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On the Valuation of Federal Loan Guarantees to Corporations

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ABSTRACT

Since 1956, Federal Loan Guarantee Programs have expanded to the point where recipients of guarantees represent most segments of the economy. Considerable debate centers on the determination of the magnitude of the liability of the Federal Government that is represented by these programs. This paper illustrates how option pricing techniques may be used to obtain estimates of the purely pecuniary costs of loan guarantees, interest saving to the firm on senior and junior debt, and implicit present value profitability indices of projects.

I. Introduction

BEFORE 1956 MOST FEDERAL loan guarantee programs were for home mortgages. Since then, these programs have diversified to the point where recipients of guarantees represent most segments of the economy. It is estimated that as of 1978 the aggregate face value of Federal loan guarantees was in excess of $300 billion and that these programs have an annual rate of growth approximating 9%.[1] A major reason for the popularity of loan guarantee programs is that they are not included in the federal budget although they represent a liability of the federal government. Considerable debate centers on the determination of the magnitude of this liability. This paper illustrates how option pricing techniques may be used to obtain estimates of the purely pecuniary costs of loan guarantees.[2]

The analysis done here is limited to a discussion of loan guarantees to corporations. This choice was made because of the increased importance of these programs (e.g., the loan guarantees to Lockheed and Chrysler) and because the valuation of corporate liabilities is particularly well suited to analysis using option

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[1] For details of loan programs see the "Catalog of Federal Loan Guarantee Programs," USGPO, September, 1977, #052-070-04263-0 issued by the Subcommittee of Economic Stabilization of the Committee of Banking, Finance and Urban Affairs, House of Representatives. It provides a detailed listing and description of most programs currently in existence. An economic evaluation of Federal Loan Guarantee programs is provided in "Federal Credit Assistance," USGAO, PAD-79-31, May 31, 1978.

[2] Note that the analysis abstracts from non-pecuniary costs and benefits. Clearly, in an overall evaluation of loan guarantee programs (i.e. a cost-benefit analysis), these factors are important and must not be ignored. However, this type of analysis is beyond the scope of this paper.

pricing techniques. Representative of many corporate loan guarantee programs is the Steel Program. It was initiated on June 23, 1978 by the Economic Development Administration of the Commerce Department to guarantee loans to firms in the basic steel industry. As of June 1979 the EDA had received applications for guarantees on loans with an aggregate face value in excess of $630 million and had approved one loan guarantee for a subordinated debt issue with a face value of $21.25 million.[3]

The financial provisions of the steel program are typical of most programs and will be used as the basis of analysis in the remainder of the paper. They include:

1. The requested financial assistance must not be available from other sources on terms and conditions that would permit the accomplishment of the project;
2. Guarantees may not exceed 90% of the value of the loan;
3. Not less than 15% of the aggregate cost of the project must be supplied as equity capital or as a loan repayable in no shorter period of time and at no faster an amortization rate than the guaranteed loan; and
4. The guarantee is provided without cost.

This paper has the following organization. Section II describes the paradigm of the paper and uses it to value pre-existing financial claims of the firm. Section III introduces a project financed with subordinated debt and discusses why a firm might not wish to undertake the project in the absence of a loan guarantee. Section IV introduces a loan guarantee. Section V presents estimates of the values of loan guarantees. Also tabulated are estimates of the interest savings to senior and junior debt holders of a firm arising out of a guarantee, and the magnitude of a guarantee required to induce a firm to undertake a project that has a present value profitability index of less than unity. Section VI presents a brief summary of the major results.

II. The Paradigm

The valuation of loan guarantees is related to a multitude of factors including the underlying riskiness of the firm, the firm's disbursement policy, the costs associated with bankruptcy, insolvency, taxes and floatation, and the variability of interest rates. In order to do meaningful analysis some abstraction is necessary. Here it is assumed that:

(A1) There exists a riskless asset paying a known constant interest rate (r);

(A2) There are no taxes, indivisibilities, transactions costs, or administrative costs;[4]

[3] On June 23, 1978 Korf Industries, Inc. became the first recipient of a loan guarantee under this program. The guarantee was for 90% of $21,250,000 loan extended by a consortium of sixteen banks. The parent of Korf supplied an additional $3,750,000 of equity capital to complete the financing of a $25 million working capital project. Korf is a privately held, foreign-owned U.S. steel maker. See references [12], [17], and [18] for further details of the Steel Program.

