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EMPIRICAL TESTS OF MULTI- FACTOR PRICING MODEL

The Arbitrage Pricing Theory: Some Empirical Results

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I. Introduction

THE ACCUMULATION of empirical evidence inconsistent with the simple one-period capital asset pricing models of Sharpe (1964), Lintner (1965), and Black (1972) indicates that alternative models of capital market equilibrium deserve investigation. A minimum requirement for any alternative model should be that it explains the empirical anomalies which arise within the simple CAPM. One such anomaly is observed when portfolios are formed on the basis of firm size (see Banz [1978] and Reinganum [1980b, c]). Small firms systematically experienced average rates of return nearly 20% per year greater than those of large firms, even after accounting for differences in estimated betas. An adequately specified model of equilibrium should eliminate these persistent "abnormal" returns.

The arbitrage pricing theory (APT) proposed by Ross (1976) is a plausible alternative to the simple one-factor CAPM. The appeal of the APT probably comes from its implication that compensation for bearing risk may be comprised of several risk premia, rather than just one risk premium as in the CAPM. Roll and Ross (1979) claim to find empirically at least three and probably four factors that are priced from 1962 to 1972. However, Roll and Ross do not offer an economic interpretation of these factors and admit that their test is a weak one.

This research investigates empirically whether a parsimonious arbitrage pricing model can account for the differences in average returns between small firms and large firms which are traded on the New York and American Stock Exchanges. If a parsimonious APT can explain these differences, then one might feel more confident in using the APT as an empirical replacement for the CAPM, even though the economic nature of the risks is not fully understood. However, if a parsimonious APT fails to account for differences in average returns, then the model ought to be rejected. There can be no justification for embracing a complicated model if it does not convey any more information than does the simple model.

II. The APT and Method of Estimation

The arbitrage pricing theory is essentially based on three assumptions. First, capital markets are perfectly competitive. Secondly, investors always prefer more

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wealth to less wealth with certainty. And, lastly, the stochastic process generating asset returns can be represented as a k-factor model of the form:

$$\tilde{R}_i = E_i + b_{i1}\tilde{\delta}_1 + \cdots + b_{ik}\tilde{\delta}_k + \tilde{\epsilon}_i \quad \text{for} \quad i = 1, \cdots, N \tag{1}$$

where:

$\tilde{R}_i$ = return on asset i;
E_i = expected return for asset i;
b_{ik} = reaction in asset i's returns to movements in the common factor $\tilde{\delta}_k$;
$\tilde{\delta}_k$ = a common factor, with a zero mean, that influences the returns on all assets;
$\tilde{\epsilon}_i$ = an idiosyncratic effect on asset i's return which, by assumption, is completely diversifiable in large portfolios and has a mean of zero;
N = number of assets.

The economic argument of the APT is a simple one. In equilibrium, the return on a zero-investment, zero-systematic-risk portfolio is zero, as long as the idiosyncratic effects vanish in a large portfolio. This economic reasoning, combined with theory from linear algebra, implies that the expected return on any asset i can be expressed as:

$$E_i = \lambda_0 + \lambda_1 b_{i1} + \cdots + \lambda_k b_{ik} \tag{2}$$

The term λ_0 can be viewed as the expected return on an asset with zero systematic risk (i.e., $b_{01} = b_{02} = b_{0k} = 0$). The weights $\lambda_1, \cdots, \lambda_k$ can be interpreted as factor risk premia, and the b_i's reflect the pricing relationship between the risk premia and asset i. In other words, the assets' expected returns are jointly based upon the assets' reaction coefficients and the common risk premia.

The jump from exposition of the theory to estimation of a model is not trivial. Unlike the "market model," in which the common factor (i.e., some market return) is directly observable, the APT does not identify the common factors. Hence, one cannot estimate the b coefficients in equation (1) using regression techniques, as one might wish to do if the δ's were known. Another estimation technique is needed. Fortunately, the stochastic process assumed in equation (1) permits one to estimate the b coefficients by using factor analysis (see Lawley and Maxwell [1963] or Harman [1976]). In the framework of factor analysis, the b coefficients are usually referred to as the factor loadings, and the k vectors, $\underline{b}_1$ through $\underline{b}_k$, are called the factor-loading vectors, where the dimension of each of these vectors is $N \times 1$.

