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The Numeraire Problem and Foreign Exchange Risk

MARK R. EAKER*

I. Introduction

TRADITIONAL EXPLANATIONS of foreign exchange exposure have generally suffered from two deficiencies: the failure to place the foreign exchange problem within the context of the firm trying to maximize shareholder wealth, and the attempt to deal with exposure management on a transaction by transaction basis. The first deficiency manifests itself in the assumption or belief that assets denominated in a home or numeraire currency are void of foreign exchange risk. Invoking that assumption ignores the impact of exchange rate changes on the home currency's purchasing power over imported goods. The second weakness is evidenced by the attempt to deal with foreign exchange decisions using a hedge or no-hedge decision rule, thus ignoring portfolio considerations.

In a recent article, Makin [7] discusses the hedge-no-hedge strategy and offers an alternative way to analyze foreign currency problems using portfolio theory. Although Makin's approach is a significant improvement on the hedge-no-hedge analysis it still suffers from the first deficiency. Makin's model is dependent upon his assumption that the home currency is a risk-free asset, an assumption that will likely bring about the inclusion in the resulting portfolio of more assets denominated in the home currency than would be the case if risk were correctly specified.

The purpose of this paper is to show that the assumption is not generally correct and to suggest an index that more accurately measures exchange risk. The index that is used is one that was developed by Kouri and Braga de Macedo [3]. It is similar in concept to a trade-weighted exchange rate and reflects the impact of exchange rate variation on the purchasing power of a unit of currency.

Section I presents a verbal description of exchange risk. The differences between the traditional view and the global view espoused here are highlighted. Section II discusses the Kouri and Braga de Macedo index of value and its applicability to the foreign exchange exposure problem. Section III demonstrates the use of the index in adjusting returns and analytically shows the error in confusing the nominal riskless asset with a real riskless asset. In section IV the characteristics of a real riskless asset are investigated.

I. *Foreign Exchange Risk and the Riskless Asset*

A simple example of the traditional view of exchange risk should serve to illustrate the nature of the numeraire related error. In the standard text book

* Edwin L. Cox School of Business, Southern Methodist University.

discussion, a home country firm is said to enter into an export or import agreement that is denominated in some currency other than that of the home country. A United States firm might import goods worth 100,000 francs, incurring an obligation of that amount which is due at sometime in the future. The dollar value of that obligation is unknown since between the time the obligation is incurred and the date of payment the exchange rate of francs for dollars might change. It is that uncertainty which is characterized as foreign exchange risk.

Since in most cases the currency of denomination is the home currency of one of the principals, it is frequently noted that one party to the transaction bears the risk whereas the other party does not. In the example given above and in the context of the standard approach to exchange risk exposure, the French exporter is said not to bear any risk because the transaction is denominated in francs. The amount of francs, which is the exporter's home currency, he will receive is fixed regardless of changes in the exchange rate. It is this certainty of the nominal cash flow in the home currency that the traditional approach describes as being devoid of foreign exchange risk. It is a characterization that Makin utilizes in order to identify the risk free asset which paves the way for his application of the separation theorem in deriving optimal currency shares.

The difficulty arises because the value of the asset is only fixed in a nominal sense. The rational firm as well as a rational individual investor should not be trying to maximize nominal profits, but real wealth or purchasing power. As a consequence when the firm makes a decision to hold any asset it should be concerned with the real value of that asset at maturity or when the asset is sold. In the earlier example, the value of the franc asset will vary as exchange rates vary. This is true because the purchasing power represented by the franc asset changes as the relative prices of other currencies change and as the prices of goods expressed in all currencies change.

The risks to the French exporter are both of a real and an opportunity cost nature. To see that, assume that after the exporter has made the sale the franc depreciates relative to the dollar. It's true that the exporter will still receive 100,000 francs but now the purchasing power of those francs over goods priced in dollars is less than it was prior to the depreciation. If the exporter consumes dollar priced goods then his wealth has decreased in a real sense. Furthermore, even if the exporter does not consume dollar priced goods, he still would have been better off ex post had he contracted to receive dollars. After the appreciation of the dollar his revenues from the sale in francs would have been greater than 100,000. Not only has the exporter experienced an opportunity cost but if his competitors had acted in such a way as to profit from the exchange rate change then he would be at a competitive disadvantage.

