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Treasury Bill Factors and Common Stock Returns

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I. Introduction

CAPITAL ASSET PRICING theory gives a linear *ex ante* relationship between a security's return and one special portfolio return, the return on all assets. This linear equation is called the capital asset pricing model. Capital asset pricing theory alone provides no empirical hypotheses because it is derived and defined wholly with expectations. Tests of empirical hypotheses with historical data require an additional proposition that relates *ex ante* and *ex post* returns. A model for this second proposition is called a return generating equation. Arbitrage pricing theory gives a more compact method for deriving a linear *ex ante* security return model. In the arbitrage theory developed by Ross [6] the return generating equation is the basis for the *ex ante* return relationship. Briefly, an approximate linear expected return equation is derived directly from the return generating equation so two separate models are not required. In addition, the *ex ante* return equation is defined in terms of an arbitrary number of special factor portfolio returns rather than a single market portfolio return. In this sense, arbitrage pricing theory has richer implications and a simpler structure than capital asset pricing.

Arbitrage pricing theory starts with a linear return generating equation for each security.

$$\tilde{r}_i = E_i + \sum_{j=1}^{k} \beta_{ij}\tilde{\delta}_j + \tilde{\epsilon}_i \tag{1}$$

In equation (1) $\tilde{r}_i$ is security i's single period actual return (a tilde denotes a random variable), E_i is the expected single period return, each $\tilde{\delta}_j$ is the actual change in a common factor with zero mean and finite variance, the β_{ij} are security i's response coefficients to changes in common factors, and $\tilde{\epsilon}_i$ is the zero mean, finite variance residual error return. The covariance between $\tilde{\epsilon}_i$ and each $\tilde{\delta}_j$ is zero.

A modest set of assumptions is required to derive the *ex ante* relationship. These include homogeneous beliefs, frictionless trading and risk aversion among investors. A no-arbitrage condition then gives convergence of the infinite series in expression (2).

$$\lim_{P\to\infty} \sum_{i=1}^{P} (\mathrm{E}_i - \rho - \sum_{j=1}^{k} \beta_{ij}\lambda_j)^2 < \infty \tag{2}$$

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The index $i = 1, 2, \cdots, P$ denumerates the set of securities. The constant ρ is the return on a zero systematic risk portfolio (or risk free security if it exists) and the constants λ_j equal $E_j - \rho$. E_j is the anticipated return on a portfolio with unit β_j and zero β_h, $h \neq j$. Ross [6] proves expression (2) with the law of large numbers. This expression means that, for a countably infinite set of securities, the sum of squared deviations of E_i from the constant ρ and the weighted constants λ_j is uniformly bounded. This implies

$$E \approx \rho e + \lambda B' \tag{3}$$

where $E = \{E_1, E_2, \cdots, E_n\}$, $n < P$ is a vector of expected security returns, $e = \{1, 1, \cdots, 1\}$ is a vector of ones, $B = \{\beta_{ij}\}$ is a matrix of response coefficients, and $\lambda = \{\lambda_1, \lambda_2, \cdots, \lambda_k\}$ is a vector of portfolio expected excess returns. The approximation is presumed close to an equality for almost any set of securities. In words, the vector of security expected returns is virtually spanned by a vector of ones and a matrix of response coefficients.

In this paper we analyze the response of common stock returns to statistical factors estimated from the weekly returns on a set of U.S. Treasury bills. Thus we get factors from one set of securities to use with a different set. We propose that the same statistical factors that influence Treasury bill returns influence share returns. In this paper's second section, we formalize this argument. Section III describes how the Treasury bill factors are estimated and gives statistical results from an analysis of Treasury bills' returns. In section IV, we describe the time series results we get by regressing common stock returns on Treasury bill factors. The results of cross section experiments that use the time series estimates are also presented. A summary completes the paper.

II. The Relationship between Returns of Treasury Bills and Common Stocks

We propose that returns on U.S. Treasury bills and common stocks respond to the same set of systematic factors and that arbitrage pricing gives a valid model representing the factor influence on share returns. The formal argument is stated as follows. Suppose the ith maturity Treasury bill's weekly return process is stationary. Then

$$\tilde{r}_i = e'_T E_i + \tilde{D}_b \beta'_i + \tilde{\epsilon}_i \tag{4}$$

is the return generating equation in matrix notation. In this format, $\tilde{r}_i = (\tilde{r}_{i1}, \tilde{r}_{i2}, \cdots, \tilde{r}_{iT})$ is a vector of weekly returns, E_i is the mean weekly return, e'_T is a vector of T ones, $\tilde{D}'_b = \{\tilde{\delta}_{jt}\}$ is a matrix of weekly percent changes in the k common or systematic factors, $\beta_i = (\beta_{i1}, \beta_{i2}, \cdots, \beta_{ik})$ is a vector of constants, and $\tilde{\epsilon}_i = (\tilde{\epsilon}_{i1}, \tilde{\epsilon}_{i2}, \cdots, \tilde{\epsilon}_{iT})$ is a vector of errors with mean zero and constant variance. All Treasury bill returns together are written in a similar format.

$$\tilde{R} = e'_T E + \tilde{D}_b B'_b + \tilde{\epsilon}. \tag{5}$$

In this notation, $\tilde{R} = \{\tilde{r}_{it}\}$ is a matrix of weekly Treasury bill returns, $E = \{E_i\}$, is a vector of mean returns, $B_b = \{\beta_{ij}\}$ is a matrix of response coefficients, and $\tilde{\epsilon}' = \{\tilde{\epsilon}_{it}\}$ is a matrix of error terms.

