THE JOURNAL OF FINANCE • VOL. XXXVI, NO. 2 • MAY 1981

Three Factors, Interest Rate Differentials and Stock Groups

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I. Introduction

MOTIVATED BY THE single period capital asset pricing model (e.g. Sharpe [33]), for years research centered on the use of a single factor model where the single factor represented the return on the market portfolio. Subsequently, multifactor models were derived in a generalized CAPM setting by Sharpe [34], Long [23], Merton [25] etc., as well as in the arbitrage pricing theoretic framework by Ross [30], [31]. Merton [25] in the context of continuous time intertemporal asset pricing derives what we might call a two-factor model where the additional factor is a portfolio which is perfectly negatively correlated with changes in the risk free rate. The general intertemporal asset pricing models developed by Long [23] or more recently by Cox, Ingersoll and Ross [5] can also be interpreted as multifactor models. On the empirical side numerous studies were finding systematic *ex post* dependence in the residuals from single factor models (e.g., Agmon and Lessard [1], Arnott [2], Basu [4], Estep [7], Farrell [8, 9], Levy [20], Rosenberg and Marathe [29], and Schipper and Thompson [32], thereby suggesting the appropriateness of a multifactor specification.

Yet, the above generalizations have outrun our empirical analysis, and therefore they have left us with a particularly interesting question, *"If there are multiple factors, what might they be?"* This question is the basis for the hypothesis and research in the present paper.

Both (1) yield curve shapes and (2) risk premia for default are known to vary over *ex post* cycles in interest rates. The two sources of variation seemed likely candidates as explanators of the *ex post* factors in returns on common stocks. Accordingly, it was hypothesized that *ex post* returns (r_{it}) on an asset in period t would be *systematically* related to the returns for a stock market index (s_t), as well as for a U.S. Government Bond index (g_t) and a corporate bond index (c_t); i.e.,

$$r_{it} = B_{i0} + B_{is}s_t + B_{ig}g_t + B_{ic}c_t + e_{it} \tag{1}$$

where $\{B_{i0}, B_{is}, B_{ig}, B_{ic}\}$ are the standard OLS estimators and e_i is the residual

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We wish to thank the Frank Russell Company for intellectual impetus and financial assistance for the present study, Dr. Martin Leibowitz of Salomon Brothers for yield series data, Haim Levy and others in the Florida Workshop for suggestions, and Richard Roll for his comments. Of course, these groups are absolved from any limitations of this study. Russell Fogler acknowledges computing support from the University of Florida. Kose John received support from a Faculty Research Grant from the Graduate School of Business Administration, New York University.

for the i^{th} asset in period t. This equation is merely for empirical testing, although it could be motivated from appropriate utility or arbitrage assumptions.

The next sections of this paper address the empirical tests of equation (1), as well as the relationship of stock factors to the explanatory variables. Section II discusses the data used, as well as presenting initial results of the relationship of returns from 100 large companies from 1959–1977 to returns from indexes representing s, g, and c. These results give us some insight into the causes of stock groups. In Section III, these three indexes will be shown to be related to the first three principal components that are derived from the returns of the one hundred company sample. Finally, Section IV summarizes these findings and draws implication for future research.

II. Data and Initial Results

To study the above hypothesis, monthly returns on 100 common stocks were extracted from the University of Chicago's Center for Research on Security Prices (CRSP) data tape. The period chosen was from January 1, 1959 through December 31, 1977. The choice of this period resulted from the availability of CRSP data through 1977, as well as reliable interest rate data from January 1, 1959. The interest data consisted of the two monthly series to be discussed below from Salomon Brothers' *Analytical Record of Yields and Yield Spreads.* The selection of stocks was based upon a desire to allow comparability with the earlier studies of Farrell [8, 9] and Elton and Gruber [6]. Accordingly, the 100 stocks are divided as follows:

Stock Sequence	
1–15	Growth Stocks (Farrell)
16–35	Stable Stocks (Farrell)
36–63	Cyclical Stocks (Farrell)
64–69	Oils (Farrell)
70–82	Pseudo-Industry #1 (Elton and Gruber)
83–93	Pseudo-Industry #2 (Elton and Gruber)
94–99	Pseudo-Industry #3 (Elton and Gruber)
100	Other[1]

The 69 of Farrell's 100 stocks were selected because of return data availability on the CRSP Tape, with the other 31 not included. The Elton and Gruber companies were selected sequentially from their list.

Next, the excess returns of each stock were related to changes in three indexes representing stocks, Government Bonds, and Corporate bonds.[2] Accordingly, the

[1] The last stock, ACF, was selected by mistake but, in the interest of computer time and since its inclusion had no consequences for our analysis, it was retained.

