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New Concepts of Aggregated Money*

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THE FEDERAL RESERVE BOARD'S recent redefinition of its monetary aggregates followed three years of intensive Board staff research on monetary aggregation.[1] While the new aggregates ultimately were selected along conventional lines, some of the staff's research was oriented towards more fundamental changes in monetary aggregation methods. This paper reviews such proposals, which are based upon the rigorous application of index number and aggregation theory and of principal components analysis in the construction of economic monetary aggregates.

The usefulness of aggregation and index number theory in acquiring price indices is widely known. The theory of the true cost of living index is an application of aggregation theory. Examples of applied index number theory include the Commerce Department's implicit price deflator, which is a Paasche index, and the Labor Department's consumer price index, which is a Laspeyres index. Somewhat less well known is the fact that dual to the theory of economic price aggregates and indices is a theory of economic quantity aggregates and indices. An example of a highly reputable quantity index is the Divisia quantity index.

The monetary aggregates traditionally are thought to measure the supplied quantity of an economic good called "money." Whenever monetary aggregates are used in the context of economic theory (rather than as simple accounting identities), the theory of economic quantity aggregation is specifically applicable. However, published monetary aggregates in all countries have always been computed as simple sums. But as Fisher [1922, p. 29] observed in the conclusion to his classic treatise on index number theory, "the simple arithmetic average produces one of the very worst of index numbers, and if this book has no other effect than to lead to the total abandonment of the simple arithmetic type of index number, it will have served a useful purpose." With an example of the simple sum index, Fisher [1922, p. 363] found that "the bias alone reaches 36 percent." Although index number theory has not yet found its way into the monetary theory literature, all of the literature on index number theory and aggregation theory for the past half century supports the result long ago deduced by Fisher [1922, p. 361] that the "simple arithmetic [index] should not be used

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[1] See Barnett, Beck, Ettin, Kalchbrenner, Lindsey, Porter, Simpson, and Tinsley [1979] and the subsequent formal redefinition.

under any circumstances." The authors' research surveyed herein provides the foundations for construction of monetary aggregates through the rigorous application of economic aggregation and index number theory.

1. The User Cost of Money

Economic quantity aggregates depend upon relative prices as well as upon component quantities. If the price of each component of the monetary aggregates were one dollar, then relative prices would never change. However, money is a durable good, and the one dollar price of a unit of its stock is applicable only for an infinite holding period. Our concern is with the flow of monetary services generated by that stock during a finite holding period. At the end of the finite holding period, the monetary asset remains in existence, and therefore the asset's lifetime services, valued at the price of the stock, have not yet been fully consumed. Within each time period, the quantity of services provided by a monetary asset (during that finite holding period) is an economic variable entering the economy's structure, and it is those services that we seek to aggregate over monetary assets. The price of those services is the user cost or "rental price" of the monetary asset.

Until recently the form of the user cost function for monetary assets was not known. This fact had hindered the application of aggregation theory to monetary aggregation. However, Donovan [1978] has deduced the user cost formula (in discrete time) by analogy with the consumer durables case, and Barnett [1979b, 1980a] has acquired the same result through a rigorous mathematical proof. Hence the way now is open to application of aggregation theory in constructing monetary aggregates.

The user cost formula is constructed as follows. Let p_t^* be the true cost of living index during period t; let R_t be the maximum available expected one period holding yield during period t; let r_{it} be the own rate of return on asset i ($i = 1, \ldots, N$) during period t; and let τ_t be the marginal tax rate during period t. Then the user cost of the services of monetary asset i during period t is

$$\pi_{it} = h(R_t, \tau_t, p_t^*)(R_t - r_{it}),$$

where $h(R_t, \tau_t, p_t^*) = p_t^*(1 - \tau_t)/[1 + R_t(1 - \tau_t)]$. Since $h(R_t, \tau_t, p_t^*)$ does not depend upon the selection of good i, that factor will be seen to cancel out of our monetary quantity indices. Hence we equivalently could view $R_t - r_{it}$ alone as the user cost.