The return generating equation for the sth common stock is

$$\tilde{r}_s = e'_T E_s + \tilde{D}_c \beta'_s + \tilde{\epsilon}_s. \tag{6}$$

Equation (6) for stock returns is analogous to equation (4) for bill returns. Each column of $\tilde{D}_c$ contains changes in a factor that influences stock returns. Formally, we propose that $\tilde{D}_b$ in equation (4) and $\tilde{D}_c$ in equation (6) are the same. Unfortunately, we cannot establish this with our tests because they are not powerful enough. But we can find out if some columns in $\tilde{D}_b$ are statistically related to $\tilde{r}_s$ and if these columns lead to special portfolios in which average returns are related to E_s. If no columns in $\tilde{D}_b$ significantly relate to share returns, part or all of our initial proposition is wrong.

We use a several step procedure to test the relationship between statistical bill return factors and share returns. First we factor analyze Treasury bill returns and use the estimated factor score time series as independent variables in equation (6). Details are described in Section III. This gives a vector β_s of estimated response coefficients for each stock. Next we form special factor portfolios of common stocks in which the portfolios, by construction, respond only to one factor and have minimum variance return. The method used is similar to that used by Black and Scholes [1]. To construct special factor portfolios, shares are randomly assigned to n intermediate portfolios. We calculate the covariance matrix of the intermediate portfolio returns and call this matrix Q. Then we solve the minimization problem

$$\begin{aligned} &\text{minimize } X_j Q X'_j \\ &\text{subject to: } X_j \beta'_j = 1; \qquad X_j \beta'_h = 0 \quad \text{for} \quad h \neq j;\; X_j e'_n = 1. \end{aligned} \tag{7}$$

The vector X_j contains the weights to attach to intermediate portfolio returns to compute special factor portfolio returns for each factor. The vector β_j contains the intermediate portfolio response coefficients computed as the average β_{sj} of shares in the intermediate portfolio. Each share enters only one intermediate portfolio. The share coefficients come from initial time series equations that regress individual stock returns on the statistical factor scores of the Treasury bills. The vector e_n is a row vector of n ones. This is the wealth constraint. If all but the last constraint are set equal to zero, we compute weights for a special portfolio that has zero response to statistical Treasury bill factors.

The minimization procedure gives $k + 1$ vectors X_j, k of which are associated with special portfolios that responds to one Treasury bill return factor and the last to no factors. Returns on these special portfolios are computed by applying the weights in the X_j's to actual intermediate portfolio returns. As a result, we construct $k + 1$ separate time series of special factor portfolio returns. (This is analogous to using equal or market value weights to construct a "market" return in standard capital asset pricing analysis). Then we re-estimate the individual security response coefficients, the β_{sj}'s, with time series equations in which the special factor portfolio returns are independent variables. This allows us to use the information in all share returns to estimate the coefficients.

A second time series equation is specified by substituting the *ex ante* share return equation into the return generating equation. The *ex ante* equation is

$$e'_T E_s \approx \rho + \Omega \beta'_s. \tag{8}$$

In this relationship, ρ is either a vector of weekly risk free returns or a vector of weekly returns on a portfolio with zero systematic risk. In our framework, the former corresponds to the time series of returns on a one week maturity Treasury bill (excluded from the initial factor analysis) and the latter is estimated with a zero response portfolio constructed with the minimization routine. The matrix Ω' has elements $\{E_{jt} - \rho_t\}$ and E_{jt} is the expected return on a portfolio in which the return responds only to factor j.

Treat expression (8) as an equation and substitute equation (6) for $e'_T E_s$. Remember that an analogous expression holds for each jth special portfolio. Cancel and collect terms to get the *ex post* equation

$$\tilde{r}_s = \rho v_s + \tilde{Z}\beta'_s + \tilde{u}_s. \tag{9}$$

In equation (9) $v_s = (1 - \sum_j \beta_{sj})$ and $\tilde{u}'_s = (\tilde{\epsilon}_{st} - \sum_j \beta_{sj}\tilde{\epsilon}_{jt})$. We use special portfolio returns in $\tilde{Z}$ so $\tilde{Z} = \{\tilde{r}_{jt}\}$ where $\tilde{r}_{jt}$ is actual return on the jth special portfolio in period t. This equation is estimated for each common stock to get a new estimate of β'_s.

