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Resolving the Agency Problems of External Capital through Options

ROBERT A. HAUGEN* and LEMMA W. SENBET**

ABSTRACT

This paper investigates the role of stock options in resolving the agency problems of external capital as originally identified by Jensen and Meckling (1976). These problems are precipitated by managerial incentives a) to consume excessive non-pecuniary benefits or perquisites beyond the optimal level for sole ownership and b) to engage in risk shifting in productive decisions so as to transfer wealth from external capital contributors. These incentive problems can be resolved through a strategy that judiciously combines call and put options retained by the owner-manager and external financiers, respectively. The resolution of the agency problems through this mechanism provides an economic rationale for the existence of managerial stock options and convertible debt.

THE WORK BY JENSEN and Meckling [7] has brought more realism to the theory of the firm. In their paper JM address agency problems in the context of a firm which is viewed as a nexus of contracts among various factors of production. The contractual relationships involve incentive conflicts arising from the pursuit of self-interest. By limiting themselves to an entrepreneur or owner-manager who retains complete control of the firm, JM have demonstrated that financing through the issuance of common stock or debt to outsiders engenders costs. These costs are precipitated in part by the manager's propensity to consume nonpecuniary benefits or perquisites (perks) by employing certain resources in excess of their optimal usage,[1] and by his propensity to engage in high risk investment projects so as to transfer wealth from bondholders to stockholders.[2]

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[1] The most obvious example of a "perk" of this type is the expansion of the owner manager's span of control of the management of the firm beyond what is optimal. The manager may value the social prestige and power accompanying his position as chief executive officer. He may be hesitant to delegate authority even when it is in the best interests of the firm to do so. Indeed, it is likely that he would be unwilling to replace himself as chief executive officer even when other managers are available who are capable of running the firm more efficiently at a similar or lower salary.

[2] Fama [3] examines the case of the corporation with diffuse ownership. The manager retains no control of the firm and his money wages are not fixed. If all markets are well-functioning and information sets are symmetric, the manager faces competition from other managers and he is

It is our purpose in this paper to examine the role of stock options in reducing the agency costs associated with raising capital externally. We show that if the entrepreneur takes positions in options to purchase and/or sell the firm at a specific price, the options play a vital role in resolving the agency problem that exists between the manager and the outside capital contributors. The paper may also constitute an important step in providing a theoretical basis for the existence of managerial stock options and convertible debt.

I. The Impact of Stock Options on the Agency Costs Associated with Perk Consumption

Agency costs arise in part because the owner-manager is unable to guarantee outside securityholders[3] that he will consume no more perquisites as a fractional owner than as a sole owner. While monitoring and bonding activities can alter the manager's opportunity to capture increased perks, it is unlikely that they would costlessly eliminate the agency problems associated with external financing.

In this section we demonstrate that stock options can align the interests of the owner manager with outside equityholders.[4] If the capital market is competitive and characterized by rational behavior, the total benefits stemming from reduced consumption of perks are capitalized into the prices of the financial claims issued to outsiders. In fact, the financial package can be designed so as to make the value of the firm revert to that value characterizing the solution for the sole owner-manager.[5] We shall at first address the perk consumption problem separately from the risk incentive problem. Then we look at the joint impact of the stock options on both problems. We should emphasize that it is the combined put and call strategy which leads to the final resolution of the agency problems, because these options engender *offsetting* agency costs associated with risk incentives.

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disciplined by the labor market. In this framework, Fama argues that the pricing of securities is independent of perk consumption. This viewpoint is well recognized by *JM* themselves (pp. 328–29). Their framework still applies, however, to those cases where the manager retains sufficient control of the firm. Examples of these firms are abundant (Getty Oil, Polaroid, Wrigley, etc.) and constitute an important segment of American industry.

[3] As recognized by *JM* (Fn. 6) this problem is associated with the issuance of risky debt as well as the issuance of common stock.

[4] It should be noted that if the manager is to retain options as well as common shares in the firm some guarantee must be provided that he will *retain* these securities at least until the relevant decisions are made and hence maintain an alliance with the interests of outside capital contributors. It should be stressed that the very same problem exists within the original *JM* framework. That is, some guarantee must be made to outside stockholders that the owner-manager will not dilute his interests in the firm or neutralize his position with a portfolio adjustment.

[5] While stock options may be used to resolve the agency problem associated with perk consumption, they would create an incentive to take high or low variance projects, thereby engendering other agency costs. However, in Section III we demonstrate how a combined call and put option strategy can be employed in order to resolve the managerial incentive to take or avoid risk.

A. The Use of a Call Option

Suppose that the terms of the settlement for external financing provide an external option for the manager to buy back (at the termination of the production period) the entire firm at an exercise value, E_C.[6] The option constitutes a part of the package of securities retained by the owner manager. The remaining part is his fractional interest in the common stock of the firm.[7] The impact of this option is that the cost to the manager of consuming \$1 of perks is no longer \$$\alpha$, where α is the fractional ownership interest held by the manager. The cost rises by the slope of the option pricing curve. The manager's incentive to consume perks is reduced accordingly. Figure 1 demonstrates this phenomenon.

The left-hand side of Figure 1 is similar to that of the original JM analysis. Line $\bar{V}\bar{F}$ depicts the total value of the firm as a function of perquisite consumption F. This line represents the relationship between dollar wealth and perquisite consumption for the manager as a sole owner. The manager, as sole owner, maximizes utility at the tangency between this wealth line and his indifference curve U_1. The manager consumes perquisites in amount F^* and the firm is valued at V^* for the optimal solution.

As JM show, if the manager issues common stock alone in order to raise an amount of external capital equal to V_S, his wealth line reverts to that of V_3Q, where the slope of this line is equal to the negative of his fractional interest in the firm. The manager now optimizes at the tangency with U_2, and he suffers a loss of utility.

Suppose, however, that the manager also retains a call option whose value (V_C) is plotted along the curve on the right-hand side of Figure 1 as a function of the value of the firm. When the manager's wealth line is augmented by the value of this option it becomes V_5Q. Note that given this package, outside capital contributors recognize that it is the manager's self interest to return to V^*F^*, since his utility will be restored.

The slope of the augmented wealth line, V_5Q, in Figure 1 is $-\left[\alpha + \dfrac{\partial^V C}{\partial V}\right]$. In general, given the option retained by the owner-manager, the market value for all securities (the manager's ownership interest as well as the option) is determined at the point where the owner-manager's indifference curve is tangent to the augmented wealth line. In order to make the value of the firm revert to that of sole ownership, the securities retained by the owner-manager should guarantee

[6] As implied in the *JM* paper (pp. 352–53), the owner-manager can bind himself contractually to hold a particular package of the securities of the firm at little or no cost. It should be stressed that the owner-manager in our framework retains an option in its true sense and *not* a warrant. In other words, the option is not valued as part of the total value of the firm, and hence it is *external* to the firm. It is not possible to *completely* control the manager through a warrant, although in the absence of agency problems and other imperfections, internal and external options must be a matter of indifference by virtue of the Modigliani-Miller irrelevance theorem.