2. Aggregation Theory

Aggregation theory is a branch of economic theory. In economic theory the structure of the economy is defined by the tastes (utility functions) of consumers and the technology (production functions) of firms. In aggregation theory, an economic aggregate exists if a function of the components of the aggregate can be factored out of the economy's structure in such a manner that the economy depends upon the components only through that function. We then say that the structure is weakly separable in the components.[2] The resulting function factored out of the structure is called an aggregator function. The current economic (or "functional") quantity aggregate is acquired by evaluating the aggregator function at the current values of its components.

Aggregation theory provides us with three capabilities. (1) We can test for weakly separable (factorable) blockings of monetary assets to select the components of economic monetary aggregates. (2) Having determined the existence and components of the aggregates, we then can test for their functional form. (3) We also can compute the values of the aggregates. The last of the three capabilities is the least useful, since the value of the economic aggregate depends upon unknown parameters, which must be estimated. Data series depending upon unknown parameters are generally not suitable for publication by government agencies. As discussed in the next section, index number theory provides a means of avoiding the unknown parameters problem arising in aggregation theory.

The first two capabilities of aggregation theory have been applied to monetary data. In applying the first capability, Barnett [1981, Chapter 7] has found empirical evidence supporting the Board's new procedure of aggregating like assets over different institution types (such as passbook savings over commercial banks and thrift institutions). Of particular interest is Barnett's [1980a] application of the second capability of aggregation theory. He tested for the simple summation form for the aggregator function and *rejected* the hypothesis.[3]

This conclusion should be no surprise. A necessary (but not sufficient) condition for simple summation aggregation is perfect substitutability between components. Over a decade of empirical evidence on elasticities of substitution between monetary assets heavily supports the conclusion of generally low substitutability between many of the components of the monetary aggregates. Having determined that simple summation aggregation must be rejected, we should consider more reputable indices.

3. *Index Number Theory*

As mentioned in the previous section, the aggregator functions of aggregation theory contain unknown parameters. Index number theory exists to permit us to approximate the current point on the aggregator function without knowledge of, or estimates of, the unknown parameters. The "statistical" index numbers of index number theory are *parameter free* and depend only upon the data.

A quantity aggregator function in aggregation theory depends upon quantities and unknown parameters, but not upon prices. A statistical quantity index number in index number theory depends upon both quantities and prices, but

[2] For all of aggregation theory to be applicable, the functions also must be linearly homogeneous. A rigorous statement of the weak separability and homogeneity conditions is contained in Green's [1964] Theorem 4 and in Barnett [1980a]. For complications that can result from aggregation over households and firms, see Barnett [1979].

[3] Barnett's test requires that monetary services (and thereby the real balances that produce those services) appear in a utility function. Such a derived utility function must exist if money is to have positive value in equilibrium. See Quirk and Saposnik [1968, p. 97] and Arrow and Hahn [1971, p. 350].

not upon unknown parameters. An important recent paper by Diewert [1976] has united index number theory and aggregation theory by identifying a class of statistical index numbers which not only perform well relative to the properties deemed desirable in statistical index number theory, but also provide high quality approximations to values of the economic quantity aggregates of aggregation theory.

For purposes of monetary quantity aggregation, the (Törnquist-Theil discrete time approximation to the) Divisia index is our preferred choice from that attractive class of index numbers.[4] As Barnett [1980a] has shown, selection between reputable index numbers (such as the Divisia or Fisher ideal indices) is of little empirical importance, since the difference between their growth rates is negligible (although their divergence from the simple sum index is large). However, interpretation of the Divisia index is attractively transparent, as could be important in any policy applications, and the Divisia index possesses a particularly desirable limiting property as measurement frequency increases and continuous time is approached. In addition, as has been shown by Diewert [1978], Divisia subindices can be nested (to within a second order approximation) within higher level Divisia indices.

The Divisia index is defined as follows. Let m_{it} be (nominal or real; total or per capita) balances of monetary asset i during period t. Let $s_{it} = \pi_{it}m_{it}/\sum_{k=1}^{N} \pi_{kt}m_{kt}$, and let $s^*_{it} = (½)(s_{it} + s_{i,t-1})$. Then the Divisia quantity index, Q_t, over the components $m_{it}(i = 1, \ldots, N)$ during period t is defined such that

$$\log Q_t - \log Q_{t-1} = \sum_{i=1}^{N} s^*_{it} (\log m_{it} - \log m_{i,t-1}).$$

Barnett [1980a] and Barnett and Spindt [1979] have shown that the velocity of Divisia monetary aggregates is particularly well behaved. As substitution is internalized through aggregation, the index is increasingly stabilized, with the remaining variation correlating with the interest rate. Those results are reproduced in the next section. The puzzling behavior of the demand for money in the United States (including the widely publicized "shifts" in the demand for money function) disappears when Divisia aggregation (or any other reputable form of aggregation) is used. In addition, Barnett and Spindt [1979, 1980] have applied information theory to compare the Divisia index with the simple sum index. They found that the Divisia index's information content *dominates* that of the simple sum index for each of their choices of components for the monetary aggregate and for each of their choices of targets for policy.