While in principle the estimation of factor loadings is simple, in practice it is not. The decomposition of just a 500×500 variance-covariance matrix of security returns, if not numerically impossible, would be exorbitantly expensive given current computer technology. The method of estimating individual security factor loadings employed in this paper is based upon a recent contribution by Chen (1980). Chen demonstrates that if one knows the factor loadings for, say, k portfolios, then one can compute the k factor loadings for all securities. By using Chen's technique, one need perform the factor analysis on only one group of securities or portfolios. Furthermore, the factor loadings for all securities will correspond to the same common factors; for example, b_{1k} and b_{Nk} will measure the sensitivities of asset 1 and asset N to the common factor $\tilde{\delta}_k$.

III. The Empirical Test of the APT

The test of the APT conducted in this paper is broken down into two stages. First, in year $Y - 1$, factor loadings are estimated for all securities. Securities with similar factor loadings are grouped into control portfolios. In year Y, excess security returns are computed by subtracting the daily control portfolio returns from the daily security returns. With excess security returns in hand, the second stage of the test involves ranking securities on the basis of the market value of their common stock at the end of year $Y - 1$. The excess returns in year Y of the firms in the bottom 10% of this ranking are combined with equal weights to form the excess returns of the smallest market value portfolio, MV1. Similarly, the excess returns of firms in the other deciles are combined to create the excess returns of the other nine market value portfolios, MV2 through MV10. Under the null hypothesis, the ten market value portfolios should possess the identical average excess returns which should be indistinguishable from zero. If the ten portfolios do not possess identical average excess returns, then the evidence would be inconsistent with the APT.

The Data

The securities selected for aalysis in this study are a subset of the stocks contained in the December 1978 version of the University of Chicago's Center for Research in Security Prices daily tape files. The CRSP daily stock return file includes all securities that have traded on the New York and American Stock Exchanges since July of 1962. The sample of firms analyzed in this study changes yearly. To qualify for inclusion in the factor analysis in a given calender year, a firm has to satisfy the following requirements. First, the initial trading date for the security must be before the beginning of the year. Secondly, the last trading date must be after that year (except for 1978). Thirdly, during the calendar year, the security needs at least one hundred one-day returns. Finally, year-end common share and price data has to be available in order to compute the market value of the common stock. If a firm meets all these criteria, then it qualifies for inclusion in the factor analysis. The number of firms which satisfied these requirements ranges from 1457 in 1963 to over 2500 in the mid-1970s.

Estimation of Factor Loadings

The first stage of the testing strategy requires estimates of security factor loadings in each of the sixteen years from 1963 through 1978. In each year, firms that qualify for inclusion in the factor analysis are randomly divided into thirty portfolios of equal size.[1] The daily returns of the securities within each portfolio are combined using equal weights to create the daily portfolio return. With the daily returns of the thirty randomly selected portfolios, a variance-covariance matrix is estimated. The factor analysis is then performed on this matrix.[2]

[1] Actually, if the number of securities that qualified for inclusion in the factor analysis is not evenly divisible by thirty, each remaining security is added to a different portfolio. Thus, some of the portfolios will possess one more security than others.

[2] The factor analysis is performed with a subroutine provided by the International Mathematical and Statistical Libraries, Inc. (IMSL). The IMSL subroutine is based upon Joreskog (1967) and

The next step in the estimation procedure is to determine the factor loadings (b coefficients) of the individual securities using Chen's technique. The details of this construction can be found in Reinganum (1980a). The basic idea of the process is simple. Once one determines the factor loadings for k assets, then the factor loadings for any other asset, p, can be calculated, as long as one knows the pth asset's covariance with each of the other k assets.

Estimating Excess Security Returns

Since Roll and Ross contend that at least three factors are priced, excess security returns will be computed based upon the three-, four-, and five-factor models. With the factor loadings for individual securities estimated in year $Y-1$, the next step is to construct control portfolios for the purpose of calculating excess security returns in year Y. The idea behind the control portfolios is to group securities with similar factor loadings together. According to the APT, securities with similar factor loadings ought to possess similar average returns. The number of control portfolios varies with the number of factors. For example, with the three-factor model, sixty-four control portfolios are formed; each control portfolio has the same number of securities.[3] Membership in one of the sixty-four control portfolios is determined as follows. First, the N factor loadings for each common factor are ranked from low to high. Then, three cutoff points (i.e., four groups) are determined for each of the three sets of factor loadings. Finally, the factor loadings for a given security fall into one of sixty-four (4^3) groups, and all securities that are in that group form a control portfolio. For the four- and five-factor models, eighty-one (3^4) and thirty-two (2^5) control portfolios are created, respectively.