[7] The requirement for the owner-manager to obtain the liquidity necessary to exercise his option at the end of the single period envisioned by the model may be considered as the direct analog to the requirement that the owner-manager obtain the liquidity necessary to pay the principal associated with any debt that is issued in order to finance the firm.

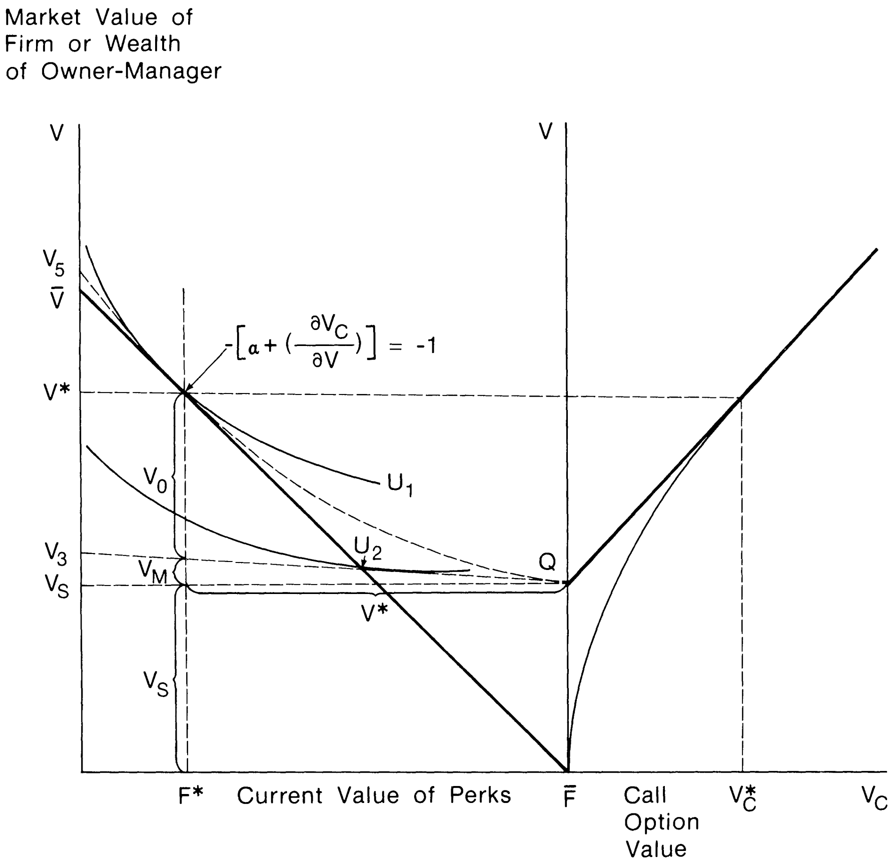

Figure 1.

to outside securityholders that he will consume no more perks as a fractional owner than as a full owner.[8] This means that the tangency of the indifference curve to the augmented wealth line must occur along $\bar{V}\bar{F}$, the value line, and at a point (V^*, F^*). This is feasible only if the augmented wealth line itself is tangent to $\bar{V}\bar{F}$ at that same point. Since $\bar{V}\bar{F}$ has a slope of -1, the augmented wealth line must also have the slope of -1 at the tangency point. These phenomena are again exhibited in Figure 1.[9]

In order to determine the nature of the call option which generates the

[8] It should be noted that this particular financial package is conceptually equivalent to the issuance of risky debt by the owner-manager where he holds all the common stock of the firm and a fraction α of the debt securities.

[9] It is conceivable that the augmented wealth line V_5Q may cross the indifference curve U_1 between the tangency point and the intersection at V_5. This depends on the relative convexity of V_5Q and U_1. If the problem exists it can be overcome either by a put option or by combined use of a call and put option, as explained in Sections *IB* and *II*. A sufficient (but not necessary) condition for the value of an option to be a convex function of the stock price is that the distribution of the returns on the stock is independent of the price level.

Our analysis does not invoke any particular option pricing model other than the convexity requirement. For instance, if the Black-Scholes model is deemed appropriate to value the options augmented wealth line, V_5Q, consider the following expressions for the lines of Figure 1. The value line, $\bar{V}\bar{F}$, can be expressed as

$$V = \bar{V} - F \tag{1}$$

the original wealth line can take the form of

$$V_W = V_S + \alpha(\bar{F} - F) = V_S + \alpha V \tag{2}$$

where

V_W = the value of the manager's wealth.

Equation (2) can be augmented by the option price line so as to generate the augmented wealth line

$$V_W = V_S + \alpha V + V_C \tag{3}$$

where

V_C = the value of the call option.[10]

The total value of the firm is still the sum of the outside and the inside ownership interests, although it can alternatively be expressed as the sum of the option value (V_C), the dollar amount provided by outsiders (V_S), and the manager's ownership interest (αV). The zero agency cost solution requires that the point (V^*, F^*) lie on the augmented wealth line, and hence the following conditions must be satisfied. First, the total wealth of the manager, including the payment from outsiders, must equal V^*.

$$\text{(a)} \quad V_W = V_S + \alpha V^* + V_C = V^*$$

or

$$V_C = (1 - \alpha) V^* - V_S \tag{4}$$

and, second, the slope of the wealth line with respect to V must equal 1 at V^*,

$$\text{(b)} \quad \left.\frac{\partial V_W}{\partial V}\right|_{V^*} = \alpha + \left.\frac{\partial V_C}{\partial V}\right|_{V^*} = 1 \tag{5}$$

At the point of the zero agency cost solution, and only at that point, the value of a package of securities held by outsiders is *insensitive* to an *instantaneous* change in the value of the firm as can be readily inferred from the requirement

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employed in this paper, the analysis is still intact. However, there are complications with the Black-Scholes model if the firm value is dependent on its capital structure. If so, a more general option pricing function is appropriate for which our analysis still holds.