In our final step, we use these new estimates of β_{sj}'s to do cross section experiments designed to test hypotheses about the arbitrage pricing model. We calculate new intermediate portfolio response coefficients, β_{pj}'s, as average of the new β_{sj}'s. In each time period we run a cross section regression (across the n intermediate portfolios) of each portfolio's actual return that period ($\tilde{r}_{p\tau}$) against $\hat{v}_p$ and $\hat{\beta}_{pj}$. constructed from the time series. The cross section equation is

$$\tilde{r}_p = \mu_\tau \hat{v}_p + L_\tau \hat{\beta}_p. \tag{10}$$

In equation (10) the τ subscript denotes a cross section in period τ rather than a time series equation. Thus we run a cross section equation in each period τ which gives T cross sections altogether. The value of μ_τ should equal ρ and $L_\tau = (L_{\tau 1}, L_{\tau 2}, \cdots, L_{\tau k})$ is an estimate of the return on the k share return factors in the period.

Hypotheses about the *ex ante* arbitrage pricing model given in expression (8) are tested by averaging the stochastic coefficients (μ_τ and L_τ) across the T cross sections. This gives

$$\bar{r}_p = \bar{\mu} v_p + \bar{L}\beta'_p \tag{11}$$

where $\bar{r}_p$ is the average return on the pth portfolio, $\bar{\mu}$ is an estimate of $\bar{\rho}$ and $\bar{L}$ is an estimate of $(E_1, E_2, \cdots, E_k)$.

A summary of the above procedure is helpful here before we go on to the statistical work. We wish to investigate the influence of statistical factors estimated from Treasury bill returns on common stock returns. We assume the arbitrage pricing model gives a valid *ex post* and *ex ante* return model for both sets of securities. We also assume common factors. We set up a five step procedure. First, we factor analyze weekly Treasury bill returns and construct time series of factor scores. Second, we use the time series of factor scores as independent variables in time series regressions with individual stock returns as the dependent variable. This yields initial estimates of stock response coefficients. In the third step we set up special stock portfolios such that a portfolio's returns respond to changes in one factor (or zero factors) only. We use intermediate portfolios in which the initial share coefficients from step two are averaged to give portfolio coefficients. From this step we get actual weekly returns on the special factor portfolios. The fourth step entails regressing individual share returns on special portfolio returns. This gives a revised estimate of share response coefficients. Finally, in step five, we do cross section regressions on the results from step four. We use the results in an averaged model to analyze the estimated *ex ante* arbitrage pricing model.

III. Statistical Factors in Treasury Bill Returns

The basis for this entire analysis is the identification of factors that affect Treasury bill returns. We observe $\tilde{r}_{it}$, the weekly return on the ith maturity bill, each week. According to theory, these returns move about in response to changes in $\tilde{\delta}_{jt}$'s, the common factors. These we do not observe. We use the multivariate technique of factor analysis (see Lawley and Maxwell [2]) to estimate the statistical factors that affect Treasury bill returns.

3.1 Treasury Bill Return Data

All Treasury bill data are from the *Wall Street Journal* weekly during the period January, 1964 to December, 1979. The data consist of Thursday closing bid-ask discounts recorded for every Treasury bill with a maturity of 1 to 26 weeks inclusive. We use Thursdays as the recording date because 13- and 26-week Treasury bills enter and leave (mature) the market on Thursdays. This assures a weekly term structure up to 26 weeks over the sample period. Cross checks on some observations were made against Merrill Lynch quote sheets. Banker's discounts are converted to prices and the prices to true rates of return. Weekly Treasury bill returns are calculated from bid-ask prices as $\ln[\bar{p}(m-1, t)/\bar{p}(m, t-1)]$ where $\bar{p}(m, t)$ is the simple average of bid and ask prices for a Treasury bill with maturity m at week t. When Thursday quotes cannot be obtained we adopt the convention of recording Fridays quotes. To keep returns consistent we adjust the resultant six and eight day returns to seven day returns by multiplying them by 7/6 and 7/8, respectively.

Summary statistics in columns (1) to (6) of Table 1 indicate that weekly Treasury bill returns are positively skewed during the sample period and frequently more peaked in the central portion of their distributions than a normal distribution. Studentized range statistics give a test for departure from normality in the tails. The studentized range values in columns (6) suggest that the weekly return distributions do not correspond to the overall shape of a normal distribution.

Sample autocorrelations for the returns computed for lags one to five weeks are presented in columns (7) to (11) of Table 1. The correlations show that all but 2 of the 130 are significantly different from zero at the 1% level. These correlations imply that a large degree of dependency may exist among weekly Treasury bill returns up to lag five. In general, the magnitude of the autocorrelations is smaller as maturity gets longer and as the lag increases. The statistical characteristics of returns during the 1964–1979 period parallel Roll's [4] findings for an earlier period.