4. *Velocity of the Index*

In this section we consider the case of an aggregate (M_3) having the following components: transaction balances, passbook savings at the three institution types and at credit unions, small time deposits at the the three institution types, and negotiable and non-negotiable large C.D.'s at commercial banks. The velocity of

[4] See Barnett [1980a, 1980c] for further details on the form and use of the Divisia index with monetary data.

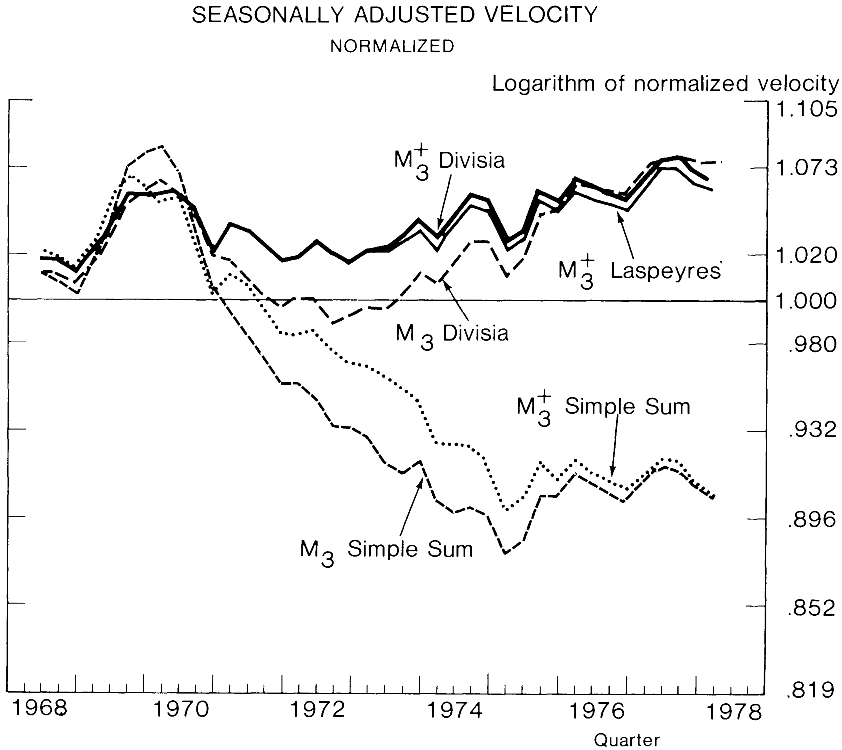

Figure 1.

the conventional simple sum index (labeled 'M_3 simple sum') and of the (Törnquist-Theil) Divisia index (labeled 'M_3 Divisia') are plotted in figure 1.

While the velocity of the Divisia index rises, the velocity of the simple sum index declines secularly from 1972(3). The Divisia index weights transaction balances more heavily than any of the other components of the aggregate, since transaction balances provide the largest share of monetary services. But the velocity of transaction balances has been rising rapidly in recent years. Hence the inadequate weighting of transaction balances in the simple sum M_3 has permitted velocity to be drawn down by the substitution effect of the increasing relative price (user cost) of transaction balances relative to less liquid monetary substitutes.