[10] The call option is presumed to be valued by the owner manager at its market value. This is consistent with the original assumption that the owner manager considers the assets at market value in making production and financing decisions. This is also in accord with the implication in the *JM* paper (pp. 352–53) in which the owner-manager is bound to hold his equity shares but that they are presumed to be valued at market (see also fn. 6). However, the analytical procedure in this paper can easily account for any adjustment in market value which reflects the manager's welfare loss in determining the true value of the package of securities he is bound to hold. This is because, in accordance with *JM*, it is assumed here that the manager's utility function is known to the market.

in (5). This package consists of the outside ownership interests and a short position in a call option, which may also be viewed as a debt claim. Although the value of this package is insensitive to an instantaneous change in the value of the firm at the zero agency cost solution point, it is not riskless in the sense that outside financiers are guaranteed the delivery of their claims at termination. For it to be riskless, the terminal value of the firm must exceed the face value of the claims under all states of nature. This calls for the truncation of the terminal value distribution at the face value. Indeed, if the amount of outside financing needed by the entrepreneur (V_S) exceeds the discounted terminal value of the firm at the truncation point, then the entrepreneur is obviously unable to raise the necessary funds unless he resorts to risk-bearing financing which certainly leads to an agency problem.

Thus, the package held by outside financiers is neither riskless in the sense discussed above nor a riskless hedge in the parlance of the Black-Scholes model. For it to be a riskless hedge, the positions must be continuously readjusted as the value of the firm changes randomly in continuous time. Our agency cost solution does not call for such continuous readjustment of positions by the outsiders once the investment-financing decision is made at the beginning of the production period. We are dealing with the Jensen-Meckling world in which decisions relating to investment, financing, and the nature of the productive process are made simultaneously at the beginning of the period and are not altered throughout the productive period. Our solution is also *generalizable to* a multiperiod framework where discrete time readjustments in contractual provisions are *feasible.* In any case, ··· given the relationship between the beginning period market value of the financial package by the entrepreneur and the beginning period market value of the firm, it is in the entrepreneur's interest to set up the productive process so that the market value of the firm is V^*. Any deviation from this value reduces his utility. Thus, the entrepreneur cannot raise the necessary funds with a riskless claim but nothing precludes him from offering the package specified in our solution. With this package he can raise the necessary funds and resolve the agency problem.

The requirement in (5) is met when $\frac{\partial V_C}{\partial V} = (1 - \alpha)$. Substituting this into (4) and solving for $\frac{\partial V_C}{\partial V}$ in terms of V_C/V^* we obtain a relationship between the slope of the option pricing line and the value of the option as a proportion of V^*.

$$\frac{\partial V_C}{\partial V} = \frac{V_S}{V^*} + \frac{V_C}{V^*} \tag{6}$$

Equation (6) provides the *zero cost constraint,* or the relationship between the call option's value and its response to a change in the value of the firm, required to restore the value of the firm to V^*. In Section II we address the question of the design of such an option, given some appropriate option pricing function.

B. The Use of a Put Option

The entrepreneur can also employ an *external* put option to align his interests with those of outside securityholders. Outsiders hold a put option to sell the

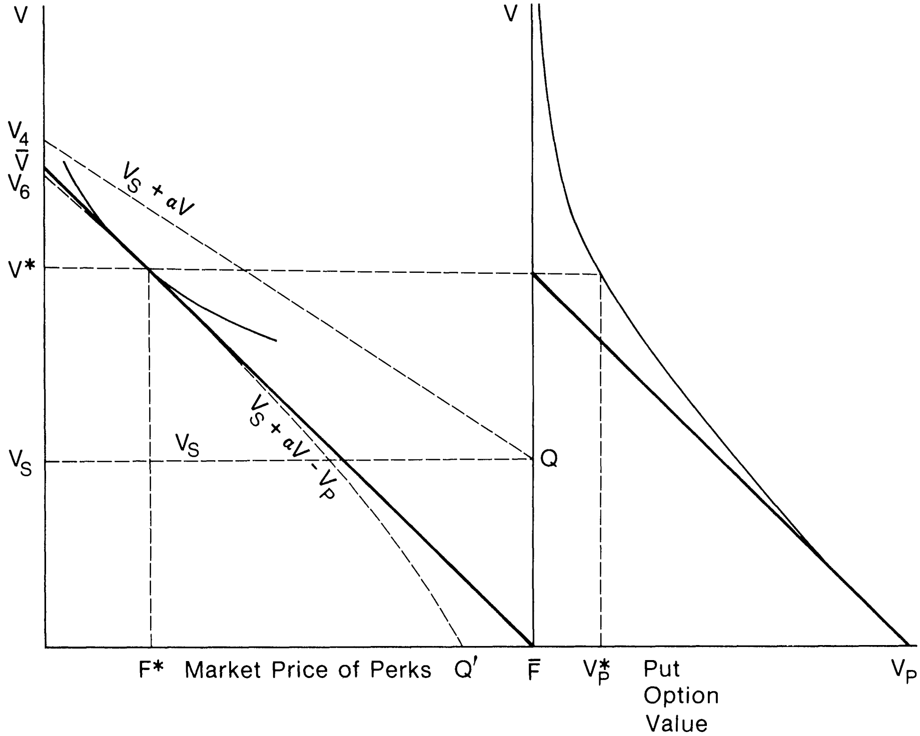

Figure 2.

entire firm at a stated price, E_p, to the owner manager at the termination of the period. The put option eliminates the owner-manager's incentive to consume perks, because perk consumption increases the manager's liability to outsiders.

Since the put option accords an additional privilege to outsiders, they pay for it in cash so that $V_S = (1 - \alpha)V + V_p$ where V_p is the value of the put option.[11] The wealth line for the manager can now be written as:

$$V_W = V_S + (1 - \alpha)V - V_p \tag{7}$$

As the entrepreneur decreases his perk consumption, the liability imposed on him by the put option decreases. The original wealth line is augmented by the value of the put option which reflects different levels of perk consumption. The augmented wealth line is labeled as V_6Q' in Figure 2. Unlike the call option, the put option augments the wealth line such that the line is now *concave* to the origin. Thus, the convexity problem (see fn. 9) which arises with the call option disappears, and again the zero agency cost solution is achieved at a point of tangency between the value line and the augmented wealth line at V^*.

At the zero agency cost solution the following two conditions must again be

[11] It should be noted that this financial package is analogous to issuing a convertible bond to outsiders. If the value of the firm exceeds the exercise price on the put option, the option is worthless, but outsiders remain as common stockholders with partial interests in the firm. This is analogous to conversion into common stock. If the value of the firm falls below the exercise price, the put is exercised and the outsiders sell the entire firm to the owner-manager at the exercise price. This event is analogous to the convertible bondholders failing to convert and exercising their fixed claim.

satisfied:

$$\text{(a)}\ V_W = V_S + \alpha V^* - V_P = V^*$$

or

$$V_P = V_S + (\alpha - 1)V^* \tag{8}$$

and

$$\text{(b)}\ \left.\frac{\partial V_W}{\partial V}\right|_{V^*} = 1 = \alpha - \left.\frac{\partial V_P}{\partial V}\right|_{V^*}$$

or

$$\left.\frac{\partial V_P}{\partial V}\right|_{V^*} = \alpha - 1 \tag{9}$$

If we combine the two conditions, we obtain the *zero cost constraint* for the put option:

$$-\frac{\partial V_P}{\partial V} = \frac{V_S}{V^*} - \frac{V_P}{V^*} \tag{10}$$

As a caveat, we wish to point out that in valuing the put option the possibility of default in payment of the exercise price must be considered in the option valuation function.