Table 1

Summary Statistics for Treasury Bill Returns

Weeks to Maturity (1)	Mean (2)	Variance (3)	Skewness (4)	Kurtosis (5)	Studentized Range (6)	$\hat{a}_1$ (7)	$\hat{a}_2$ (8)	$\hat{a}_3$ (9)	$\hat{a}_4$ (10)	$\hat{a}_5$ (11)
1	10.7691	1.290327	.9816*	3.4029	5.16	.9857*	.9728*	.9632*	.9543*	.9431*
2	10.3749	1.245500	1.1467*	4.1479*	6.05	.9516*	.9356*	.9285*	.9215*	.9016*
3	10.3434	1.249442	1.2213*	4.5329*	6.47	.9020*	.8742*	.8735*	.8664*	.8379*
4	10.4950	1.356798	1.3548*	5.3897*	7.08	.8437*	.7993*	.8030*	.8095*	.7743*
5	10.8540	1.529611	1.4281*	6.1368*	7.90*	.7898*	.7292*	.7340*	.7565*	.7078*
6	11.0450	1.672629	1.5495*	7.3089*	8.21*	.7395*	.6583*	.6747*	.6975*	.6563*
7	11.0893	1.811549	1.5675*	7.7974*	8.81*	.6882*	.5948*	.6109*	.6439*	.5998*
8	11.1685	1.978334	1.4262*	6.8875*	8.38*	.6415*	.5399*	.5579*	.6013*	.5568*
9	11.4399	2.166188	1.4463*	7.1889*	8.02*	.6117*	.5036*	.5149*	.5432*	.5058*
10	11.4972	2.460026	1.4641*	8.1040*	9.84*	.5588*	.4273*	.4514*	.5023*	.4565*
11	11.4669	2.664464	1.5386*	9.4606*	10.68*	.5085*	.3916*	.4061*	.4537*	.4028*
12	11.4106	2.901886	1.5435*	10.2548*	11.35*	.4472*	.3556*	.3919*	.4033*	.3595*
13	11.5993	3.351620	1.6445*	12.1666*	12.11*	.3496*	.3054*	.3050*	.3105*	.3373*
14	11.1371	4.444268	1.7119*	11.4885*	10.98*	.3707*	.2320*	.2571*	.2660*	.2776*
15	11.3560	4.025916	1.5018*	11.4859*	11.59*	.3781*	.2310*	.2508*	.2643*	.2372*
16	11.5017	4.180589	1.3338*	10.7883*	11.91*	.3860*	.2274*	.2404*	.2590*	.2239*
17	11.6442	4.532509	1.3190*	10.5378*	11.35*	.3993*	.2243*	.2436*	.2430*	.1975*
18	11.8209	4.703504	1.1513*	9.1975*	10.47*	.3986*	.2369*	.2480*	.2286*	.1684*
19	11.8819	4.991895	.9901*	8.6911*	10.29*	.3714*	.2274*	.2329*	.2066*	.1546*
20	11.8799	5.279392	.8909*	8.7622*	11.04*	.3523*	.2126*	.2146*	.1742*	.1289*
21	11.9538	5.666303	.7400*	8.0347*	10.31*	.3351*	.2154*	.1901*	.1573*	.1101*
22	12.0138	6.047242	.8070*	8.0616*	10.62*	.2940*	.1955*	.1827*	.1604*	.1174*
23	11.9821	6.694850	.8073*	7.5685*	10.33*	.2779*	.1929*	.1747*	.1396*	.1112*
24	12.0193	7.467323	.9287*	8.2326*	10.64*	.2635*	.1739*	.1526*	.1241*	.0790
25	11.9921	8.209247	.7705*	7.6155*	10.57*	.2185*	.1591*	.1398*	.1148*	.0861
26	12.1535	8.812960	.6740*	6.7943*	9.53*	.1733*	.1459*	.1658*	.1206*	.1012*

The sample period is January, 1964 to December 1979 giving 834 weekly return observations for each Treasury bill. Means are multiplied by 10^4 and variances by 10^7. Skewness is measured by the third moment from the mean divided by the second moment to the 3/2 power. Kurtosis is computed by dividing the fourth moment about the mean by the square of the second moment about the mean. The studentized range is the range of observations in the sample divided by the square root of the sample variance. $\hat{a}_1$, $\hat{a}_2$, $\hat{a}_3$, $\hat{a}_4$, $\hat{a}_5$ are sample autocorrelations for lags one to five, respectively. Asterisks represent values which are significant at the 1% level.