This insight into the values of foreign assets has been developed in two separate bodies of literature. Within the balance of payments literature, this insight has led to the development of trade-weighted exchange rates. Hirsh and Higgins [2], and Rhomberg [9] discuss the use and structure of various indexes. Several others including Grauer, Litzenberger, and Stehle [1], Kouri and Braga de Macedo, and Kwai [4] have used the concept in order to derive international asset valuation models. At the same time, however, discussions of risk management in recent

textbooks and journal articles have failed to grasp the need for a global purchasing power view of exchange risk.

II. *An Index of Value*

In order to formalize the verbal description of exchange risk given above, Kouri and Braga de Macedo's index of value will be utilized. To construct their index, assume that a firm has a Cobb-Douglas utility function of the following form:

$$U(x_1, x_2, \cdots x_n) = \prod_{i=1}^{n} x_i^{\alpha_i} \tag{1}$$

where α_i is the proportion of total expenditures spent on the x_i^{th} good and $\Sigma\alpha_i = 1$.

Related to the n goods is a vector of prices $(P_1, P_2, \cdots P_n)$ such that the purchasing power of a unit of the currency in which the prices are expressed is measured as:

$$\prod_{i=1}^{n} P_i^{-\alpha_i}. \tag{2}$$

Expression (2) is a price index that reflects the consumption pattern of the individual firm for goods whose prices are denominated in that single currency. However, the firm is not restricted to purchases in any one currency. Units of a currency can be exchanged for other currencies in order to purchase goods and services whose prices are expressed in one of the other currencies. The value Q_i, of a unit of the currency of country i can be expressed as its purchasing power over goods priced in all currencies and the exchange rates of other currencies for currency i.

$$Q_i = \prod_{j=1}^{n} (P_j S_{ij})^{-\partial j}, \tag{3}$$

where P_j is the price level in country j for a composite bundle of goods,[1] S_{ij} is the exchange rate of currency j in terms if i, ∂j is the firm's expenditure share represented by currency j, and $S_{ii} = 1$.

Q_i is an expenditure weighted exchange rate. It is constructed using a geometric mean although an arithmetic mean could have been used and the global purchasing power effect still would have been captured. Such an index would be derived from a different utility function. The important point is that this index reflects the fact that the purchasing power of a unit of currency is a function of the price level in each country, the exchange rate between that currency and every other currency and the expenditure pattern of the firm or individual holding the unit of currency. Since the value of each currency is a function of its exchange rate relative to each other currency in the expenditure basket no currency is immune from exchange risk and no real riskless asset necessarily exists.

[1] Although P_j is described as a national price index such as the GNP deflator, it more accurately should be considered an index that includes all goods whose prices are denominated in currency j. Magee and Rao [6] discuss findings that a substantial portion of trade is not contracted in the exporter's currency. Therefore goods acquired in countries other than j might be priced using the jth currency.

The assumption that each firm or individual has constant ∂j's is not entirely satisfactory. It assumes that the decision maker spends a constant proportion of total expenditures on goods priced in the j^{th} currency. A more complex model might take into account that the ∂j's are not exogenous but are themselves a function of the P_j's and S_{ij}'s. However, the partial equilibrium approach followed here is not totally unrealistic. Decision makers such as multinational firms are often tied to long-term purchasing agreements. This is particularly true if one realizes that included in the P_j's is the price of labor services and other non-traded goods. The expenditure on those items is largely determined by plant location which commits the firm to certain purchasing patterns for relatively long periods of time.

