothesis differs from the covered interest parity theorem enunciated by Keynes [11]. While covered interest parity equates the nominal interest differential between financial centers to a known premium or discount on forward foreign exchange, and is thus in principle riskless, Fisher parity involves risk in an essential way. The two conditions are equivalent only when forward exchange rates equal expected future spot rates. Recent theoretical work, for example that of Grauer, Litzenberger, and Stehle [8], Kouri [12], and Frankel [6], shows that this need not be the case when market participants are risk averse. Thus, while covered interest parity must always hold exactly in an informationally efficient asset market with no transactions costs, Fisher parity need not. Nominal interest differentials may reflect risk premia in addition to expected exchange-rate movements.[2]

It should be emphasized at the outset that the tests conducted in this paper are really joint tests of the Fisher hypothesis and a weak form of foreign exchange market informational efficiency. Levich [14] has argued that tests of market efficiency are always joint tests involving some hypothesized structure of "normal" equilibrium returns. Below, we assume informational efficiency and instead test Fisher's hypothesized arbitrage condition that nominal interest differentials equal expected percentage changes in exchange rates.

I. The Fisher Hypothesis

The Fisher hypothesis states that when two bonds are similar in all respects except currency of denomination, asset-market equilibrium requires that any nominal rate-of-return differential between them be offset by an expected ex-

[1] The hypothesis is discussed in Fisher [5].

[2] If covered interest parity holds exactly, the extensive empirical literature on the relationship between forward exchange rates and subsequently-observed spot rates has implications for the validity of the Fisher parity. Recent contributions to this area include work of Hakkio [9], Hansen and Hodrick [10], and Meese and Singleton [16]. The findings of these authors are consistent with those reported below.

change rate change over the holding period. More formally, let s_t be the logarithm of the spot exchange rate, defined as the domestic-currency price of foreign exchange; let ${}_ts_{t+1}$ be the expected value of the logarithm of next period's exchange rate, conditional on information available at time t; and let r_t and r_t^* be the one-period nominal yields (between t and $t + 1$) on the domestic and foreign bonds, respectively. The Fisher parity condition may then be written as:

$$\begin{aligned} {}_ts_{t+1} - s_t &= ln(1 + r_t) - ln(1 + r_t^*) \\ &\simeq r_t - r_t^* \end{aligned} \tag{1}$$

To obtain an empirical test of condition (1), we require two assumptions. The first assumption is that ${}_ts_{t+1}$ is a mathematical conditional expectation, based on the true probability distribution underlying the events impinging on financial markets. The second is that the foreign exchange market is weakly efficient, in the sense that market expectations of future spot rates are always conditioned on an information set that includes past forecast errors.

The first assumption allows us to write the realized spot exchange rate as its expected value plus a forecast error having zero mean:

$$s_{t+1} = {}_ts_{t+1} + \epsilon_t, \qquad E(\epsilon_t) = 0 \tag{2}$$

The second assumption, taken in conjunction with the first, allows us to assert further that the stochastic process $\{\epsilon_t\}$ is a white noise process, so that the forecast errors are mutually uncorrelated at all lags.

Combining Equations (1) and (2), we obtain

$$s_{t+1} - s_t - r_t + r_t^* = \epsilon_t \tag{3}$$

Since all variables on the left-hand side of (3) are observable ex post, we may test the Fisher hypothesis by testing whether the process $\{\epsilon_t\}$ is white noise.[3]

Under a mean-variance theory of capital-asset pricing with risk-averse investors, ϵ_t is the sum of an expectational error *and* a risk premium that accounts for a divergence between the interest differential and the expected rate of depreciation over the holding period.[4] In such a setting, domestic and foreign bonds will not be perfect substitutes. If covered interest parity holds identically, this risk premium may be expressed as the difference between the logarithm of the one-period forward rate and the expected logarithm of the exchange rate one period hence,

$$f_t - {}_ts_{t+1} = \text{risk premium}$$

[3] Levich [13] suggests a similar procedure. It is equivalent to testing the whiteness of forward-rate forecast errors when covered interest parity holds. A referee has pointed out that Equation (3) may also be obtained, under certainty, by assuming that real interest rates are equal across countries, that purchasing-power parity holds, and that nominal interest rates differ from real interest rates one-for-one with the inflation rate. Roll [20] derives an equation similar to (3) in this manner and interprets the test for whiteness of his series as a test of purchasing-power parity.