3.2 Factor Analysis of Treasury Bill Returns

We start with a matrix of weekly Treasury bill returns for maturities 2 to 26 weeks inclusive. We do not include the one week maturity bill because we wish to use it later as a risk free rate. *In addition, we subtract the return on the one week bill from each other maturity's weekly return.* This eliminates most of the return serial correlation shown in columns (7) to (11) of Table 1. $\tilde{W}' = \{\tilde{r}_{it} - \tilde{r}_{1t}\}$ is the matrix of observed bill excess returns and B_b is the matrix of response coefficients based on $\tilde{W}'$ (see equation (5)). In the factor analysis procedure, we wish to find an orthogonal factor loading matrix $B_b B_b'$ such that

$$\sum = B_b B_b' + V \tag{12}$$

where $\sum$ is the covariance matrix of excess returns on different maturities of bills and V is a diagonal matrix of residual error variance of excess returns. The B_b matrix is estimated with the equation

$$S\hat{V}^{-1}\hat{B}_b = \hat{B}_b(I + \hat{B}_b'\hat{V}^{-1}\hat{B}_b) \tag{13}$$

in which S is sample covariance matrix of excess returns, the matrix $\hat{B}_b$ is the estimate of the $B_b = \{\beta_{ij}\}$ response coefficients, and $\hat{V}$ is the estimated matrix of residual error variances of excess returns. Equation (13) is a maximum likelihood procedure if Treasury returns are multivariate normal. Alternatively, equation (13) derives from an approach that minimizes the partial correlations among factors even if returns are not multivariate normal (see Morrison [3, p. 330]).

Once a rule is adopted that fixes the dimension of $\hat{B}_b$, the elements of $\hat{B}_b$ are estimated. These are termed factor loadings. We use the factor loadings to represent zero-mean factor scores $\hat{D}_b$. These factor scores are our estimates of the time series of statistical factor realizations. The factor scores are computed with $\tilde{W}$, $\hat{B}_b$, and $\hat{V}$ as

$$\hat{D}_b = \tilde{W}(\hat{B}_b\hat{B}_b' + \hat{V})^{-1}\hat{B}_b \tag{14}$$

The matrix $\hat{D}_b'$ contains elements, $\{\hat{\delta}_{jt}\}$. The $\hat{\delta}_{jt}$ are the independent variables used in equation (6) as an estimate of $\tilde{D}_c$, the time series of stock factors. This leads to initial estimates of stock return coefficients through time series regressions of $\tilde{r}_s$ on $\hat{D}_b$.

We factor analyze excess returns separately for one factor through ten factors. Determining the appropriate number of factors required is difficult to do precisely because the return data are probably not multivariate normal (See Table 1). This means that standard statistical tests for deciding on the factor structure are not strictly valid. We use five factors because we want the residual variation as small as possible without having the factor model become too cumbersome for later analysis. To obtain some perspective on the residual variation still remaining, a Tucker and Lewis [7] reliability ratio is computed. It represents a ratio of explained covariation to total variation. A five factor model explains all but about 7% of the covariation in the excess Treasury bill return data from Table 1.

3.3 Stock Return Data

Weekly common stock data are from the CRSP Daily Returns File which includes individual securities listed on the New York Stock Exchange (NYSE) and the American Stock Exchange (ASE). The weekly returns cover the same period as the Treasury bill data, (i.e. January, 1964 to December, 1979). Weekly returns are calculated with continuous compounding from Thursday to Thursday closing prices plus any dividends to correspond exactly with the Treasury bill data week by week. Therefore, as with the Treasury bill returns, if a holiday occurs on a Thursday (or the market is closed) then by convention Friday's close is used and returns in the week before and after are adjusted to seven day returns. All returns are adjusted for any splits or capital changes.

Intermediate portfolios of the stocks are formed by ordering the NYSE *and* ASE securities alphabetically into portfolios of thirty securities each. For convenience, the ending date is fixed as the last sample week and all securities are excluded that do not have at least twelve years of data. This results in 639 sample observations for 42 portfolios of 30 securities. Although we select 1260 securities as do Roll and Ross [5] it comes about more by accident than design. There may or may not be similarity in the composition of our portfolios compared with theirs.

Summary statistics for the intermediate portfolios are computed as in Table 1 but are not shown. The weekly stock portfolio returns are typically symmetric but leptokurtic and hence non-normal as indicated by significant studentized range values at the 1% level for all 42 portfolios.

3.4 Formation of Special Factor Portfolios

A factor score matrix $\hat{D}_b$ is constructed using equation (14). Time series regressions are run between all 1260 stock returns and the five Treasury bill factor scores over the sample period. A count is made of the number of individual securities for each factor score that have a t-value on an estimated coefficient significant at the 1% and 5% levels under a null hypothesis of zero. For the 1% level it is expected by chance alone that at least 1260(.01) = 12.60 of the securities have significant t-values on any given factor score. At a 1% significance level, the first factor score has about 28% significant while the second, third, fourth, and fifth factor scores have 2%, 3%, 2%, and 2%, respectively. Turning to the 5% level, on average factor scores 1, 2, 3, 4, and 5 have more significant coefficients than the approximate number of 63 expected by chance alone (619, 105, 144, 98, 79). These statistical findings seem to suggest that at least one and up to four or five Treasury bill factor scores may be related to individual stock returns. However, it is difficult to interpret the t-values because the underlying stock return data is leptokurtic.