II. The Option Pricing Constraint

The zero cost constraint, Equation (6), for the *call option* is shown in Figure 3 with an intercept of V_S/V^*, the proportion of the total value of the firm that must be financed through outside equity, and a slope equal to unity. Note that the scales are bounded by 0 to 1 on both the vertical and horizontal axes.

The values $\partial V_C/\partial V$ and V_C/V^* must also satisfy the constraint imposed by the nature of an appropriate option pricing function. Given some assumed option pricing function, the relative values of $\partial V_C/\partial V$ and V_C/V^* change with the exercise price, given the expiration date for the option, the current market value for the firm, the distribution for the terminal value of the firm, and the appropriate rate of opportunity cost. This is easily demonstrated by considering the special case where the terminal value of the firm is known with certainty. If V^* exceeds the present value of the exercise price, the option is certain to be exercised and hence $\partial V_C/\partial V = 1$. At the other extreme, when the present value of the exercise price exceeds the current value of the firm, the option is worthless. Accordingly $\partial V_C/\partial V = 0$. We can generalize this simple situation by specifying the option value as $V_C = \text{Max}(0, V^* - E_C e^{-rt})$, where E_C is the exercise price, r is the riskless rate of interest and t is the length of time to expiration. If $E_C e^{-rt} > V^*$, the relative values of V_C/V and $\partial V_C/\partial V$ are zero; and we are at a point 0. One moves to point E if $E_C e^{-rt} = V^*$, and to midpoint B if $E_C e^{-rt} = ½V^*$. Thus, as the exercise price E_C (and hence its discounted value) approaches zero the relative values of $\partial V_C/\partial V$ and V_C/V^* also approach point C. Consider the case where the

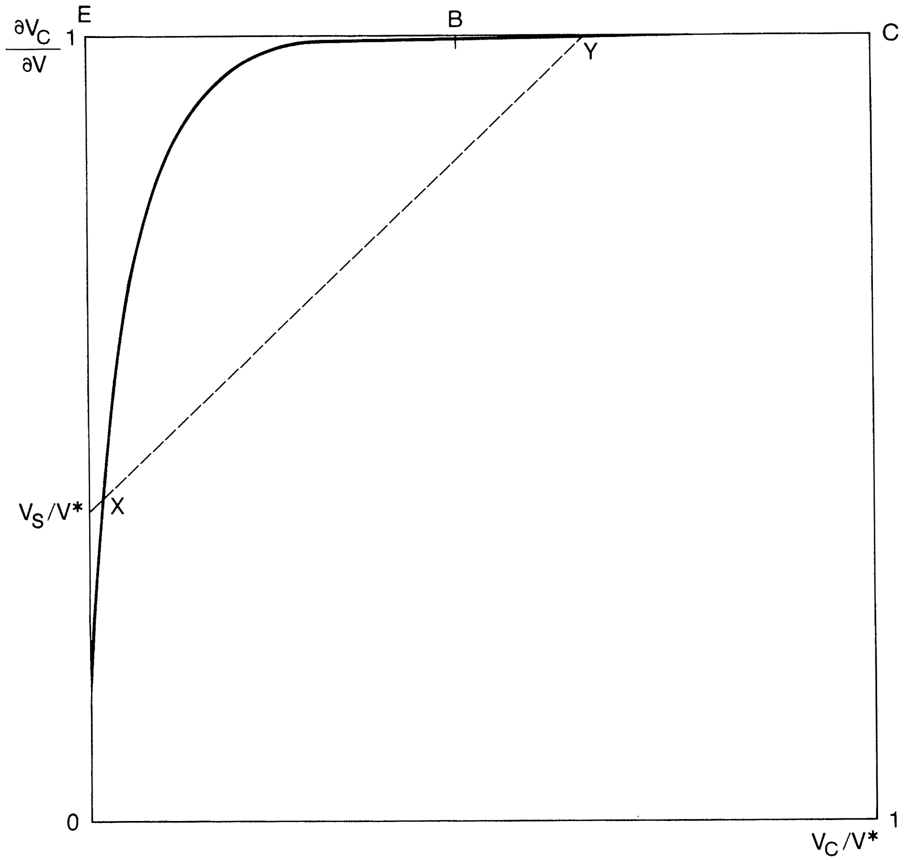

Figure 3.

terminal value is not known with certainty. A nonlinear curve such as OXYC emerges. The precise nature of this curve is determined by the nature of the distribution for the terminal value of the firm and the assumed option pricing function.[12] In any case holding all other variables constant, as the exercise price increases from zero and approaches infinity, we move along the curve from C toward 0. We refer to this curve as the *option pricing constraint.*

Given a concave function such as that depicted in Figure 3, the zero cost constraint intersects the option pricing constraint at two points, X and Y. The solutions at X and Y satisfy two conditions: (a) they restore the value of the firm to V^*; and (b) they assure that, given the assumed option pricing function, an option exists with the required relationship between its value relative to V^* and $\partial V_C/\partial V$.[13]

[12] The curve depicted in Figure 3 happens to hold for the option pricing function as defined by Black and Scholes [1], where the exercise period is 1, $\sigma = .1$ and $r = .1$.

[13] Note that we are assured of an intersection in Figure 3 only up to some maximum value of V_S/V^* which in general increases as the variance of the returns to the firm's assets decreases. For the case of a put option, however, we are assured of an intersection irrespective of the value of V_S/V^* [see Equation (10)].

The zero cost constraint for the put option can also be represented in Figure 3 if the vertical axis is taken to be $-\frac{\partial V_p}{\partial V}$. The zero cost constraint for the put option is a linear function intercepting at V_S/V^* with slope equal to minus one. For the put option, there exists a single simultaneous solution to the zero cost and option pricing constraints.