[4] The reviewer of this article noted that "the existence of corporate taxes may lead to a less obvious cost to the government of a loan guarantee in that guarantees typically lead a firm to use more tax-deductible debt and less equity in its capital structure than it would have in the absence of the guarantee (assuming that the project would have been undertaken in any case)."

(A3) Trading takes place continuously;

(A4) The rate of return of the market value of the underlying assets of the firm (V) follows a diffusion. Specifically, $(dV/V) = \mu dt + \sigma dz$ where V is the value of the firm, μ and σ are respectively, the known, constant instantaneous expected rates of return and standard deviation of the rate of return of the assets of the firm,[5] and dz is a Gauss-Wiener process with mean zero and variance dt;

(A5) All bonds issued by the firm are discount bonds maturing in year T;

(A6) Stockholders will receive constant proportional dividends (δ) throughout the life of the corporation; and

(A7) The firm liquidates at time T at which time bond holders are to receive the full face value of their bonds, and equity holders are to receive any residual value. However, if the value of the firm is insufficient to redeem all debt issues, the assets of the firm are distributed to the bond holders in order of priority until all assets are exhausted.

This set of assumptions allows financial claims of the firm to be valued using Merton's [11] proportional dividend option pricing model.[6,7] Before introducing a project and obtaining the value of a loan guarantee it is convenient to illustrate how this paradigm may be used to value the pre-existing financial claims (the debt and equity) of the firm.

Consider a corporation that has issued stock and bonds where at time $t = t_0$ bonds represent $100^*(1 - s)\%$ of the value of the firm. s is the fraction of equity in the firm's capital structure ($0 \leqslant s \leqslant 1$, when $s = 1$ the firm is all equity).

At time t_0, let:

$S(t_0)$ = the market value of the firm's common stock;

$B(t_0)$ = the market value of the firm's bonds, $B(t_0) \equiv (1 - s)V(t_0)$; (for simplicity, in the remainder of the paper, it will be assumed that the firm initially has only one class of bonds outstanding); and

$V(t_0)$ = the aggregate market value of the firm, $V(t_0) \equiv S(t_0) + B(t_0)$.

An implication of the model constructed here is that the position of the stockholders, $S(t)$, is equivalent to a European call option on the value of the firm plus a claim to all future dividends. That is, when the firm terminates, shareholders have the option to purchase the assets of the firm from the bond holders at the face value of the debt; in addition, throughout the life of the firm, they will receive dividends. If X is the face value of all outstanding debt, X will be identical

[5] While holding the variance constant may not be a bad assumption for large firms, if the analyses of this paper were to be applied to value loan guarantees supplied to small businesses, a model that accounts for changing variance might be more appropriate.

[6] Two obvious criticisms of the chosen paradigm are: (1) by not requiring periodic interest and dividend payments the potential for early default is ignored; and (2) no account is taken of potential interest rate fluctuations. Techniques exist for handling parts of each of these problems (see for example Brennan and Schwartz [4], Cox, Ingersoll and Ross [6], Cox and Ross [7], Schwartz [14] and Smith [15]) and perhaps should be examined before detailed policy implications are drawn. However as a first approximation, the chosen paradigm seems quite adequate.

[7] Option pricing techniques have been used by several authors to value financial claims. See for example Black and Cox [2], Black and Scholes [3], Merton [8, 9], Ramaswamy [13], and Smith [16].

to the exercise price of the call. Letting $C[V(t_0), X, \tau]$ denote the value of this call at time $t = t_0$, X may be determined implicitly as the solution to equation (1),

$$S(t_0) \equiv sV(t_0) = C[V(t_0), X, \tau] + [1 - e^{-\delta\tau}]V(t_0), \tag{1}$$

where τ is the term of the option ($\tau \equiv T - t_0$) and second term on the far RHS is the present value of the future proportional dividend stream.