Based on the time series results with individual stocks, the intermediate stock portfolio returns are regressed against the factor scores over the 639 weeks to yield response coefficients. In essence, the response coefficients for each portfolio are a simple weighted average of the coefficients of each of the thirty individual securities in the portfolio.

The sample covariance matrix for the intermediate stock portfolio returns and these response coefficients are used in the minimization routine discussed in Section II (see expression (7)) to calculate weights for the special factor portfolios. Special factor portfolio return series are obtained by multiplying each intermediate stock portfolio by its weight and summing over all stock portfolios. This is done each week for each special factor. These portfolio weights allow us to construct special factor portfolio return series which satisfy the requirements of the minimization procedure. That is, the sum of the weights is unity and each special factor portfolio is uncorrelated with every other by construction. The sixth special factor portfolio is designed to represent the minimum variance—zero beta portfolio.

IV. Propositions about Common Stock Returns and Treasury Bill Factors

We mention in Section II that a joint proposition underlies our work. In this section we define five subsidiary propositions that are consistent with the underlying proposal. Empirical hypotheses are defined in terms of the five propositions.

4.1 *Time Series of Common Stock Returns are Correlated with Special Factor Portfolio Returns*

One important advantage of the arbitrage pricing theory (APT) in comparison with the standard capital asset pricing model (CAPM) is that APT does not require a special market portfolio as the only factor that influences share returns. This proposition is designed to test if APT is correct and if Treasury bill returns are a source of factor information. Our null hypothesis is that stock returns are correlated with returns on the special factor portfolios.

We run a time series regression for each share in which $\tilde{r}_{st}$ is the dependent variable and the special factor portfolio returns ($\tilde{r}_{jt}$'s) are independent variables. The 639 sample observations give estimates of v_s and β_s in equation (9) for each of the 1260 stocks. Results are reported in columns (2) and (3) of Table 2. Panel A results include the risk free rate (r_f, the one week bill return) and Panel B results include the minimum variance zero beta portfolio (z_0, computed using the minimization procedure). The numbers in the table give a count of the number of coefficients significantly different from zero at the 1% and 5% levels for each factor. If one factor is significant in all equations, the count is 1260. Comparing the reported numbers versus chance, all the special factor portfolios have more than chance numbers of significant coefficients at both 1% and 5% levels.

In sum, the time series results do not allow us to reject the proposition that stock returns are correlated with returns of special factor portfolios. This implies the APT is correct because several of the special factor portfolios appear to have a significant influence and none of these portfolios represent the market portfolio.

4.2 *The Mean of the Cross Section Coefficients Equals the Sample Mean of Special Factor Portfolio Returns*

Cross section equations are run to verify the time series results for the *ex ante* model (equation (8)). A cross section equation is run each week with the model in equation (10). The dependent variable is $\tilde{r}_{p\tau}$, the intermediate portfolio return

Table 2

Time Series Regression Results

Special Factor Portfolio (1)	Including Either r_f or ρ: Number Different from Zero at the 1% Level (2)	Number Different from zero at the 5% Level (3)	Also Including the Market Portfolio: Number Different from Zero at the 1% Level (4)	Number Different from Zero at the 5% Level (5)
A. Including the risk free rate r_f				
Constant	55	239	7	38
$\hat{z}_1$	1077	1152	88	209
$\hat{z}_2$	113	252	93	232
$\hat{z}_3$	330	547	89	192
$\hat{z}_4$	96	236	101	227
$\hat{z}_5$	50	153	90	203
$\hat{r}_f$	47	191	8	45
$\hat{r}_m$	—	—	1257	1260
B. Including the minimum variance—zero beta rate ρ				
Constant	0	3	1	7
$\hat{z}_1$	1207	1231	156	307
$\hat{z}_2$	470	698	124	254
$\hat{z}_3$	855	1018	129	254
$\hat{z}_4$	129	257	108	244
$\hat{z}_5$	407	647	104	242
$\hat{z}_0$	1251	1256	270	437
$\hat{r}_m$	—	—	1250	1255
Number Expected by Chance Alone	12.6	63.0	12.6	63.0

$\hat{r}_f$, $\hat{z}_0$, $\hat{r}_m$ denote the risk free security, the minimum variance—zero beta portfolio, and the equally-weighted market portfolio, respectively. Time series regressions are run between the returns of each individual stock and the returns on the special factor portfolios with and without the equally-weighted market portfolio. The numbers in the table are a count of the number of significant coefficients at the 1% and 5% levels.

during week τ and the dependent variables are the estimated coefficients $\hat{v}_p$ and $\hat{\beta}_p$. Thus we have 42 observations each week and we repeat the procedure 639 times. A portfolio approach is used to eliminate biases in individual coefficients. The resulting series of α_τ (the intercept), μ_τ, and L_τ are used to estimate mean coefficients and standard deviations. We use the estimates to make inferences with equation (11) about the true values of $\bar{L}$ in this proposition and $\bar{\alpha}$ and $\bar{\mu}$ in later propositions.