[2] Although equation (1) was written in returns, excess returns (returns minus a riskless rate) were used throughout the study for two reasons. First, by using excess returns, one insures that no bias is introduced because of dependency between the riskless rate and the explanatory variables. Second, principal components analysis (factor analysis with orthogonal factors) fits a plane through the origin and thus excess returns are the appropriate measurement where the intercept is hypothesized as zero. Finally, the riskless rate was the yield on a one-month U.S. Government Bond.

following time-series regression was estimated for each stock:

$$r_{it} = \hat{B}_0 + \hat{B}_1 x_{1t} + \hat{B}_2 x_{2t} + \hat{B}_3 x_{3t} + e_{it} \tag{2}$$

where

r_{it} = the excess return on the i^{th} stock in time period t;

x_{1t} = the excess return on the CRSP value weighted index in period t;

x_{2t} = the excess return on a three month U.S. Treasury Bond in period t;[3]

x_{3t} = the excess return on a long-term Aa utility bond with a deferred call in period t;

$\hat{B}_0, \hat{B}_1, \hat{B}_2, \hat{B}_3$ = OLS coefficient estimates;

e_{it} = residual error in period t.

Table 1 contains the results of estimating the coefficients in equation (2) using monthly data, over the time period of Farrell's study (from 1/61 through 12/69). The major observation from Table 1 is that the estimated coefficients on the stock market return (x_1) are generally significant, while those on the other variables are not. This result probably explains part of the wide acceptance of the diagonal version of the CAPM. Of course, because the stock market is an average, it will already contain the value weighted average sensitivity of its component stocks to both x_{2t} and x_{3t}; this type of multicollinearity would be expected to lessen the significance of these latter variables.

In examining Farrell's groups, several characteristics provide insight into the determinants of these groups. First, growth stocks had coefficients which were generally negative on both the Government Bond (x_2) and Aa-Utility (x_3) return series.[4] This result would suggest that investors may expect these companies to

[3] x_2 and x_3 are bond returns, not bond yields. The Salomon Brothers yield-series data were converted to returns to provide consistency with the requirements for the arbitrage pricing model. The procedure was to treat each bond as if its yield were also its coupon, and then to calculate the resultant price change as market yields changed during the period. This price change plus the original yield was divided by the original price, thus providing the bond's return for that period.

Because returns from each of these series exhibit a high degree of multicollinearity, several previous studies (e.g., Friend, Westerfield, and Granito [14], Lloyd and Schick [22]) have "orthogonalized" the returns from the bond variables. The procedures used were equivalent to using only the residuals from a regression of the bond return series against the other explanatory variable or variables.

Such orthogonalization procedures are equivalent to a transformation which extracts away the common dependence between the variables, assigning such dependence to the explanatory variables in each orthogonalizing regression equation. This procedure will change the OLS estimators and their standard errors on the variables which are orthogonalized, but it *does not add any additional explanatory power*. Also, unless there is an economic rationale behind the choice of which variables are orthogonalized, subsequent interpretation of the results is difficult. For a further discussion of this see Fogler and Ganapathy [11, pp. 60–63]; also, since this procedure and its effect is similar to the implications of deriving principal components, the interested reader should review Maddala [24, pp. 193–194].

Finally, although the arbitrage pricing model is specified in terms of orthogonal factors, such factors can obviously be derived from dependent variables by the above methods. Thus, whether empirical tests are conducted on orthogonalized variables is of no consequence.

[4] Although many of the coefficients are not statistically significant, this would be expected for individual stock coefficients, versus portfolio coefficients. Yet, the results are surprisingly strong in terms of number of coefficients with the same sign within groups. This result also appears in other parts of the study. Nonparametric tests could be performed, although we have not done this, as we believe that the results in Section III are sufficiently strong for conclusions to be drawn.