The put and call options can, of course, be used in a combined strategy with the entrepreneur taking a positive position in the call and a negative position in the put. The zero cost constraint, under such a strategy, is given by

$$\frac{\partial V_C}{\partial V} - \frac{\partial V_p}{\partial V} = \frac{V_S}{V^*} + \frac{V_C}{V^*} - \frac{V_p}{V^*}$$

Figure 4 depicts feasible sets of combinations of put and call values which satisfy the zero cost constraint. The graph assumes a fixed financing requirement or a given value for V_S/V^*. Each of the parallel 45° lines shows the combined values of V_P/V^* and V_C/V^* which satisfy the zero cost constraint of Equation (11), given V_S/V^* and a particular value for the net derivative, $\partial VC/\partial V - \partial V_P/\partial V$. As one moves to the southeast, these lines represent increasingly lower values for $(\partial V_C/\partial V - \partial V_P/\partial V)$. Thus in the figure $K_2 < K_1$. Each of the broken curves in the figure depicts combined values of the put and call options which satisfy the option pricing constraint. That is, given an appropriate option pricing function and a particular value for $\partial V_C/\partial V - \partial V_P/\partial V$, each curve designates the locus of points for put and call values, pairs of which correspond to the value for the net derivative. Consider, for example, curve *EBF*, which is presumed to represent the same value for $(\partial V_C/\partial V - \partial V_P/\partial V = K_1)$ as does line *ABD*. Movement from point *E* toward point *F* is consistent with increasing the exercise prices for both options, thus decreasing the price of the call and increasing the price of the put. Also as we move along *EBF* the partial derivatives of *both* options fall so as to maintain the net difference at a constant level. These properties may not necessarily hold for any arbitrary option pricing model, but it is sufficient to assume that the underlying return distribution is independent of its price level following Merton [8]. Under this assumption the option price is (a) a convex function of the underlying asset value; and (b) homogeneous of degree one in the asset price and the exercise price. This ensures that the partial derivatives of both the put and the call options fall as the exercise prices are increased.[14] Curves which are closer to the origin are presumed to represent smaller values for the net derivative.

Now consider the intersection of the curves *EBF* with line *ABD* at point *B*. At this point both the zero cost and option pricing constraints are satisfied for a

[14] If the option price is homogeneous of degree one in the firm value and the exercise price, we have

$$V_C = \frac{\partial V_C}{\partial V} V + \frac{\partial V_C}{\partial E_C} E_C$$

Accordingly,

$$\frac{\partial V_C}{\partial E_C} = \frac{\partial^2 V_C}{\partial V \partial E_C} V + \frac{\partial V_C}{\partial E_C} + \frac{\partial^2 V_C}{\partial E_C^2} E_C$$

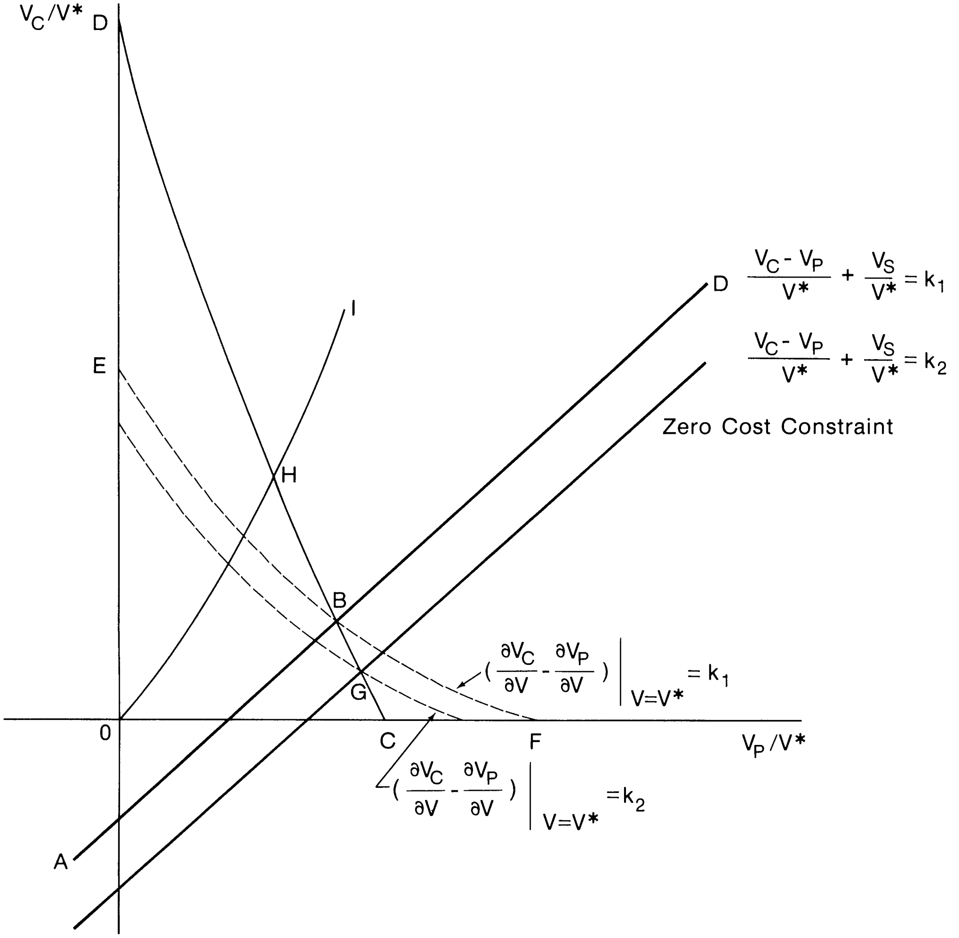

Figure 4.

given value for the net derivative. The intersection at point G satisfies both constraints at a lower value for $(\partial V_C/\partial V - \partial V_P/\partial V)$. Curve $DBGC$ represents the locus of these points of intersection and provides the combinations of call and put values which satisfy both constraints, as the value of the net derivative is varied.

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so that

$$\frac{\partial^2 V_C}{\partial E_C \partial V} = -\frac{E}{V}\frac{\partial^2 V_C}{\partial E_C^2} < 0$$

Thus, the partial derivative of the call option price with respect to the firm value falls as the exercise price increases. Similarly $\partial V_P/\partial V$ must decrease with E_P, since the put is a complement of the call. The reader should note that the above properties hold for the Black-Scholes model as a special case so that the partial derivates of the call option price with respect to firm value and the exercise price can be replaced by $N(d_1)$ and $e^{-rt}N(d_2)$. In the parlance of this model $\partial N(d_1)/\partial E_C < 0$, and intuitively, since the probability of exercise is a positive function of $N(d_1)$, it decreases with an increase in the exercise price.

III. The Effect of Stock Options on the Incentive to Take or Avoid Risk

While the issuance of stock options may be used to resolve the agency problem associated with the consumption of perquisites, there may remain an incentive for the manager to engage in either high or low risk investment programs. This is the well-known wealth transfer problem associated with the existence of risky debt in the capital structure. With the issuance of a call option to buy the entire firm, the equity capital of the firm is conceptually equivalent to risky debt. The wealth transfer problem is analyzed in detail by JM who view stockholders in a levered firm as holding a European call option to buy back the entire firm at an exercise price equal to the face value of the debt. Since the value of this call option is an increasing function of the variance of the cash flows of the underlying asset (the firm), stockholders have incentive to engage in high risk activities at the expense of the debtholders.