Results for the cross sections are summarized in columns (1) to (5) of Table 3. Column (1) identifies the independent variable. In Panel A μ_τ should equal the average risk free rate coefficient and in Panel B the average zero beta rate coefficient (see proposition 4.4). The means and standard deviations of the cross section coefficients are given in columns (3) and (4). If our proposition is valid and APT is correct, the average coefficients for L_1, L_2, $\cdots$, L_5 in column (3) should not differ significantly from the mean returns on special factor portfolios given in column (2).

Table 3

Cross Sectional Regression Results

Indepen-dent Variale (1)	Hypothesized Value (2)	Including Either r_f or ρ: Average Coefficient (3)	Standard Deviation (4)	t-value (5)	Also Including the Market Portfolio: Average Coefficient (6)	Standard Deviation (7)	t-value (8)
A. Including the risk free rates r_f							
α_τ	0	.001620	.017938	2.281*	.001876	.019577	2.420*
L_1	$\bar{r}_1 = .046554$	.035062	4.777322	−.061	.006999	4.253646	−.235
L_2	$\bar{r}_2 = .081180$	.131739	4.335385	.295	.122255	4.269195	.243
L_3	$\bar{r}_3 = .090410$	.124308	4.667575	.183	.124538	4.667422	.185
L_4	$\bar{r}_4 = .095959$	.125144	3.563616	.207	.120330	3.354495	.174
L_5	$\bar{r}_5 = .104118$	.137861	4.131380	.206	.150815	3.999773	.295
μ_τ	$\bar{r}_f = .001174$	.000088	.001425	−19.267*	.000079	.001371	−20.191*
M	$\bar{r}_m = .000789$	—	—	—	−.001087	.031873	−1.488
B. Including the minimum variance—zero beta rate ρ							
α_τ	0	−.000061	.035660	−.043	−.008954	.359621	−.629
L_1	$\bar{r}_1 = .046554$	.075493	4.893293	.149	.014601	4.208520	−.192
L_2	$\bar{r}_2 = .081180$	.147060	4.347474	.381	.131426	4.277538	.297
L_3	$\bar{r}_3 = .090410$	.125299	4.671805	.189	.131160	4.663347	.221
L_4	$\bar{r}_4 = .095959$	.149997	3.583969	.381	.142059	3.519530	.331
L_5	$\bar{r}_5 = .104118$	.120217	4.176623	.097	.155791	4.023647	.325
μ_τ	$\rho = .001782$	.001093	.031025	−.561	.005874	.226499	.457
M	$\bar{r}_m = .000789$	—	—	—	.004811	.240690	.422

The regressions are run each of the 639 weeks for the 42 common stock portfolios and a sample mean and standard deviation computed for each of the regression coefficients over the 639 weeks. Hypothesized values are average returns during the sample period. $\bar{r}_m$ denotes the average return on an equally-weighted portfolio containing all 1260 securities. The t-values in column (5) are the difference between column (3) and column (2) times $\sqrt{639}$ divided by column (4). The t-values in column (8) are the difference between column (6) and column (2) times $\sqrt{639}$ divided by column (7). Asterisks indicate significant t-values at the 5% level.

The t-values in column (5) are for tests of the difference between the average cross section coefficients and the mean special factor portfolio returns. These values are computed as column (3) minus column (2) times $\sqrt{639}$ and divided by column (4). With a two-tailed test and a significance level of 5%, we cannot reject the hypothesis that cross section mean coefficients and time series mean returns are equal for all factor portfolios.

In sum, it appears that the proposition that the mean of cross section coefficients equals the sample mean portfolio returns cannot be rejected. This implies that the APT gives a correct *ex ante* return model and Treasury bill returns are a source of information about factors. However, since the hypothesized values in column (2) are very close to zero this test is rather weak.

4.3 *The Market Index for Common Stocks is a Source of Additional Factor Information*

An equally weighted market portfolio return is formed by taking the simple average of the 1260 individual share returns each week. We test whether this portfolio return influences share returns when included with Treasury bill factors.

Table 2 presents time series results in columns (4) and (5). These regressions are similar to those in proposition 4.1 but include the market portfolio return as an independent variable ($\tilde{r}_m$). From Table 2, almost all of the 1260 market portfolio coefficients are significant at 5% and 1% levels. In addition, all five of the special factor portfolios have more coefficients significant at 5% and 1% levels than expected by chance alone. This is more evidence of the APT's validity. However, inclusion of the market return reduces the number of significant coefficients on other special factor portfolio returns. This is probably due to the colinearity between $\tilde{r}_m$ and $\tilde{r}_j$'s and $\tilde{\rho}$. Partial correlations between the market return and other independent variables are −.3434*, .0163, −.0426, −.1310*, .0517, .4456*, respectively for returns on factor portfolios one through five and the zero beta portfolio. Starred values are significantly different from zero at the 1% level.