Table 1

Equation (2), Excess Return Regressions 1/61 to 12/69

Group	Company Name:	B_0	B_1	B_2	B_3
GROWTH — FARRELL	1. Burroughs	.02*	1.20**	3.82	.68#
	2. Eastman Kodak	.01	.93**	−4.21	−.09
	3. IBM	.00	1.22**	8.93	−.51**
	4. International Telephone	.00	1.48**	−21.08#	−.62**
	5. Merck	.01*	1.06**	−26.54*	−.28
	6. Mion. Mining and Manufacturing	.00	1.17**	−32.36**	−.07
	7. Motorola	.00	1.61**	−20.36	−.49
	8. NCR	.00	1.29**	−.31	−.83**
	9. Pan American	.01	1.87**	−13.70	.07
	10. Polaroid	.01	1.45**	−1.90	−1.08**
	11. Sears	.00	.87**	8.68	−.32
	12. Texas Instruments	.00	1.38**	−12.61	−.86*
	13. Trane	.00	1.02**	2.00	−.26
	14. UAL. Inc.	.00	1.70**	14.15	.02
	15. Zenith	.00	1.46**	−13.18	−.49
STABLE — FARRELL	16. American Electric Power	.00	.81**	12.01	.50**
	17. American Home Products	.00	1.20**	−4.85	−.37
	18. C.I.T.	.00	.76**	16.29	.39
	19. CPC International	−.01	.94**	1.91	−.13
	20. Central and Southwest	.00	.64**	20.29	.33
	21. Coca-Cola Company	.01*	.91**	−8.05	−.21
	22. Columbia Gas	.00	.38	15.92*	.19
	23. Federated Department Stores	.00	.97**	−3.50	−.16
	24. Florida Power Corp.	.00	.88**	12.86	−.54*
	25. General Foods	.00	.94**	10.49	−.07
	26. Gillette Company	.00	1.13**	−10.3	−.01
	27. Hershey	.00	.87**	21.34	−.08
	28. Household Finance	.00	1.07**	32.33*	−.08
	29. Kraftco	.00	.91**	13.72	−.33#
	30. National Biscuit	.00	.59**	5.06	.03
	31. Proctor and Gamble	.00	.87**	−3.36	−.12
	32. Quaker Oats	.01	.91**	−7.97	.02
	33. Reynolds	.00	1.10**	−13.58	−.10
	34. Transamerica	.00	1.71**	13.88	−.02
	35. Virginia Electric	.00	.73**	10.40	.40#
CYCLICAL — FARRELL	36. American Can	.00	.68**	10.03	−.05
	37. American Standard	.01	1.04**	5.95	.16
	38. Bethlehem Steel	.00	1.00*	−10.75	.51**
	39. Borg-Warner	.00	1.10**	−23.27*	.39*
	40. Burlington Industries	.01*	1.01**	−12.95	.01
	41. Caterpillar	.01	1.09**	−11.13	.46
	42. Cinn. Milling	.01	1.50**	−10.23	.11
	43. Clark Equipment	.00	1.25**	15.35	−.19
	44. Continental Can	.01*	.79**	−3.13	.21
	45. Deere	.01	.79**	−12.31	.30
	46. Eaton, Yale & Towns	.01*	1.27**	−29.49*	.60**
	47. Gardner Denver	.00	1.09**	5.71	−.12
	48. Georgia Pacific	.01	1.34**	−7.11	.10
	49. Goodyear	.01	1.06**	−30.72*	.20
	50. Ingersoll-Rand Corporation	.00	1.22**	−8.80	.31

Group	Company Name:	B_0	B_1	B_2	B_3
	51. International Harvester	.00	.64**	−.75	.11
	52. International Paper	.00	1.27**	18.55	−.14
	53. Johns-Manville	.00	.95**	−15.78	.44*
	54. Joy Manufacturing	.01	1.75**	−36.66#	.24
	55. Kennecott	.01	.98**	−16.26	.27
	56. Mohasco	.01	1.43**	8.36	.21
	57. Monsanto Co.	.00	.89**	.92	.23
	58. National Lead	.00	.77**	−5.16	.46*
	59. National Steel Corporation	.00	1.30**	−22.70	.16
	60. Pullman	.00	1.67**	−14.75	.22
	61. Square D	.01	1.39**	−10.85	.25
	62. Sunbeam	.00	.94**	11.81	−.24
	63. Timkin	.00	.83**	−21.16*	.65**
OIL-FARRELL	64. Gulf Oil	.00	.79**	10.54	−.06
	65. Mobil	.00	.73**	26.18#	.29
	66. Shell Oil	.00	.88**	−3.79	.03
	67. Standard of California	.00	.70**	2.84	.11
	68. Standard Oil (Indiana)	.00	.93**	1.24	.07
	69. Texaco	.00	.87**	10.14	.02
ELTON AND GRUBER — PSEUDO 1	70. Allegheny Ludlum	.00	1.24**	−7.18	.16
	71. Armco Steel	.00	.94**	−15.07	.41*
	72. Bliss and Laughlin Industry	.01	1.04**	−2.90	−.25
	73. Brunswick Corporation	−.01	1.44**	−14.90	.70
	74. Chicago Pneumatic	.00	1.17**	−14.02	−.42#
	75. Clevite Corporation	.00	.55**	5.95	.38*
	76. Federal Mogul	.00	.97**	−9.79	.39#
	77. Interlake Steel	.00	.95**	−2.92	.80**
	78. Mesta Machine	.00	1.06**	−26.22#	.38
	79. Midland Ross	.01	.99**	−18.31	.45#
	80. Quaker State,	.02#	.77**	13.38	−.39
	81. Republic Steel	.00	1.11**	−19.29#	.31
	82. Smith, Kline and French	.00	1.21**	−6.90	−.43#
ELTON AND GRUBER — PSEUDO 2	83. Abbot Laboratories	.01	.74**	−11.01	−.35
	84. Allied Chemicals	.00	.96**	−18.83	.64**
	85. American Cyanamid	.00	1.15**	−10.29	.41#
	86. Celanese Corporation	.01	1.47**	−5.42	.18
	87. Dow Chemical	−.01	1.07**	14.04	−.31
	88. Dupont	.00	.91**	−3.68	.29#
	89. FMC Corporation	.00	1.50**	−4.1	.12
	90. Hercules	−.01	1.16**	−13.31	.07
	91. Johnson & Johnson	.01**	1.02**	−14.13	.00
	92. Libby-Owens Ford Class	.00	.89**	−12.18	.50**
	93. Pfizer Uchasid & Coct.	.01	1.37**	−2.46	.01
ELTON AND GRUBER — PSEUDO 3	94. Cities Services	.00	.70**	−5.74	.01
	95. Getty Oil	.01	1.43**	−18.32	−.01
	96. Kerr McGee Corporation	.00	1.49**	5.93	−.50#
	97. Lukene Steel	.00	1.66**	−29.37	.05
	98. Marathon Oil	.00	.60**	11.66	.18
	99. Phillips Petroleum	.00	1.16**	5.95	.01
EXTRA	100. ACF Industries	.02#	1.23**	−10.37	.33