In this section we examine the conditions under which the issuance of a financial package containing a put and a call option can resolve the agency cost associated with the risk incentive problem. Assume that the manager is financing investment opportunity A, but that an alternative opportunity B is available where $\sigma_B > \sigma_A$ and where $\Delta V = (V_B - V_A) < 0$. The investment outlay is assumed to be equal for both opportunities. He attempts to finance by issuing the financial package discussed in Section I, the package employing combined used of a put and call option. Because of the greater variance of project B the value of the manager's call option is greater if this project is adopted. Outsiders will recognize this and may price the financial package offered them with the presumption that the lower valued project, B, will be adopted. Thus, the cost associated with the risk incentive problem is entirely borne by the owner-manager. However, this would be the case only if the entrepreneur is unable to provide a contract that neutralizes his risk incentive. We shall show that the same put and call option strategy which resolves the perk consumption problem has the potential to neutralize the risk incentive problem.

In the presence of the risk incentive problem, if the manager shifts to high risk (but low value) project B, the value of the call option may increase if the positive effect of the increase in the variance offsets the negative effect of the decline on the value of the underlying asset. Note that the increase in the value of the call is, in fact, the essence of the problem. It may give the manager an incentive to shift into the low value project. Note also, however, that if he makes the shift, the value of the put option must increase. This is due to the fact that both the increase in variance and the decrease in the value of the firm enhances its value. Since, in the presence of the problem, both options increase in value, their exercise prices can be designed so that the change in prices are equal and therefore offset in the managers' portfolio. The manager has a positive position in the call and a negative position in the put. Consider first the extreme case where $E_P = 0$, E_C approaches infinity, and hence $V_C = V_P = \Delta V_C = \Delta V_P = 0$. Increasing the exercise price on the put and lowering the exercise price on the call by some sufficient amount may result in positive values for both options and the condition $\Delta V_C = \Delta V_P > 0$. In fact there exists combinations of E_C and E_P values which generate the locus of points OHI in Figure 4, whereby for any point on this locus $\Delta V_C = \Delta V_P$. As one moves up *OHI*, the exercise price increases for the put and decreases for the call forcing the values of both options upward as a percent of the value of the firm but maintaining the condition $\Delta V_C = \Delta V_P$.

At some E_C, ΔV_C reaches a maximum that may fall short of the maximum ΔV_P as a function of E_P. This situation is represented by point *I* on curve *OHI*. Notice again that this is because ΔV_P must continue to increase with increasing E_P when the entrepreneur shifts to high variance (but low value) investment opportunity, but ΔV_C increases with decreasing E_C only for some range and reaches its maximum at some point. (For instance, if $E_C = 0$, V_C *decreases* to the extent of the decrease in the underlying asset value.) The maximum point represents the highest put option value that is consistent with satisfying the zero risk incentive constraint. That is, the ΔV_P consistent with this value is equal to the maximum ΔV_C over all E_C.

One of the positions on *OHI* may also satisfy the zero perk constraint. This occurs at the intersection of *OHI* and *DBGC* at *H*. This point represents the zero agency cost solution which restores the value of the firm to V^* while providing no incentive to either increase or decrease the variance of its underlying distribution. The reader should note, however, that under some circumstances a point of intersection may not exist because *OHI* may bend backward before intersection with *DBGC*.[15] In this case options offer no simultaneous solution to the problems. However, in the next section we show that for at least one feasible option pricing function, a simultaneous solution does in fact exist.[16]

IV. The Zero Agency Cost Solution for a Particular Option Pricing Function: An Illustration

We shall illustrate a zero agency cost solution by using a particular pricing function for the call and put options. The solution must satisfy three conditions: (a) the manager consumes no excess perquisites beyond what is optimal for the sole owner [the perk constraint]; (b) the total wealth of the manager, including receipts from outsiders, must sum up to total firm value [the budget constraint];

[15] We use the term "variance" to signify an index of increased uncertainty in the sense of Merton [8, Theorem 8] where he proves that the more uncertain are the outcomes on the underlying asset, the more valuable is the option. A special case of this is the Black-Scholes world in which the variance in its traditional notion is a measure of increased uncertainty.

[16] It should be emphasized that our solution is not costless, unless the agent is risk neutral, because he is forced to hold a package of securities that may be suboptimal in terms of diversification. If he can identify another firm or portfolio of firms that is highly and positively correlated with his own firm, however, he can substantially reduce the risk associated with random fluctuations in the value of his firm by taking a short position in the other firm or portfolio. This is true even if he is constrained with respect to the investment of the proceeds of the short sale. Note that this opportunity does not mitigate the power of the option contract to neutralize the manager's incentives to consume perquisites and take risk, for it can be safely assumed that the manager's decisions in this respect have no impact on the value of his short position. The manager can also use the proceeds, V_s, to further diversify his holdings. Neither our solution nor the original exposition by Jensen and Meckling, however, *explicitly* addresses the costs associated with suboptimal risk sharing on the part of the entrepreneur. For a more explicit discussion of this issue in the context of the economic theory of agency, see Harris and Raviv [5], Shavell [9], and Holmström [6], although they ignore *market* opportunities for risk diversification.

(c) there exists no incentive for the manager to avoid value maximizing projects in preference for high (low) variance (but suboptimal) investment opportunities [the risk incentive constraint]. The objective is to solve for three variables which satisfy the preceding conditions. The three variables are (a) the manager's fractional ownership interest in the firm (α); (b) the exercise price (E_C) for the call option retained by the manager; and (c) the exercise price (E_P) for the put option retained by external capital contributors.

In deriving the pricing functions for the call and put, we employ two simplifying assumptions. First, the capital market is characterized by investors who are risk-neutral so that the expected return on all assets is equal to the riskless rate of interest. Second, the terminal value of the firm is uniformly distributed over the interval $[V_1, V_2]$, where V_2 is the limiting value in the uniform distribution. Thus, the values of the (European) call and put can be specified as:

$$V_C = \phi^{-1}\left[\frac{1}{R}\int_{E_C}^{V_2} (X - E_C)\, dX\right] = \frac{(V_2 - E_C)^2}{2\phi R} \tag{12}$$

$$V_P = \phi^{-1}\left[\frac{1}{R}\int_{V_1}^{E_P} (E_P - X)\, dX\right] = \frac{(E_P - V_1)^2}{2\phi R} \tag{13}$$

where

ϕ = one plus the riskless rate of interest
X = a variable of integration for the terminal value of the firm
$R = V_2 - V_1$; $\frac{1}{R}$ = the density function which is constant for a uniform distribution

The options are assumed to be European-type with a single period to expiration.