Table 3 summarizes results of cross section regressions on time series coefficients with the market portfolio (factor M) included (see columns (6) to (8)). The t-values for factor M indicate that the mean cross section coefficient is not significantly different from the time series average of $\tilde{r}_m$ at the 5% level. This is true for both the risk free and zero beta rate specifications.

Overall, the proposition that the market return adds an additional significant dimension to the factor model cannot be rejected with our data. But this does not lead to rejection of our joint proposition that APT is valid and Treasury bills give factor information. It simply means the Treasury bills returns do not appear to contain all the relevant information for stock returns.

4.4 *The Average Coefficients on the Weekly Risk Free Rate and the Weekly Zero Beta Return Equal $\tilde{r}_f$ and $\tilde{p}$.*

We test the hypothesis that the average coefficient $\bar{\mu}$ in equation (11) equals its theoretical value. The t-values of −19.267 and −20.191 for μ_τ in Table 3's Panel A are significant at the 5% level and indicate the time series mean $\bar{r}_f$ exceeds its *ex* *ante* value. The t-values of μ_τ in Panel B are not significant at the 5% level. We cannot reject the proposition that $\bar{\mu}$ equals $\bar{\rho}$. Together these tests imply that the minimum variance zero beta portfolio return calculated from Treasury bill factors may yield a better specified arbitrage model.

4.5 *Time Series and Cross Section Regressions Have Zero Intercepts*

An important test of the arbitrage pricing theory is that the *intercepts* from the time series in Table 2 are zero for all stocks. We investigate this issue here. As seen in Table 2, with the risk free rate there are more constants significantly different from zero at the 1% and 5% levels (55 and 239) than expected by chance but there are less (0 and 3) using the zero beta rate. Both regression sets have fewer significant constants than expected when the market portfolio is included.

To jointly test the equality of constants an analysis of variance is performed using the intercepts underlying Table 2. An F-test is used. Resulting F-values without and with the market portfolio for Panel A are .642, .783 and for Panel B are .481, .499, respectively. None of these F-values are statistically significant using a 1% or 5% level. Thus, we conclude that the time series intercepts are the same for all stocks.

The arbitrage pricing theory predicts that the *ex ante* intercept in the regression equivalent of equation (10) is zero. A mean and standard deviation of estimated intercepts α_τ from the cross section equations are reported in Table 3. We test the null hypothesis that $\bar{\alpha}$ equals zero. With the risk free rate equations the t-value of 2.281 means α_τ is significantly different from zero at the 5% level. For equations with the zero beta rate the t-value of $-.043$ means the intercept is not significantly different from zero. If the market portfolio is also included the constant term in Panel A continues to be significantly different from zero ($t = 2.420$) at the 5% level while in Panel B it still is not ($t = -.629$). Thus, we cannot reject the null hypothesis that the average cross section intercept is zero only for the regressions including the zero beta rate. This implies again that the zero beta rate may give a better specification for the *ex ante* model.

V. Summary and Conclusions

In summary, we propose that the arbitrage pricing model is a correct specification of *ex post* and *ex ante* security returns. We also propose that Treasury bill returns provide a source for identifying statistical factors that influence common stock returns. Five propositions with specific empirical hypotheses are defined to investigate this argument. The results of several statistical analyses are consistent with the validity of the joint proposition.

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DISCUSSION

IRWIN FRIEND:* Profs. Fogler, John and Tipton (F-J-T) state that the purpose of their paper is "to assign economic meaning to stock market factors and to determine the extent to which these factors were related to the prices of capital in the bond market." They conclude "For the most part, we feel that our analysis has achieved these goals." It is not clear to me that the two goals mentioned have been achieved.

F-J-T hypothesize that three factors explaining market returns on individual common stocks are or can be measured by the returns on the stock market as a whole, on a 3-month Treasury obligation and on a long-term AA-Utility bond, with the last two factors specifically representing interest-rate and default risks. Their initial analysis consists of regressing excess returns for each of a sample of stocks on excess returns for each of the three factors. Their tentative conclusion from this part of their analysis is that while only the stock market return factor is generally significant, the sample regressions suggest that the classification of stocks into "growth", "stable" and "cyclical groups" "may be related to their relative sensitivity to bond market returns for interest rate risk and default risk."

The relevant data presented in Table 1, however, do not provide any indication that interest rate risk as measured by the 3-month rate significantly affects relative stock returns. Of the 100 stocks in the sample of companies covered, the 3-month rate had a significant effect on only one company at the .01 level of significance, on 7 companies at the .05 level and 5 more at the .10 level, numbers which do not appear to differ significantly from chance results. Nor does there seem to be much justification to the F-J-T statement that the more generally negative effect of the 3-month rate of returns of "growth" stocks as well as similar results for the AA rate "would suggest that investors may expect growth companies to be stronger financially and therefore able to survive relatively better than other companies during times of economic stress." The generally negative effect of the 3-month rate in "growth" stocks (10 out of 15 companies) seems very little different from that for "cyclical" stocks (20 out of 28 companies) while for "stable" stocks which might be expected to fare much better than "cyclical"

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