** at .01 significance level
* at .05 significance level
# at 10 significance level

be stronger financially and therefore able to survive relatively better than other companies during times of economic stress. By contrast, the cyclical group exhibits positive coefficients with the Aa-Utility returns generally, thereby suggesting that cyclical stocks may fluctuate more during times when stock index declines are accompanied by fears of defaults. The stable stocks show very little consistent pattern within their coefficients.[5] By contrast, Farrell's Oil group has consistently positive coefficients on both bond return variables, as well as low betas on the stock series, thereby suggesting characteristics somewhat opposite to the growth stocks. Finally, Elton and Gruber's groups appear to have little pattern except for the general tendency toward positive coefficients on the Aa-Utility series, thereby suggesting that most of the stocks are somewhat similar to Farrell's cyclicals; this is not surprising, however, when the company composition in each of these pseudo-industries is considered. Yet, while these conclusions are appealing intuitively, their relationship to stock market factors needs statistical substantiation and this task is approached in the following section.

III. Imparting Economic Meaning to Stock Return Factors

To begin testing, principal components were analyzed for the 100 stock returns studied in the previous section.[6] Table 2 presents an overall summary of results about the proportion of variance explained by each of the first seven principal components in each of five time periods. In contrast to Elton and Gruber's finding that "... the first three principal components account for such a disportionate amount of the variance.... [6, p. 1210]," the results appear more in line with Farrell's finding that the market index explained about 30% from 1961–1969 while the other four factors explained an additional 15%. This difference from Elton and Gruber's findings may be due to the use of later time periods, but more likely, it is the result of a broader set of companies. It appears that the incremental effect from subsequent causes of variance declines in a rather continuous fashion.

Table 3 shows the correlation coefficients after varimax rotation of each stock's return with the first three principal components for the time period of Farrell's study.[7] The underlined coefficients are the highest coefficients for the respective

[5] There may be some slight tendency to have positive coefficients on the Government Bond variable, as well as negative coefficients on the Aa-Utility variable (i.e., 13 out of 20 in both cases), but this effect would be too weak to be of any statistical significance.

[6] Principal components analysis may be conceptualized intuitively as a mathematical procedure which creates the best "indexes" to explain a set of returns. The first principal component would be that index which explains the greatest amount of the total variation in the returns. The second principal component would be that index which explains the greatest amount of the remaining (residual) variation in the returns, and which is totally independent of (orthogonal to) the first index. The third principal component is the best explanator of the remaining unexplained, and is orthogonal to the first two, and so on for the other principal components. Because of the mathematical conditions, all principal components are derived simultaneously, not sequentially as the intuitive explanation might suggest. Also, the components are derived such that the return on any one security can be written as a linear combination of the components—plus an error term. For further and nonmathematical discussion of this, see the SPSS Manual [26]; for more technical discussion, see Johnston [17].

[7] Once the number of orthogonal coordinates have been specified, it is a simple calculation to rotate the coordinates while keeping them orthogonal; thus it is common to rotate the axis so as to maximize the explanatory power of the coordinates (factors).

Table 2

Proportion of Excess Stock Return Variance Explained By First Seven Principal Components

Principal Component	1961–69	1959–65	1966–71	1972–77	1959–77
1	34.8	33.2	36.1	37.8	35.3
2	4.8	5.2	5.4	8.8	4.8
3	4.1	4.8	4.6	5.7	4.2
4	3.0	3.1	3.3	4.6	3.9
5	2.6	2.8	2.9	2.9	2.0
6	2.3	2.5	2.8	2.4	1.9
7	2.1	2.4	2.6	2.2	1.7

stock, and as such, indicate which principal component each stock is most highly correlated with (e.g., for the first stock, 0.50 indicates that $(0.50)^2$ of its variance is accounted for by the third principal component). Since Farrell's clustering algorithm was based upon the correlation matrix of the stock returns, the high correspondence between the underlinings with his groups is to be expected. In fact, if our study had used exactly his stocks, as versus introducing some from Elton and Gruber's study, the underlinings would have matched exactly if four factors were computed. When four factors are computed, the oil stocks switch from loading on the second principal component to loading on the fourth principal component; thus, by extending past three factors, industry loadings begin to appear.