In year Y, the daily returns of a control portfolio are calculated by applying equal weights to the daily returns of the securities within that portfolio. Excess security returns are computed by subtracting the daily returns of the appropriate control portfolio from the daily returns of the security. Since all securities within a control portfolio possess similar factor loadings, the average excess daily return over time for any security should be near zero. Since the excess daily security returns are, in effect, risk-adjusted returns, they are used to test the APT's ability to explain the firm size effect.[4]

The Test of the APT Using Market Value Portfolios

To test whether the firm size effect "disappears" within the APT, a simple procedure, similar to the one employed by Reinganum (1980c) is followed. First,

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Clarke (1970). The initial factor analysis is also performed on a stratified sample of thirty individual securities. The stratification is based upon firm size. Results are not significantly altered using factor loadings estimated with this technique [see Reinganum (1980a)].

[3] If the number of securities is not evenly divisible by 64, then some control portfolios have one more security than others.

[4] If stock market values are highly correlated with the loadings of any one factor, then excess returns computed with control portfolios might very well be related to firm size, even though one of the factors, in fact, accounts for the size effect. As an empirical matter, though, the correlation between stock market values and the loadings on factors ranges between −.09 and .07. Thus, this potential problem does not appear to be empirically significant.

the market value of the common stock outstanding is calculated at the end of year $Y - 1$ for all firms that qualified for the factor analysis in year $Y - 1$. Firms are than ranked on the basis of their stock's total market value. The excess returns of the firms in the bottom 10% of this distribution in year Y are equal-weighted to form the daily excess return of the lowest market value portfolio, MV1. The excess returns of firms in the other nine deciles are similarly combined to create the daily excess returns of portfolios MV2 through MV10. Each year the composition of securities within the ten market value portfolios changes as the rankings change and as firms are added and deleted from the sample. Each year the average excess return for each market value portfolio is calculated. If the ten average excess returns jointly equal zero, then the empirical evidence supports the APT.

The results of the tests indicate that the evidence is inconsistent with the APT. Table 1 contains the mean excess returns for the ten market value portfolios based upon the three-, four-, and five-factor models.[5] In these tables, the statistics are computed using daily portfolio excess returns from 1964 through 1978. The results might be interpreted as illustrating the average firm size effects over the entire fifteen-year period.

The average excess returns of the ten portfolios are clearly not equal to zero, regardless of whether a three-, four-, or five-factor model is employed to account for APT risk. The small firm portfolio, MV1, possesses a positive and statistically significant average excess return. On the other hand, the large firm portfolio, MV10, is characterized by a negative, but statistically significant, average excess return. In fact, the difference in point estimates of the mean excess returns between the small and large firm portfolios (MV1–MV10) is about .1 percent per trading day. Given that each year contains about 250 trading days, a conservative estimate of the mean difference in excess returns between the small and large firms might be about twenty-five percent ($250 \times .1\%$) on an annual basis. Obviously, this phenomenon is of economic as well as statistical significance.

One also observes in Table 1 that the point estimates of the mean excess portfolio returns of MV1 through MV10 are perfectly ordered with firm size. That is, the rank correlation between average portfolio excess returns and medium stock values within a portfolio is exactly -1.0. While the mean excess returns between portfolios appear to differ, one can formally test for equality between the mean excess returns. The test can be formulated within Zellner's seemingly unrelated regression framework. The advantage of such an approach is that it takes into account any contemporaneous correlations between portfolio returns. Furthermore, the test does not require that the variances of the ten portfolios be identical. A potential drawback with this test statistic is that only asymptotically does its distribution converge to a known one. However, if one ignores the approximation of the true contemporaneous covariance matrix by the sample one, then the appropriate test statistic has an F distribution under the null hypothesis (see Theil [1971], p. 314). The test for equality between the ten portfolio means is implemented by imposing nine linear constraints across equations and assessing the adequacy of these restrictions. The 5% and 1% limits for

[5] Results based on security factor loadings estimated in the same year as excess returns are measured are similar to the ones reported in the text [see Reinganum (1980a)].

Table 1

Mean Excess Returns of the Ten Market Value Portfolios Based on the Three-, Four-, and Five-Factor Models

Portfolio	Three Factor Model	Four Factor Model	Five Factor Model
*MV*1	.671	.656	.719
	(.069)	(.067)	(.073)
*MV*2	.244	.228	.255
	(.049)	(.046)	(.051)
*MV*3	.130	.120	.131
	(.040	(.040)	(.040)
*MV*4	.007	−.005	.005
	(.037)	(.031)	(.033)
*MV*5	−.048	−.059	−.063
	(.030)	(.030)	(.030
*MV*6	−.076	−.086	−.099
	(.030)	(.029)	(.030)
*MV*7	−.138	−.145	−.164
	(.028)	(.028)	(.029)
*MV*8	−.154	−.162	−.175
	(.031)	(.031)	(.032)
*MV*9	−.228	−.299	−.247
	(.036)	(.036)	(.037)
*MV*10	−.317	−.301	−.346
	(.046)	(.046)	(.050)

Control portfolio returns are subtracted from security returns to calculate excess returns for the factor analytic models. A mean excess return is based upon 3756 trading day returns from 1964 through 1978. The mean excess returns are multiplied by 1000 for reporting purposes. *MV*1 is the smallest market value portfolio; *MV*10 is the largest market value portfolio. Standard errors are in parentheses.