III. *Adjusting the Nominal Return*

When a firm assumes an exposed asset position in a currency it is concerned with both the expected real return and the risk related to that position. The real return on that asset can be approximated by the difference in the nominal return and an adjustment for the change in purchasing power of the currency of denomination:

$$\tilde{R}_i \cong \tilde{r}_i - \tilde{m}_i, \tag{4}$$

where R_i is the real return of an asset denominated in currency i, r_i is the nominal return, and m_i is the adjustment factor.

The approximation is derived as follows:

$$1 + \tilde{R}_i = \frac{(\tilde{A}_i^{t+1})/(\tilde{Q}_i^{t+1})}{(A_i^t)/(Q_i^t)}, \tag{5}$$

where A_i^t is the price of an asset denominated in the i^{th} currency at time t and Q_i is the index of value defined above.

$$1 + \tilde{R}_i = \frac{(\tilde{A}_i^{t+1})}{(A_i^t)} \cdot \frac{(Q_i^t)}{(\tilde{Q}_i^{t+1})} \tag{6}$$

$$1 + \tilde{R}_i = \frac{1 + \tilde{r}_i}{1 + \tilde{m}_i} \cong 1 + \tilde{r}_i - \tilde{m}_i, \tag{7}$$

where $\tilde{m}_i = \dfrac{\tilde{Q}_i^{t+1} - Q_i^t}{Q_i^t}$, from which expression (4) follows.

Makin's major contribution was to place the exchange risk problem into a portfolio context. In doing so, he was extending and improving on an approach taken by Lietaer [5]. That approach indicates that the risk related to a foreign asset position is measured by its contribution to the overall variance of returns of the portfolio of asset positions taken by the firm. In line with earlier discussion in this paper, the proper measure of return is once again the real, and not nominal, yield.

In that context, the risk of a two asset portfolio consisting of assets denominated in currencies i and j can be expressed as

$$V_p = X_i^2 \operatorname{Var}(\tilde{R}_i) + X_j^2 \operatorname{Var}(\tilde{R}_j) + 2X_iX_j \operatorname{Cov}(\tilde{R}_j\tilde{R}_i), \tag{8}$$

where X_i, X_j are the percentages of the total asset portfolio denominated in each currency, after conversion into a single currency at investment value, and $x_i + x_j = 1$.

The error of assuming a riskless asset can be seen if the adjustment represented by expression (4) is substituted into (8), and the result is expanded.

$$V_p = X_i^2 \operatorname{Var}(\tilde{r}_i - \tilde{m}_i) + X_j^2 \operatorname{Var}(\tilde{r}_j - \tilde{m}_j) + 2X_iX_j \operatorname{Cov}(\tilde{r}_i - \tilde{m}_i, \tilde{r}_j - \tilde{m}_j) \quad (9)$$

$$\begin{aligned} V_p = {} & X_i^2[\operatorname{Var}(\tilde{r}_i) + \operatorname{Var}(\tilde{m}_i) - 2\operatorname{Cov}(\tilde{r}_j, \tilde{m}_j)] \\ & + X_j^2[\operatorname{Var}(\tilde{r}_j) + \operatorname{Var}(\tilde{m}_j) - 2\operatorname{Cov}(\tilde{r}_j, \tilde{m}_j)] \\ & + 2X_iX_j[\operatorname{Cov}(\tilde{r}_i, \tilde{r}_j) + \operatorname{Cov}(\tilde{m}_i, \tilde{m}_j) - \operatorname{Cov}(\tilde{r}_i, \tilde{m}_j) - \operatorname{Cov}(\tilde{r}_j, \tilde{m}_i)]. \end{aligned} \quad (10)$$

Notice that even if both assets offer riskless nominal returns, there is still variance in the real return of the portfolio. The $\operatorname{Var}(\tilde{m}_i)$, $\operatorname{Var}(\tilde{m}_j)$ and $\operatorname{Cov}(\tilde{m}_i, \tilde{m}_j)$ terms remain, reflecting the global purchasing power risk. Since that risk is not included in the traditional specification of exchange risk, the simulation results of Makin and Lietaer can be called into question. Their variances incorrectly measure the risk related to holding assets in any currency. The portfolios that they derive using those risk measures will in most cases include a greater proportion of home currency assets than is optimal because the relative riskiness of such assets is underestimated.