The existence of the principal components extracted above, however, does not guarantee that they are *systematic* factors, as shown by the following example. Assume that there is only one true market factor, but by examining a sample of 100 companies, a second principal component is found which explains 2% of the overall sample variation. Such a chance result might be highly insignificant statistically, and might result from random shocks on several stocks (especially if several stocks are in the same industry). If another sample were taken, and a second principal component were found, there is no reason to believe that this principal component would be caused by the same random shock that caused the second principal component in the first sample. Thus, there is a difficult problem analyzing the significance of the principal components since one cannot really know if they are the same principal component from one sample to the next. For a fuller discussion of the statistical difficulties of testing for this effect, see Roll and Ross [27].

If the principal components exist, however, and if they can be shown to be a linear combination of other economic variables, then by rather straightforward linear algebra, the economic variables make a rather strong case for the existence of *systematic* factors, especially if the economic variables are indexes based on other types of asset portfolios. This problem might be viewed as attempting to find a statistically significant relationship between two "set" variables, each of which itself is a linear function of other variables; more specifically, in the present context, the problem is to find vectors a and b such that there is a statistically

Table 3

Varimax Factor Coefficients for Three Components 1/61–12/69

	F_1	F_2	F_3
1. Burroughs	.27	−.08	.50
2. Eastman Kodak	.22	.27	.59
3. IBM	.22	.22	.76
4. International Telephone	.36	.25	.63
5. Merck	.14	.34	.56
6. Minn, Mining and Manufacturing	.30	.26	.59
7. Motorola	.44	−.05	.55
8. NCR	.19	.12	.65
9. Pan American	.59	.04	.37
10. Polaroid	.18	.03	.63
11. Sears	.20	.25	.51
12. Texas Instruments	.45	−.06	.45
13. Trane	.40	−.03	.45
14. UAL, Inc.	.51	.12	.41
15. Zenith	.41	−.04	.53
16. American Electric Power	.12	.74	.22
17. American Home Products	.24	.37	.49
18. C.I.T.	.29	.29	.26
19. CPC International	.23	.44	.39
20. Central and Southwest	.02	.62	.26
21. Coca-Cola Company	.16	.34	.62
22. Columbia Gas	.15	.57	.14
23. Federated Department Stores	.21	.29	.52
24. Florida Power Corp.	.00	.51	.44
25. General Foods	.17	.49	.49
26. Gillette Company	.19	.27	.54
27. Hershey	.29	.31	.32
28. Household Finance	.22	.42	.39
29. Kraftco	.26	.48	.35
30. National Biscuit	.09	.37	.31
31. Procter and Gamble	.17	.41	.48
32. Quaker Oats	.17	.44	.30
33. Reynolds	.36	.29	.39
34. Transamerica	.36	.62	.33
35. Virginia Electric	.04	.64	.25
36. American Can	.46	.31	.17
37. American Standard	.47	.20	.19
38. Bethlehem Steel	.72	.25	.18
39. Borg-Warner	.58	.24	.34
40. Burlington Industries	.52	.06	.30
41. Caterpillar	.57	.07	.31
42. Cincinnati Milling	.58	.23	.20
43. Clark Equipment	.50	.22	.29
44. Continental Can	.53	.31	.09
45. Deere	.45	.11	.20
46. Eaton, Yale and Towne	.58	.33	.23
47. Gardner Denver	.62	.23	.17
48. Georgia Pacific	.46	.30	.40
49. Goodyear	.51	.30	.28
50. Ingersoll-Rand Company	.67	.31	.28