The aggregate does not include many negotiable money market instruments, although many assets subject to governmental rate regulation are included. We should therefore expect substitution (disintermediation) to occur out of this aggregate into unregulated substitutes during periods of rising interest rates and high inflation, if M_3 approximates an economic good. That is, velocity should *rise.* Comparing figure 1 with the ten-year government bond rate in figure 2, we see that when the Divisia index is used, the interest elasticity of money demand has the right sign. Internalizing more money market substitution by aggregating over additional money market instruments can be expected to further stabilize the

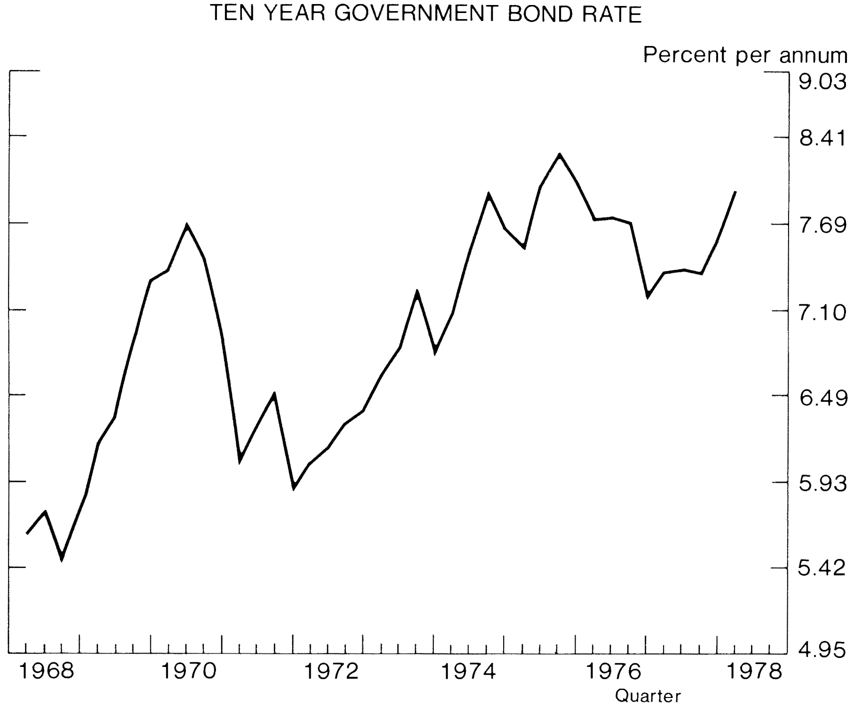

Figure 2.

velocity of the Divisia index, since economic aggregation perfectly internalizes substitution effects between components (see Barnett [1980a]).

To further verify our interpretation, we incorporate elements of the unregulated money market [dealer and directly placed commercial paper, repurchase agreements (RP's) of commercial banks with the non-bank public, bankers' acceptances, and negotiable Treasury securities with less than one year remaining to maturity] into M_3 to create M_3^+. In figure 1 we plot the velocity of M_3^+, with M_3^+ computed as a simple sum index (labeled "M_3^+ simple sum"), as a Divisia index (labeled "M_3^+ Divisia"), and as a chained Laspeyres index (labeled "M_3^+ Laspeyres"). Internalizing these segments of the money market further stabilizes the velocity of the Divisia index. The velocity of the simple sum index continues to trend in the wrong direction. The Laspeyres index is seen to provide a far better approximation than the simple sum index, despite the fact that the Laspeyres index provides only a first-order approximation to the value of the underlying economic aggregator function. The slight variations remaining in the velocity of the Divisia index continue to correlate with the ten-year bond rate, since some elements of the unregulated money market remain outside of the aggregate.

The simple sum index is a Laspeyres quantity index with the weights erroneously set to be equal. Clearly the erroneous weighting destroys the index's critical independence of substitution effects (within the aggregate), and hence the simple sum index *cannot* approximate the economic aggregate.

5. *Liquidity Characteristic*

When the user cost of monetary services is imputed to the components of a Divisia monetary aggregate, the resulting quantity index is an index of the quantity of services provided by the aggregate. The index then evaluates an economic structural variable, as is required in monetary policy. However, for some specialized research purposes, measurement of only one of the elementary services provided by the assets can be more informative. In such cases, transformation to elementary consumption characteristics is required.

Lancaster [1971] has postulated that market goods are functions of elementary consumption characteristics. In monetary theory, we might expect that those characteristics would include "liquidity" and perhaps "means of payment" and "store of value." In some cases one of those characteristics may be of independent interest. Lancaster's formulation views market goods as being functions of measurable characteristics, while we seek to deduce consumption characteristic quantities from measurement of market goods. Theil [1976, chapts. 12–13] has provided a means of computing such an inverse of the Lancaster transformation under the plausible (if not necessary) assumptions that elementary consumption characteristics are preference independent (strongly separable) and that the number of characteristics does not exceed the number of goods.