It might be argued, and with some merit, that the use of real returns has changed the ground rules within which others have worked. Most people using the CAPM assume either implicitly or explicitly that no inflation exists or that inflation affects all returns equally. Therefore nominal returns can be used. Once uncertain inflation is allowed, then even nominally riskless assets are risky.

It is beyond the scope of this paper to argue the validity of the above inflation assumptions for domestic applications of portfolio theory. However, it will be argued that such assumptions are not valid in an international context.

The essence of foreign exchange risk is that the value of currencies fluctuates. Economic theory indicates that those fluctuations are at least in part caused by relative price changes between countries. To assume away or ignore such price movements is inconsistent with a correct specification of the problem and can lead to the type of error discussed above. In the next section the interrelationship between price changes and exchange rate fluctuations is discussed in the context of describing an asset that is riskless in terms of its real return.

IV. *A Riskless Real Asset*

As has been indicated above once the zero inflation assumption is dropped then the nominally riskless asset is no longer necessarily riskless in real terms. That holds for a purely domestic discussion as well as an international one. In order to concentrate on the foreign exchange problem the derivations of this section will be for the special case in which there is zero inflation in the home country. The loss of generality is compensated for by the slightly less cumbersome mathematics and, more importantly, the explicit linking of the risk free assumption to a basic economic theory of exchange rate determination.

The theory that is of relevance is the Purchasing Power Parity Theory (PPPT) of exchange rates (see Officer [8]). It states that exchange rates are determined by the relative prices of similar goods between countries. This leads to the law of one price: similar goods should sell for the same price in all countries once the local currency price is converted into some numeraire price. This implies that changes in relative prices between countries will be reflected in changes in the exchange rates in such a way as to maintain real price equivalency.

To see the importance of PPPT[2] in defining the riskless real rate of return, assume that there are two countries, 1 and 2 such that the index of value for the currency of country 1 is

$$Q_1 = P_1^{-\partial_1} (P_2 S_{12})^{-\partial_2}. \tag{11}$$

Country 1 is the home country and is assumed to experience zero inflation. This results in all changes in the index of value being related to changes in prices in country 2.

$$\frac{dQ_1}{dP_2} = P_1^{-\partial_1} (-\delta_2)(P_2 S_{12})^{-\partial_2 - 1}\left(S_{12} + P_2 \frac{dS_{12}}{dP_2}\right). \tag{12}$$

However, if PPPT holds, then $(S_{12} + P_2 dS_{12}/dP_2)$ is equal to zero as is dQ_1/dP_2. This implies that $Q_1^t/Q_1^{t+1} = 1$ and therefore that the real return in expression (7) is equal to the nominal return.

Under the restrictive conditions that there is zero inflation in the home country, that PPPT holds precisely, and that the nominal return is itself riskless then so is the real return.

A second set of conditions that is sufficient to establish an asset as riskless in real returns has been developed by Grauer, Litzenberger, and Stehle (GLS). Using a state preference approach, they define a real bond as "an asset whose payoff is indexed to provide a constant real return across states" ([1] p. 239). Unlike the approach followed here where the index of value does not change, the approach taken by GLS assumes that the change in nominal returns offsets the change in the index.

Their approach can be demonstrated using the framework of this paper. Rearranging expression (6) shows the price of the asset at maturity as a function of its original price, the stated return, and the change in the index of value between time t and $t + 1$.

$$\tilde{A}_1^{t+1} = \frac{A_1^t(1 + \bar{R}_1)(Q_1^t + \Delta\tilde{Q}_1^t)}{Q_1^t} \tag{13}$$

The real bond can be viewed as a commitment on the part of the issuer to redeem the bond at maturity at a price that guarantees the stated or fixed real

[2] Discussions of purchasing power parity center on the behavior of national price indexes. Unless the firm's consumption bundle in a particular currency is identical to the bundle in the index or all prices in the index change by the same amount then the impact on the firm is not precisely described by the changes in the national index. The implication is that even when PPPT holds the firm bears some exchange risk. This point further emphasizes one raised below that a bond promising a real return should be constructed differently for each firm or investor.

return $\bar{R}_1$. The nominal price at maturity is then an uncertain random variable. The uncertainty is generated not by uncertainty in the return, but by changes in prices and exchange rates which affect the index of value.