Group		F_1	F_2	F_3
	51. International Harvester	.60	.16	.03
	52. International Paper	.52	.39	.33
	53. Johns-Manville	.69	.20	.18
	54. Joy Manufacturing	.57	.27	.19
	55. Kennecott	.47	.27	.19
	56. Mohasco	.58	.13	.30
	57. Monsanto Company	.54	.18	.15
	58. National Lead	.67	.30	.00
	59. National Steel Corporation	.66	.16	.31
	60. Pullman	.61	.19	.21
	61. Square D	.47	.41	.24
	62. Sunbeam	.37	.22	.21
	63. Timkin	.68	.25	.16
OIL-FARRELL	64. Gulf Oil	.22	.60	.14
OIL-FARRELL	65. Mobil	.23	.63	−.03
OIL-FARRELL	66. Shell Oil	.35	.53	.11
OIL-FARRELL	67. Standard of California	.27	.28	.01
OIL-FARRELL	68. Standard Oil (Indiana)	.26	.64	.03
OIL-FARRELL	69. Texaco	.28	.70	.10
ELTON AND GRUBER — PSEUDO 1	70. Allegheny Ludlum	.62	.24	.24
ELTON AND GRUBER — PSEUDO 1	71. Armco Steel	.61	.18	.30
ELTON AND GRUBER — PSEUDO 1	72. Bliss and Laughlin Industry	.36	.29	.25
ELTON AND GRUBER — PSEUDO 1	73. Brunswick Corporation	.36	.01	.42
ELTON AND GRUBER — PSEUDO 1	74. Chicago Pneumatic	.42	.15	.41
ELTON AND GRUBER — PSEUDO 1	75. Clevite Corporation	.14	.63	.13
ELTON AND GRUBER — PSEUDO 1	76. Federal Mogul	.49	.32	.19
ELTON AND GRUBER — PSEUDO 1	77. Interlake Steel	.68	.20	.15
ELTON AND GRUBER — PSEUDO 1	78. Mesta Machine	.67	.02	.26
ELTON AND GRUBER — PSEUDO 1	79. Midland Ross	.60	.26	.12
ELTON AND GRUBER — PSEUDO 1	80. Quaker State	.26	.23	.12
ELTON AND GRUBER — PSEUDO 1	81. Republic Steel	.68	.24	.28
ELTON AND GRUBER — PSEUDO 1	82. Smith, Kline and French	.24	.33	.53
ELTON AND GRUBER — PSEUDO 2	83. Abbot Laboratories	.13	.37	.22
ELTON AND GRUBER — PSEUDO 2	84. Allied Chemicals	.69	.18	.19
ELTON AND GRUBER — PSEUDO 2	85. American Cyanamid	.55	.36	.25
ELTON AND GRUBER — PSEUDO 2	86. Celanese Corporation	.54	.27	.34
ELTON AND GRUBER — PSEUDO 2	87. Dow Chemical	.52	.18	.35
ELTON AND GRUBER — PSEUDO 2	88. Dupont	.51	.43	.27
ELTON AND GRUBER — PSEUDO 2	89. FMC Corporation	.49	.40	.43
ELTON AND GRUBER — PSEUDO 2	90. Hercules	.34	.41	.32
ELTON AND GRUBER — PSEUDO 2	91. Johnson & Johnson	.35	.12	.49
ELTON AND GRUBER — PSEUDO 2	92. Libby-Owens Ford Glass	.58	.35	.16
ELTON AND GRUBER — PSEUDO 2	93. Pfizer Uchasid & Coct.	.26	.51	.51
ELTON AND GRUBER — PSEUDO 3	94. Cities Services	.39	.52	.17
ELTON AND GRUBER — PSEUDO 3	95. Getty Oil	.39	.27	.31
ELTON AND GRUBER — PSEUDO 3	96. Kerrr McGee Corporation	.24	.45	.39
ELTON AND GRUBER — PSEUDO 3	97. Lukens Steel	.54	.07	.32
ELTON AND GRUBER — PSEUDO 3	98. Marathon Oil	.35	.48	.16
ELTON AND GRUBER — PSEUDO 3	99. Phillips Petroleum	.42	.57	.12
	100. ACF Industries	.47	.34	.17

ote: Underlined Coefficient is largest; if tie, both underlined.

significant relationship between the vectors u and v where

$$u_t = a_1x_{1t} + a_2x_{2t} + a_3x_{3t}$$

$$v_t = b_1F_{1t} + b_2F_{2t} + b_3F_{3t}$$

and x_{1t}, x_{2t}, and x_{3t} are the three indexes defined in Section II and F_{1t}, F_{2t}, and F_{3t} are the values of the first three principal component indexes for period t. As such, this can be viewed as a standard canonical correlation problem.

Table 4 contains the results of using canonical correlation to determine whether a linear combination of the first three principal components (F_1, F_2, and F_3) was related to a linear combination of x_1 (stock market), x_2 (3-month Government) and x_3 (Aa Utility). Also, principal components indexes for four principal components (labelled G_1, G_2, G_3 and G_4) were tested for a linear relationship to x_1, x_2, and x_3. The body of the table contains statistics for the eigenvalues, Wilk's lambda which is Chi-square distributed, degrees of freedom, significance level of Wilk's lambda, and coefficients of the linear combinations (i.e., the components of the vectors a and b above). For example, the first canonical correlations were for the seven-year period 1959–65, and two independent linear combinations were found between the three principal component indexes (F_1, F_2, and F_3).[8] Approximately 97% of the variance of the first linear combination of the three principal components was related to variance of the linear combination of the three return indexes. Of the remaining unexplained variation in each set of variables, a second set of linear combinations was found which explained 9% of the remaining variation and which was statistically significant at the 11.7% level.