F (9, ∞) are 1.88 and 2.41, respectively. However, since each portfolio contains 3756 observations, the 1% significance level is probably a more appropriate criterion for testing the hypothesis than the 5% level. The computed values of the test statistic are 11.43, 10.92, and 11.63 for portfolio excess returns based on the three-, four-, and five-factor models, respectively. Thus, the evidence indicates the presence of a substantial size effect, even after controlling for APT risk.

Table 2 presents the mean differences in daily excess returns between the low and the high market value portfolios on a year-by-year basis. The mean differences are calculated using excess returns from the three-, four-, and five-factor models. The annual mean differences seem to corroborate the findings based on the overall period. In at least nine of the fifteen years, the estimated mean difference is more than two standard errors greater than zero. In only one of the fifteen years is the estimated mean difference more than two standard errors less than zero. Thus, the year-by-year results appear to confirm the conclusions drawn

Table 2

Mean Differences in Daily Excess Returns Based on the Three-, Four-, and Five-Factor Models Between the Low Market Value and High Market Value Portfolios on a Yearly basis

Year	Three Factor Model	Four Factor Model	Five Factor Model
1964	.427	.378	.333
	(.199)	(.199)	(.204)
1965	1.008	1.016	1.247
	(.217)	(.220)	(.240)
1966	.288	.351	.343
	(.313)	(.328)	(.333)
1967	2.875	2.902	3.102
	(.311)	(.310)	(.320)
1968	2.602	2.513	2.808
	(.364)	(.354)	(.393)
1969	−.582	−.658	−.738
	(.269)	(.275)	(.288)
1970	−.013	−.133	−.185
	(.433)	(.459)	(.463)
1971	.690	.676	.707
	(.299)	(.309)	(.329)
1972	−.130	−.147	−.096
	(.283)	(.272)	(.291)
1973	−.392	−.387	−.411
	(.440)	(.425)	(.462)
1974	1.684	1.671	1.833
	(.676)	(.671)	(.723)
1975	2.291	2.358	2.409
	(.497)	(.505)	(.537)
1976	1.604	1.509	1.845
	(.424)	(.416)	(.503)
1977	1.377	1.297	1.504
	(.303)	(.304)	(.326)
1978	1.254	1.167	1.456
	(.421)	(.401)	(.462)

A mean excess return is based upon 3756 trading day returns from 1964 through 1978. The mean excess returns are multiplied by 1000 for reporting purposes. Standard errors are in parentheses.

from the analysis of the entire period. Namely, the low market value portfolio experiences excess returns significantly greater on average than those of the high market value portfolio, regardless of whether excess returns are constructed with a three-, four- or five-factor model.

IV. Conclusion

The arbitrage pricing theory offers a parametric alternative to the simple one-period capital asset pricing model. A minimum empirical requirement for an alternative model should be that it accounts for empirical anomalies that arise within the CAPM. The evidence in this paper indicates that a parsimonious APT fails this test. That is, portfolios of small firms earn on average 20% per year more than portfolios of large firms, even after controlling for APT risk. This result is detected regardless of whether APT risk is measured with a three-, four-, or five-factor model.

While the evidence is inconsistent with the arbitrage pricing theory, one ought to realize that several component hypotheses are being jointly tested, and that the tests do not definitely reveal which hypothesis or hypotheses cannot be supported. For example, the stochastic process generating returns of financial securities may not be linear (for example, see Jarrow and Rudd [1980]). Or one may not be able to completely diversify away the idiosyncratic variances. Thus, the factor loadings need not be cross-sectionally related to security expected returns, even if the linear stochastic process assumed is true. Another possibility is that arbitrage profit opportunities may have existed on the New York and American Stock Exchanges during 1964 through 1978. Thus, while the evidence does not support the APT, the tests do not pinpoint exactly the source of error. Nonetheless, one cannot conclude from the evidence that the arbitrage pricing theory is an adequate model of asset pricing. Furthermore, the empirical mystery that motivated this research seems no closer to a solution. The difference in average returns obtained by grouping portfolios on the basis of firm size is still not accounted for by empirical representations of capital market equilibrium.

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