In principle, Theil's transformation can be applied to any estimated integrable demand system (at a point) to reveal the basic preference independent consumption characteristics (at least at that point). In addition to generating the characteristics, the transformation provides the income elasticities of each characteristic and the composition matrix of share contributions of each market good to each consumption characteristic. The income elasticities and composition matrix can be computed at each data point, since the properties of the preference independent goods are usually not constant over time.

Since Theil's methodology is intended to be general, the transformation alone is not capable of attaching a name ("flavor," "nutrition," "liquidity," etc.) to the revealed consumption characteristics. However, the income elasticities and composition matrices generally provide sufficient information to permit deduction of the identities of most, if not all, of the revealed consumption characteristics. Since the quantities of the revealed consumption characteristics depend upon estimated parameters and upon the specification of a demand system in market goods, the resulting indices of revealed consumption characteristics are not usable for publication as quantity data by governmental agencies. Nevertheless, the information provided by the elementary consumption characteristics can be informative for some research purposes.

Offenbacher [1980b] has estimated a household demand system for monetary assets with holdings by firms subtracted out. He applied Theil's preference independence transformation and found that one of the elementary consumption characteristics appeared to be liquidity, which was the dominant characteristic.[5]

In applications in which *data* on the supply of economic monetary services is required, as is true in monetary policy, the Divisia index described in Section 3

[5] Another characteristic appeared to be store of value "for a rainy day."

should be preferred. When knowledge solely of the liquidity characteristic of the components of that aggregate is required, the preference independence transformation can be applied, as has been illustrated by Offenbacher [1980b].

6. *Rates of Return*

The real user cost of a monetary asset depends upon the one period holding yield of that asset and the maximum available expected one period holding yield on any asset. All of the above methods depend upon availability of user costs and therefore on those own rates and the maximum rate. Issues exist in the operational definition of those rates.

The explicit rate of return on demand deposits (other than NOW accounts) is zero in the United States. But an implicit rate of return is received by many firms holding demand deposits. In principle, that implicit rate of return should be evaluated and used in a Divisia index of monetary services to the degree that the implicit return is itself not in the form of a "monetary service."[6]

In addition, an implicit rate must be imputed to demand deposits, if the substitutability between currency and demand deposits is to be estimable. Offenbacher [1980a] imputed to demand deposits both the full competitive rate and 40% of that rate and estimated the translog model in monetary assets.[7] He found that substitutability between currency and demand deposits is very low in both cases. Unless 40% of the competitive rate greatly overestimates the non-monetary-services implicit return to demand deposits, Offenbacher's result implies that even transaction balances should be aggregated by Divisia aggregation rather than by simple summation. The proper implicit rate imputation to demand deposits remains an important open issue.

In addition, two reputable competing methods exist for computing the maximum available expected holding period yield.[8] The selection between those two methods remains another open issue. Recent work on these issues is described in Barnett and Spindt [1980]. However, such data refinements are always possible and pose no fundamental problems in implementing aggregation theory.

7. Conclusion

Simple sum monetary aggregates are appropriate for accounting purposes, as in monitoring the liability structure of banks. But when monetary aggregates are used as economic variables, the aggregate should be computed using appropriate methods from aggregation or index number theory. For such purposes, the parameter-free (Törnquist-Theil) Divisia index is particularly well suited.

[6] A nonmonetary service paid to a demand deposit holder could be better-than-market terms on a commercial loan. For further discussion of this issue, see Offenbacher [1980c] and Barnett [1980b].

[7] Startz [1979] estimates the implicit return at about 50% of the competitive rate. Interestingly, firms' share in demand deposits is about 40%. Hence 40% of the competitive rate is a potentially attractive imputation.

[8] Term structure theory, rational expectations theory, and perfect arbitrage dictate use of the maximum available short rate (plus a probably small maturity premium). However, recent work by Shiller [1979] dictates maximization over available rates at all maturities. Our recent empirical work marginally supports Shiller's result, although our results are robust to either selection.

When a single component of monetary services (such as "liquidity") must be isolated for research purposes, the preference independence transformation can be used.

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