As can be seen from the coefficients on the set of market indexes, the first linear combination was due almost entirely to the stock market variable (x_1); this variable affected all three principal components (the absolute value of the coefficients may be thought of as relative importance of each variable in the linear combination).[9] Further, the second linear combination was mostly the result of the additional impact of the three-month government return variable (x_2). Finally, when the market return indexes were related to the indexes of the first four principal components, only one significant linear combination was found in the first two periods.[10]

[8] An intuitive way to view canonical correlation is to think of two new variables being created, each of these being a linear combination of the previous sets of variables. Each linear combination will such that it maximizes the correlation between the two new variables. The amount of the variance explained in one of the new variables by the other new variable is the correlation coefficient squared, or equivalently the eigenvalue. Further, as with principal components analysis, further linear combinations can be computed based upon the residual variation unexplained by the (previous) linear combinations. The mathematical procedure insures that each linear combination will be independent of (orthogonal to) the other linear combinations. The statistical significance of each linear combination can be tested by Wilk's lambda as explained in Bartlett [3]. For an elementary non-mathematical introduction see Nie et. al. [26], and for a more mathematical presentation Johnston [17].

[9] It should be remembered that these principal components were calculated after varimax rotation, and thus the components can be thought of as being rotated so as to explain stock groupings. Thus, the above results indicated that the stock market affected each type of stock group, which is the result that would be expected.

Because the movements of stock groups were the focus of our analysis, the varimax rotation was appropriate. Another approach, which was not undertaken, however, would be to rotate the principal components to reflect the underlying independent forces (e.g., shifts in expectations for stocks versus Government bonds versus corporate bonds). In such an analysis, one would expect coefficients to load in (1, 0, 0) combinations on the components while the return index variables would have more evenly weighted coefficients. Such a result would suggest that changes in one of the expectations variables affects all types of market returns.

[10] While it might seem that four components would be easier to fit, this is not necessarily so. Because the components were constrained to be orthogonal, if only three linear relationships were truly present, then the creation of the fourth component would cause the indexes to be artificial in their movement.

Table 4

Canonical Correlations on Rotated Factors

		Eigen-Value	Wilk's λ	x^2	df.	Significance	F_1	F_2	F_3	G_1	G_2	G_3	G_4	x_1	x_2	x_3
						a. Initial Tests										
1959–65																
Linear Comb.	#1(F)	.97	.025	293.9	9	.001	.60	.65	.46					1.00	.01	−.00
	#2(F)	.09	.911	7.4	4	.117	−.78	.35	.53					.10	1.02	−.18
Linear Comb.	#1(G)	.97	.025	292.3	12	.001				.54	.57	.44	.43	1.00	.01	−.00
1966–71																
Linear Comb.	#1(F)	.98	.020	263.7	9	.001	.57	.56	.61					1.00	−.00	−.01
	#2(F)	.10	.900	7.2	4	.128	−.57	−.26	.78					.49	−.51	−.79
Linear Comb.	#1(G)	.98	.015	280.9	12	.000				.51	.50	.48	.50	1.01	−.01	−.01
1972–77																
Linear Comb.	#1(F)	.98	.016	277.5	9	.001	.66	.61	.43					1.00	.02	−.01
	#2(F)	.21	.785	16.3	4	.003	.74	−.43	−.52					.10	1.02	−.17
Linear Comb.	#1(G)	.98	.013	288.5	12	.001				.56	.48	.58	.34	1.00	.02	.01
	#2(G)	.24	.700	23.9	6	.001				.76	.01	−.51	−.39	−.04	.98	.10
	#3(G)	.08	.919	5.7	2	.059				−.31	.79	−.03	−.53	.55	.32	−1.15
						b. Subsequent Tests										
Jan. 1959–Dec. 1962																
Linear Comb.	#1(F)	.99	.011	142.0	9	.001	.43	.69	.37					1.00	.04	−.03
Linear Comb.	#1(G)	.99	.010	143.4	12	.001				.40	.60	.27	.34	1.01	.04	−.03
	#2(G)	.16	.746	9.1	6	.169				.85	.68	−37	.28	.12	.98	−.61
	#3(G)	.11	.892	3.5	2	.171				.41	.40	−.98	−.08	.07	.38	.83
Jan. 1966–Dec. 1970																
Linear Comb.	#1(F)	.98	.017	227.1	9	.001	.55	.56	.69					1.00	−.02	.00
	#2(F)	.19	.807	11.9	4	.018	−.38	−.41	.78					.46	−.11	−1.05
Linear Comb.	#1(G)	.98	.013	238.3	12	.001				.49	.51	.50	.51	1.01	−.02	−.00
	#2(G)	.19	.804	12.0	6	.062				−.44	−.47	.37	.59	.46	−.11	1.04
Jan. 1972–Dec. 1975																
Linear Comb.	#1(F)	.98	.013	188.4	9	.001	.66	.64	.43					1.00	.01	−.00
	#2(F)	.25	.753	12.3	4	.015	.72	−.41	−.48					.21	1.03	−.26
Linear Comb.	#1(G)	.98	.010	196.9	12	.001				.56	.41	.61	.36	1.00	.01	.01
	#2(G)	.29	.604	21.7	6	.001				.81	−.18	−.49	−.26	−.02	.94	.24
	#3(G)	.15	.851	7.0	2	.031				−.15	.75	−.06	−.63	.51	.41	−1.10