The above relations in (12) and (13) obtain when E_C and E_P lie within the interval $[V_1, V_2]$. When the exercise prices lie outside the interval, the values of the call and the put collapse to:

$$V_C = \begin{cases} V - \phi^{-1}E_C & \text{if } E_C < V_1 \\ 0 & \text{if } E_C > V_2 \end{cases}$$

$$V_P = \begin{cases} 0 & \text{if } E_P < V_1 \\ \phi^{-1}E_P - V & \text{if } E_P > V_2 \end{cases}$$

where

$$V = \phi^{-1}\left(\frac{V_1 + V_2}{2}\right) = \text{the current value of the firm}$$

Thus, exercise prices outside the interval imply either worthless options or options that are sure to be exercised. However, neither of these is feasible for the zero agency cost solution. For instance, a call option to buy back the entire firm at a specific price, which is sure to be exercised, implies that the external capital is completely riskless, and there exists no agency problem with riskless external financing. Hereafter, we seek solutions which lead to the exercise prices that lie in the interval $[V_1, V_2]$, and hence utilize the expressions in (12) and (13) for our analytical framework.

It is instructive to recast the call and put option values in terms of V, the current value of the firm, so that

$$V_C = \frac{(\phi V + R/2 - E_C)^2}{2\phi R} \tag{14}$$

$$V_P = \frac{(E_P + R/2 - \phi V)^2}{2\phi R} \tag{15}$$

In deriving (14) and (15) we have employed the fact that $R = V_2 - V_1 = 2(\phi V - V_1)$. The sensitivity of the option values with respect to the underlying value (V) can be expressed as

$$\frac{\partial V_C}{\partial V} = \frac{2\phi V + R - 2E_C}{2R} \tag{16}$$

$$\frac{\partial V_P}{\partial V} = \frac{2\phi V - R - 2E_P}{2R} \tag{17}$$

Given that E_C and E_P lie within the interval $[V_1, V_2]$, it is easily seen that V_C is an increasing function of V, while V_P is a decreasing function of V. Indeed, the *RHS* of (16) collapses to $(V_2 - E_C)/R$, which is a positive quantity, whereas the *RHS* of (17) is $(V_1 - E_P)/R$, a negative amount. We shall employ these expressions interchangeably in subsequent analysis.

Similarly, we can express the sensitivity of the option values with respect to variance or standard deviation. The variance of the uniform variable in this case is $R^2/12$ which is merely the square of the range scaled by a constant. Consequently, without loss of generality, we employ R^2 or R as an index of dispersion. Thus, the sensitivity can be expressed as

$$\frac{\partial V_C}{\partial R} = \frac{1}{2\phi R^2}(R/2 + A)(R/2 - A) = \frac{R^2 - 4A^2}{8\phi R^2} \tag{18}$$

$$\frac{\partial V_P}{\partial R} = \frac{1}{2\phi R^2}(R/2 + D)(R/2 - D) = \frac{R^2 - 4D^2}{8\phi R^2} \tag{19}$$

where

$$A = \phi V - E_C$$

$$D = \phi V - E_P$$

Again it can be shown that (18) and (19) are both positive, given that E_C and E_P lie in the interval $[V_1, V_2]$.[17] Therefore, as expected, the values of the call and the put are increasing functions of R.

[17] Alternative expressions exist for (18) and (19) such that

$$\frac{\partial V_C}{\partial R} = \frac{(V_2 - E_C)(E_C - V_1)}{2\phi R^2} > 0 \tag{18a}$$

We can now express the constraints underlying the zero agency cost solution in terms of the specific option pricing functions derived above.

The Perk Constraint

$$\alpha + \left.\frac{\partial V_C}{\partial V}\right|_{V^*} - \left.\frac{\partial V_P}{\partial V}\right|_{V^*} = \alpha + \frac{(V_2 - E_C)}{R} - \frac{(V_1 - E_P)}{R} = 1 \tag{20}$$

where

the partials are evaluated at $V^* = \bar{V} - F^*$ = the total firm value at the zero agency cost solution. For our purpose, $V^* = (V_1 + V_2)/2\phi$.

The perk constraint is intended to ensure that the manager consumes no excess perquisites. In other words, the cost to the manager of consuming a dollar of perks is entirely borne by him in the form of a commensurate change in his wealth (V_W) at a utility maximizing point (V^*, F^*) in Figure 1. The second order condition will depend on the manager's utility function, and we assume that the condition is satisfied.

The Budget Constraint

$$V_W = V_S + \alpha V^* + V_C(V^*) - V_P(V^*) = V^*$$

or

$$V_S + V^*(\alpha - 1) + \frac{(\phi V^* + R/2 - E_C)^2 - (E_P + R/2 - \phi V^*)^2}{2\phi R} = 0 \tag{21}$$

The managerial wealth, including receipts from external securityholders, must sum up to total firm value, V^*, at the zero agency cost solution. Notice that the options are evaluated at V^*.

The Risk Incentive Constraint

The call option retained by the manager creates an incentive for him to shift to high risk (and possibly suboptimal) investment opportunities. On the other hand, the put option retained by outside capital contributors creates a countervailing risk incentive. We wish to specify a constraint so that the marginal wealth changes due to changes in the call and put option values is zero when the manager shifts to an unwarranted risk. We assume that the investment opportunity set available to the entrepreneur is known to the market. This set can be mapped into its corresponding risk (R) so that we can specify the value of an investment opportunity as $V = f(R)$. There exists no cause and effect relationship between V and R, but we assume for simplicity a continuum of project values and their

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and

$$\frac{\partial V_P}{\partial R} = \frac{(E_P - V_1)(V_2 - E_P)}{2\phi R^2} > 0 \tag{19b}$$

These alternatives are employed in subsequent analysis whenever useful.

corresponding risks. Assume that the investment opportunities are arranged in an increasing order of R. If the manager shifts to an unwarranted risk, the option values change not only due to the change in R, but also due to the change in V. Thus, we can express the risk incentive constraint as:

$$\frac{dZ}{dR} = \left[\frac{\partial V_C}{\partial V} f'(R) + \frac{\partial V_C}{\partial R}\right] - \left[\frac{\partial V_P}{\partial V} f'(R) + \frac{\partial V_P}{\partial R}\right] = 0$$

where

$$Z = V_C[f(R), R] - V_P[f(R), R]$$

Invoking all the results in (16) up to (19) and evaluating them at V^*, we obtain the risk incentive constraint in the following simplified form

$$(E_C^2 - E_P^2) + 2\,(E_P - E_C)[\phi V^* - \phi R f'(R)] - 2\phi R^2 f'(R) = 0 \tag{22}$$

For purposes of checking the second order condition we establish the following expression after considerable simplification of partials and cross-partials:

$$\frac{d^2Z}{dR^2} = \frac{2}{R^2}\,(E_C - E_P) f'(R) + \frac{(E_P - E_C + R)}{R} f''(R) + \frac{(A^2 - D^2)}{\phi R^3} \tag{23}$$

As we shall see later, the second order condition is satisfied. (See fn. 15).