[4] Porter [18] examines the Fisher hypothesis using evidence on the Canadian-U.S. dollar exchange rate. He uses the estimated standard deviation of the previous year's exchange rate around its mean as a measure of exchange risk. This measure is inadequate, for it does not distinguish between diversifiable and nondiversifiable risk. Only the latter gives rise to a risk premium.

where f_t denotes the logarithm of the forward rate.[5] Although the risk premium is not explicitly modeled here, we interpret serial correlation in $\{\epsilon_t\}$ as providing indirect evidence of the existence of such a premium. Of course, the risk premium may be positive or negative and does not have to be stable over time. Thus, a failure to detect serial correlation in $\{\epsilon_t\}$ is consistent with the existence of a risk premium that is itself serially uncorrelated.

This test differs in an important way from tests for the randomness of exchange rate movements which have been carried out periodically since the original work of Poole [17].[6] The random-walk model of exchange rate behavior is usually justified on the grounds that "new information arrives randomly and independently of information received in the past" and speculators use this information to "bid the current price to a level equal to the expected future spot price."[7] But as (3) shows, this statement is true in a world of perfect asset substitutability only when interest rates are equal across countries so that *any* exchange rate changes are unanticipated. In a world where central banks pursue independent inflation targets, the random-walk test says nothing about market efficiency or asset substitutability, for it fails to account properly for the opportunity cost of holding wealth in the form of one currency rather than another.[8]

II. Data and Methods

The test of the Fisher hypothesis proposed above involves the sequence of random variables $\{\epsilon_t\}$ defined in (3), which, under the null hypothesis of Fisher parity, is just the sequence of unanticipated movements of the log of the exchange rate. To carry out this test, we use weekly observations on exchange rates and interest rates, so that ϵ_t is a one-week ahead forecast error when Fisher parity holds.[9] s_t is taken to be the natural logarithm of the U.S. dollar price of foreign currency, r_t the one-week yield on seven-day London Eurodollar deposits, and r_t^* the one-week yield on seven-day London Euro-deposits denominated in the foreign currency. Eurocurrency rates are employed because of the similar political risk characteristics of Eurocurrency deposits.

We test for Fisher parity between U.S. dollar deposits and deposits denominated in Canadian dollars, French francs, Deutsche marks, Netherlands guilders,

[5] See Cornell [3].

[6] See Upson [22], Burt, Kaen, and Booth [2], Cornell [3], and Cornell and Dietrich [4].

[7] Poole [17], p. 468.

[8] The random-walk theory of exchange-rate behavior has been stated both as a proposition concerning percentage changes in exchange rates (or, approximately, first differences of the *logarithms* of exchange rates) and as one concerning first differences of the *levels* of exchange rates. The model seems to originate in a misunderstanding of Samuelson's [21] martingale theorem, which concerns the sequence of expected values of a *single* future price, rather than the sequence of expected values of a sequence of future prices. Upson [22] provides an example of the confusion surrounding the empirical application of Samuelson's result. Rogoff [19] applies it correctly to the market for foreign-exchange futures.

[9] The advantage of using weekly rather than monthly data is that more observations are available. A disadvantage is that percentage transactions costs on one-week deposits may be large when considered on an annual basis. However, McCormick [15] provides evidence that transactions costs are much smaller than has been widely believed.

Swiss francs, and pounds sterling. Exchange rates are Friday closing bid rates, as reported by the Harris Bank (Chicago) in its *Weekly Review.*[10] Data on seven-day interest rates in the London Eurocurrency market come from the *Financial Times*, and are Friday closing bid rates as well.[11] The data run from 5 July, 1974 to 27 June, 1980, giving 312 nonoverlapping observations for each currency.

The first statistical test we perform on the series $\{\epsilon_t\}$ is the standard Q test for whiteness. The Q-statistic is defined as

$$Q(k) = n \sum_{i=1}^{k} \hat{c}_i^2$$

where $\hat{c}_i = \sum \epsilon_t \epsilon_{t-i} / \sum \epsilon_t^2$, n is the sample size and k is an increasing function of n. $Q(k)$ is asymptotically distributed as χ^2 with k degrees of freedom under the null hypothesis that the ϵ_t are serially uncorrelated.[12]

Geweke [7] has suggested a likelihood ratio test having greater asymptotic power against some types of serial correlation than does the Q test. For this second test, we assume that the disturbances ϵ_t are normal with mean zero under either hypothesis and are generated by the autoregressive model

$$\Lambda(L)\epsilon_t = \xi_t \tag{4}$$

where $\Lambda(L) = \sum_{i=0}^{\infty} \rho_i L^i$ is a polynomial in the lag operator, $\rho_0 = 1$ and $\{\xi_t\}$ is a white noise process. The null hypothesis is just the hypothesis that $\Lambda(L) = 1$.