Part (a) of Table 4 provides evidence for certain important conclusions. First, as would be expected, the stock market variable is always the major influence in explaining the movement of stock groups. This influence is evident in all of the periods under study, as shown jointly by the high eigenvalues and near 1.0 coefficients on the x_1 variable. Second, effects due to the returns on a three-month Government were a second independent influence on the movements of the stock groups, with the movements being different for groups loading on the second and third principal components (F_2, F_3) versus those loading on (F_1). Third, the Aa-Utility Bond variable (x_3) did not have a statistically significant influence except in the third period, and then only for the tests on four principal components.

Three potential reasons for statistical insignificance of the Aa-Utility bond came to mind. First, because the Aa-Utility series had a longer maturity than the Government series, changes in the shape of the yield curve are not being adequately captured by the short-term Government series; to test this, other runs with longer maturity series were done, but the same conclusions were reached with the only differences suggesting the high sample sensitivity due to multicollinearity. Second, because of the high multicollinearity between the two interest rate variables, the second one would not be expected to contribute very much *additional* explanatory power. Third, and really a subset of the first reason, the yield differential between quality grades exhibits wide changes only near the top and bottoms of market "cycles" (which are an "ex post" phenomenon in the eyes of the sampler).

To test whether cyclical multicollinearity was covering-up the impact of x_3, selected subperiods were chosen. The chosen periods were designed to capture a near-peak to a near-trough phase of the stock market cycle (1/59–12/62, 1/66–12/70, and 1/72–12/75). To avoid the criticism of sampling bias from selecting subperiods (since by trying enough subperiods eventually statistically high significance would be expected due to chance), the principal components indexes were those which had been estimated already for the three *total* periods in part (a) of the Table 4. Thus, if statistical significance were found during the subperiods, this would be based upon linear combinations of the principal components which were calculated *from all observations in the period*, not just from the stock group movements during the shorter down-market phase.

Part (b) of Table 4 provides the statistical results for the shorter down-market phases. Here, a third linear combination showed up in two of the three subperiods if one is willing to accept significance at the 17% level (however, given the expected multicollinearity this may not be unreasonable). However, although the coefficients of x_3 do indicate that it is the variable affecting this third linear combination, the coefficients on the G-variables are less consistent as to any consistent impact on the factor.

In summary, the contents and analysis of Table 4 provide important findings for future research. First, it documents the role of interest rate variables as important causes of the differential movements of stock groups. Second, it points up the immense problems of multicollinearity for efficient estimation in using multibeta models to test risk/return tradeoffs in a multibeta context; while Litzenberger and Ramaswamy [25] have taken an important step in this direction with two independent variables, the step to three independent variables may be nearly impossible statistically.[11] Finally, because the results appear to be sensitive to the part of an ex-post cycle studied, bias in beta estimation would be expected in studies which used arbitrary periods for estimating beta if these periods did not provide an *ex-post* sample space which exactly coincided with *ex ante* expectations (Sharpe [34]). The possibility of such bias, or equivalently the "nonstationarity" of the estimated betas is being examined in a separate paper [12], but initial results suggest that nonstationarity still exists even after the interest rate variables are added to the estimating equation.

V. Conclusions and Implications

The purpose of this study was both to assign economic meaning to stock market factors and to determine the extent to which these factors were related to the prices of capital in the bond market. We feel that our analysis has achieved these goals. The returns from stock groups such as Farrell's stables-cyclical-and-growth were shown to relate to returns in the Government bond market and to corporate bonds with default risk. Further, the returns of bond market variables were found to relate the stock market factors derived from all 100 stocks, although those bonds with default risk show a very weak relationship.

Some interesting questions are raised by this research for further investigations:

(1) Does the existence of a relationship between *ex post* factors and *ex post* return series indicate that there is an *ex ante* risk/return relationship with the risk space being a function of the three beta coefficients?;
(2) What are the *economic* determinants of the changed expectations which are reflected in the *ex post* returns of the three *financial* series used in this study?;[12]
(3) Does a multi-beta model with interest rate and bankruptcy variables reduce beta non-stationarity such as found by Kon and Lau [23]?;
(4) What are the implications of the foregoing for portfolio performance measurement?

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[11] In fact, except for the method used here (i.e., statistical testing on subperiod observations estimated from the whole period), the multicollinearity seems to be a formidable problem. While we did try some variable transformations (e.g., the bankruptcy risk was approximated by the change in the yield differential between the Aa series and the Government series), the results were similar to those in the tests discussed above.

[12] This question is raised because the formulations of Long [23], Grauer and Litzenberger [15], and Fisher [10] certainly suggest that the economic variables of concern may be investor expectations of *real return*, *inflation rate*, and *additional premium for risk*.

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