As the index of value changes then the price of the asset at time $t + 1$ will change in order to maintain the real return. Again assuming that P_1 is constant while P_2 changes, and making use of the following approximation:

$$\Delta \tilde{Q}_1^t \cong \frac{d\tilde{Q}_1}{dP_2} \Delta \tilde{P}_2, \text{ then}$$

$$\tilde{A}_1^{t+1} \cong \frac{A_1^t(1 + \bar{R}_1)\left(Q_1^t + P_1^{-\partial_1}(-\partial_2)(P_2 S_{12})^{-\partial_2 - 1}\left(S_{12} + P_2 \frac{d\tilde{S}_{12}}{dP_2}\right)\Delta \tilde{P}_2\right)}{Q_1^t} \tag{14}$$

$$\cong \frac{A_1^t(1 + \bar{R}_1)\left(Q_1^t - \partial_2 \frac{Q_1^t}{P_2 S_{12}}\left(S_{12} + P_2 \frac{d\tilde{S}_{12}}{dP_2}\right)\Delta \tilde{P}_2\right)}{Q_1^t}$$

$$\cong A_1^t + A_1^t \bar{R}_1 - (\partial_2) \frac{(A_1^t + A_1^t \bar{R}_1)}{P_2 S_{12}}\left(S_{12} + P_2 \frac{d\tilde{S}_{12}}{dP_2}\right)\Delta \tilde{P}_2$$

$$\Delta \tilde{A}_1^t \cong \tilde{A}_1^{t+1} - A_1^t = A_1^t \bar{R}_1 - (\partial_2) \frac{(A_1^t + A_1^t \bar{R}_1)}{P_2 S_{12}}\left(S_{12} + P_2 \frac{d\tilde{S}_{12}}{dP_2}\right)\Delta \tilde{P}_2 \tag{15}$$

Expression (15) gives the change in the price of the asset necessary to provide the stated real yield. If purchasing power parity holds or if there is no change in prices, then the second term on the right hand side equals zero and the change in asset price is the amount necessary to provide a yield of $\bar{R}_1$ (i.e. $A_1^t \bar{R}_1$). When purchasing power parity does not hold, then $\Delta \tilde{A}_1^t$, consists of the $A_1^t \bar{R}_1$ term plus an adjustment that reflects the impact of price and exchange rate changes on the purchasing power of both the principle, A_1^t, and the interest $A_1^t \bar{R}_1$. Although PPPT is a sufficient condition for the real bond, it is not necessary. Even if PPPT is not valid, the change in price represented by expression (15) will still provide the required real return.

It is important to note that the size of the adjustment necessary to maintain the real return is dependent on the expenditure weight, ∂_2. That follows from the construction of the index of value as a weighted average and implies that for each individual or firm the indexing procedure would have to be different.

V. *Conclusion*

It has been shown that the assumption that an asset with a return fixed in the home country currency is a riskless asset is generally incorrect. Consequently, models in which a riskless asset is required can not be appropriately used for problems involving exchange risk unless certain restrictive conditions are invoked. Those conditions include assuming purchasing power parity or constructing the asset so that it is a real bond.

On the other hand, the realization that the global purchasing power effect is relevant to the decision maker supports the arguments of Makin and Lietaer that

a portfolio approach to exchange risk management is desirable. It is important that future studies looking at the implementation of this approach also look at its implications for other international operating decisions and for the accounting treatment of exchange gains and losses.

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