To test this hypothesis, we assume that we may truncate the autoregressive process (4) at lag length k, where k is increasing in n, and test whether $\rho_1 = \rho_2 = \cdots = \rho_k = 0$ in the autoregression

$$\epsilon_t = \sum_{i=1}^{k} - \rho_i \epsilon_{t-i} + \xi_t \tag{5}$$

The appropriate test statistic is $n \cdot ln(\hat{\sigma}_\epsilon^2 / \hat{\sigma}_\xi^2)$, which is minus twice the logarithm of the likelihood ratio and is asymptotically distributed $\chi^2(k)$. $\hat{\sigma}_\epsilon^2$ is the estimated variance of the process $\{\epsilon_t\}$, while $\hat{\sigma}_\xi^2$ is the sample variance of the residuals obtained from estimating (5) by ordinary least squares. The likelihood ratio is large when past values of ϵ_t provide significant help in predicting future values; this leads to rejection of the null hypothesis.

If the null hypothesis is not rejected for an autoregression of order k, we test the hypothesis $\rho_k = 0$ using the standard normal test, and if this is not rejected, we again perform the likelihood-ratio test for an autoregression of order $k - 1$. We continue in this manner until the log likelihood ratio exceeds the critical value we have chosen. It is important to realize that this procedure results in a nested sequence of tests, and that the probability of committing a Type I error at each

[10] The series of Canadian exchange rates, however, is taken from the International Monetary Fund's *International Financial Statistics*.

[11] One-months rates were used in the few instances when one-week rates were not reported. The fact that interest rates and exchange rates, while observed on the same day of the week, are observed in different financial centers, gives rise to an alignment problem that may bias the tests.

[12] See Box and Pierce [1].

stage must be calculated accordingly. If the critical value for each normal test on individual coefficients is chosen so that its significance level would be α if that test were performed in isolation, and if the critical value for the likelihood ratio tests is chosen so that the probability of Type I error is β, then, under the null hypothesis, the probability of Type I error when one rejects at the j th stage is $1 - (1 - \beta)(1 - \alpha)^{j-1}$. In practice, we take $\alpha = .001$ and $\beta = .025$.

III. Empirical Results

Table I reports the results of the Q test for serial correlation. The statistics are computed for 16 lags, and therefore are distributed asymptotically as $\chi^2(16)$.

Fisher parity was rejected at the 5 percent level in all cases save that of the pound sterling. In the cases of Canada, France, and the Netherlands, rejection at the 1 percent level was possible. The high serial correlation in the series of ex post nominal return differentials suggests that they are not simply series of forecast errors, as they would be under conditions of perfect asset substitutability.

The outcomes of the maximum likelihood tests appear in Table II. For all countries, the nested sequence of tests began with a 16-week autoregression. In all cases except that of the United Kingdom, rejection of the Fisher hypothesis at the 2.5 percent level occurred at once. Since no nesting of hypotheses took place, the probability of Type I error for the test procedure as a whole was also .025 for these currencies.

Immediate rejection at the 2.5 percent level was not possible for the United Kingdom, but occurred only after the imposition of zero constraints on the original sixteenth-order autoregression. Although the Fisher hypothesis was eventually rejected for the pound, the probability of Type I error was .035, somewhat higher than for the other currencies.

As indicated by the earlier Q test, the evidence against the hypothesis of perfect

Table I

Results of the Q Test

Test Statistic	U.S./Canada	U.S./France	U.S./Germany	U.S./Netherlands	U.S./Switzerland	U.S./U.K.
$\chi^2(16)$	38.87	33.77	28.10	40.75	27.28	25.97[a]

[a] Not significant at the 5 percent significance level. The critical value for tests at the 5 percent level is 26.3; the critical value for tests at the 1 percent level is 32.0.

Table II

Results of the Likelihood-Ratio Test

	U.S./Canada	U.S./France	U.S./Germany	U.S./Netherlands	U.S./Switzerland	U.S./U.K.
Order of χ^2 statistic	16	16	16	16	16	6
Value of χ^2 statistic	29.18	31.97	29.36	37.78	30.26	15.02
Critical value	28.8	28.8	28.8	28.8	28.8	14.4
Significance level of test	.025	.025	.025	.025	.025	.035

asset substitutability is weakest in the case of the United Kingdom. But even so, the significance level of 3.5 percent is quite low by conventional standards.

To summarize, the two statistical procedures we have employed yield essentially the same conclusions. Both suggest that the Fisher-parity relationship does not hold. Deviations from Fisher parity appear to be highly autocorrelated and so do not behave like expectational errors. The observed behavior of the series of deviations may be interpreted as evidence favoring the existence of a foreign exchange risk premium for most major currencies. These findings lend support to recent theories suggesting that foreign exchange market efficiency is consistent with the existence of risk premia at equilibrium.

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