The Zero Agency Cost Solution

Now we have a system of three equations [(20, (21), and (22)] in the three unknowns E_C, E_P, and α. The amount of external financing needed by the entrepreneur (V_S) is fixed. We solve for α from (20) as

$$\alpha = \frac{(E_C - E_P)}{R} \tag{24}$$

Substituting (24) for α in (21) we combine the perk and the budget constraints, and simplify the combined expression to arrive at

$$(E_P^2 - E_C^2) + R(E_C + E_P) - 2\phi R V_S = 0 \tag{25}$$

Now we have a system of two quadratic equations, (25) and (22), in the two unknowns, E_C and E_P. Before we proceed with our solution we wish to argue that the option contract must be designed so as to force the entrepreneur to adopt a value maximizing strategy. Among a continuum of projects in the function $V = f(R) = 0$ and $f''\ (R)$ V is maximized when $f'(R) = 0$ and $f''(R) < 0$. Given this rationale, (22) collapses to

$$(E_C - E_P)(E_C + E_P - 2\phi V^*) = 0 \tag{26}$$

Two conditions satisfy (26), namely

(a) $E_C = E_P$

(b) $E_C + E_P = 2\phi V^*$

The first condition is unacceptable, because it implies that the manager surrenders all his ownership interests [see (24)] by retaining a package which includes a positive position in the call option and a negative position in the put option with identical parameters as the call. This combined position is equivalent to holding an option which is sure to be exercised. Under this condition we can solve for $E_C = E_P = \phi V_S$ from (25), implying that the net value of the assets held by the manager is $V^* - V_S$. Notice that with the option contract, the original equity is essentially converted into debt so that, in this case, the manager holds a 100% of "equity" (option). Since the option is sure to be exercised, it must mean that debt is riskless. Thus, the first condition satisfying (26) sterilizes the agency problem of external financing. Consequently, we find condition (a) unacceptable.

We can now move to condition (b) and use it along with (25) to solve for E_C and E_P. Thus,

$$E_C = \frac{2\phi V^{*2} + R(V^* - V_S)}{2V^*} \tag{27}$$

$$E_P = \frac{2\phi V^{*2} - R(V^* - V_S)}{2V^*} \tag{28}$$

Now (24), (27), and (28) constitute the zero agency cost solution. Under this solution the manager retains a positive fractional interest (α), because $E_C > E_P$ as we see from (27) and (28). Indeed, the *RHS* of (24) is equivalent to $(1 - V_S/V^*)$. This in turn implies that the option contract must be designed so as to equate the values of the call and the put. Since the positions in the options cancel out in terms of current value, we can say that the original equity has not been transformed into debt as previously expected. Its form remains intact.[18]

V. Contemporary Use of Call and Put Options in Capital Structures

Our use of the call option is analogous to the actual use of executive stock options in managerial compensation schedules. In a typical qualified stock option the executive is granted an option for a given number of shares at a given exercise price which is usually equivalent to the market value of the stock at the date of the grant. The shares under the option are usually nonexercisable before specific dates distributed over a five year period. The option is also typically nonmarketable. A slightly different form of compensation commonly found in industry is the stock appreciation right. These give the right for an executive to receive compensation based on the difference between the market value of the stock and a pre-established price at a specific expiration date. The compensation may be in the form of cash, stock or some combination of the two. Stock appreciation rights are sometimes issued in conjunction with nonqualified stock options in which case the exercise of either right or the option cancels the other. Some stock appreciation rights limit the amount of appreciation for which the executive is eligible. Such a limitation is usually not present in a stock option plan. Stock

[18] The second order condition for the risk incentive constraint can be checked from (23). As we argued earlier, $f'(R)$ is zero at the solution point. Thus, we ignore the first term of the *RHS* of (23) and concentrate on the two subsequent terms. The second term is equivalent to $(1 - \alpha)f''(R)$ [see Eq. (24)] and hence negative since $f''(R)$ is negative at the solution point. Also it is easy to see that the last term of the *RHS* is negative, since $E_C > E_P$ in our solution. Consequently, the entire expression in (23) is negative, and the second order condition is, thus, satisfied for the risk incentive constraint.

appreciation rights avoid the necessity of a cash outlay by the executive and also avoid the holding period rule that applies to stock *ownership* under the rules of the Securities and Exchange Commission as defined in the Internal Revenue Code 422(b).

While put options are not commonly observed in the capital structures of actual firms, our use of the put option is remarkably similar to a financing strategy that is, in fact, commonly employed. The put option, in combination with the common stock of the firm, can be thought of as a surrogate for the convertible bond. If the terminal value of the firm exceeds the exercise price of the put, the put is worthless and outsiders remain as common stockholders holding a fractional interest in the common stock of the firm. This outcome represents conversion of the bond into a fractional interest $(1 - \alpha)$ of the firm's common stock. On the other hand, if the terminal value of the firm falls below the exercise prices, the put has value and the outsiders can exercise it by selling the firm to the entrepreneur for the exercise price. This is analogous to the situation where conversion is unprofitable and bondholders exercise their fixed claim.

VI. Conclusion

This paper has analytically demonstrated that stock options can play a vital role in resolving agency problems associated with external financing which arise in the Jensen-Meckling framework. Indeed, complex facets of existing capital structures, such as the use of executive stock options in executive compensation and conversion privileges associated with prior claim securities, may emerge as solutions to the agency problems analyzed by Jensen and Meckling.

REFERENCES

1. F. Black and M. Scholes. "The Pricing of Options and Corporate Liabilities." *Journal of Political Economy* (May/June 1973).
2. M. Brennan. "The Pricing of Contingent Claims in Discrete Time Models." *Journal of Finance* (March 1979).
3. E. Fama. "Agency Problems and the Theory of the Firm." Center for Research in Security Prices, *Journal of Political Economy* (April 1980).
4. E. Furubotn and S. Pejovich. "Property Rights and Economic Theory: A Survey of Recent Literature." *Journal of Economic Literature* (December 1972).
5. M. Harris and A. Raviv. "Some Results on Incentive Contracts." *The American Economic Review* (March 1978).
6. B. Holmström. "Moral Hazard and Observability." *The Bell Journal of Economics* (Spring 1979).
7. M. C. Jensen and W. H. Meckling. "Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure." *Journal of Financial Economics* (October 1976).
8. R. Merton. "The Theory of Rational Option Pricing." *Bell Journal of Economics and Management Science* (Spring 1973).
9. S. Shavell. "Risk Sharing and Incentives in the Principal and Agent Relationship." *The Bell Journal of Economics* (Spring 1979).
10. H. Simon. "Theories of Decision Making in Economics and Behavioral Science." *American Economic Review* (June 1959).
11. H. Stoll. "The Relationship Between Put and Call Option Prices." *Journal of Finance* (December 1969).