Under assumptions (A1)-(A7), Merton has illustrated that the value of the call will be,

$$C[V(t), X, \tau] = V(t)e^{-\delta\tau}N\{d_1\} - Xe^{-r\tau}N\{d_2\}, \tag{2}$$

where $N\{\cdot\}$ is the cumulative normal distribution function, $d_1 = [\ln(V(t)/X) + (r - \delta + \sigma^2/2)\tau]/\sigma\sqrt{\tau}$, and $d_2 = d_1 - \sigma\sqrt{\tau}$. Knowledge of X, (the existence and uniqueness of which is guaranteed since $\partial C[\cdot]/\partial X < 0$) allows computation of the firm's nominal interest rate on debt, k_B, as $k_B \equiv (1/\tau)\ln[X/B(T)]$. Of course, for an all equity firm, $s = 1$ and $X = 0$.

III. The Introduction of a Project Financed with Subordinated Debt in the Absence of a Loan Guarantee

Guarantees typically arise when projects are politically desirable but financially infeasible. That is, projects for which guarantees are proposed typically have a present value profitability index of less than unity;[8] this is the essence of the financial provision (1) presented in the introduction. This phenomenon will be incorporated into the analysis by introducing a discrepancy between the cost and the market value of a project. Thus, the firm will be *required* to take on an investment of cost I that has a market value of ξI where it is assumed that $0 \leq \xi \leq 1$; ξ is the present value profitability index of the project. (Given a choice, value maximizing firms would only take on projects where $\xi \geq 1$). It is intuitive that the value of a loan guarantee increases with an increase in the percent of the loan that is guaranteed. Therefore, this characterization of the value of a project allows an evaluation of the functional relationship between the percentage guarantee required to make a firm just indifferent to undertaking a project and ξ, when $\xi < 1$. This relationship is analyzed in section V.

In general, when a firm undertakes a new project its underlying riskiness is altered. However, it is often the case that projects simply expand the scale of operations of the firm. Because it simplifies the analysis, throughout the remainder of the paper only projects that do not alter the underlying variance of the rate of return of the firm's assets are considered.

When undertaken, the project will be financed with a fraction γ, $0 \leq \gamma \leq 1$, of subordinated debt and with a fraction $(1 - \gamma)$ of equity (for the Steel Program, maximum $\gamma = 0.85$).

[8] For example, the expressed purpose of the Korf guarantee was to help preserve the jobs of 2,000 workers at steelrod mills in Georgetown, South Carolina and Beaumont, Texas. It is implicit that without the guarantee the firm would not have found it financially advantageous to maintain the jobs.

At time $t = t_0$, after financing the project, the value of the firm's underlying assets becomes:

$$V_A(t_0) \equiv V(t_0) + \xi I = B_A(t_0) + D(t_0) + S_A(t_0), \tag{3}$$

where $D(t_0)$ denotes the initial market value of the subordinated debt, and subscript A signifies value after financing the project. The assumption of a competitive loan market implies that $D(t_0) = \gamma I$.

In this setting, by undertaking the project, the firm decreases the value of equity held by pre-existing share holders. This decrease in value has two causes: First, by assumption, the project has a present value profitability index of less than unity. The loss, $(1 - \xi)I$, will be borne by pre-existing shareholders as long as the protective covenants of the senior debt prevent confiscatory activities. Second, failure to finance a scale expanding project using fractional contributions from bond and stock holders at their pre-project market value proportions will redistribute value between the bond and share holders of the firm. In the paradigm of this section, value is conserved and hence the gains and losses to claim holders would be offsetting. In the example described by relation (3), the addition of junior debt increases the likelihood that senior debt holders will be paid off in full at time T and therefore increases the value of the senior debt at time t_0. In order to float new debt and new equity, each issue must have a market value that is no less than its contribution to the firm. Therefore, the increase in value of the senior debt is at the expense of the original equity holders. Of course, if the firm were initially all equity (i.e., $s = 1$), no redistribution could take place as long as the new issues were fairly priced.

By assumption the equity holders cannot avoid the loss caused by $\xi < 1$. However, it is irrational for equity holders to knowingly redistribute wealth away from themselves, and, therefore, in the remainder of the paper, it is assumed that the firm *recapitalizes* at the time it takes on a new investment. Assuming that the protective covenants of senior debt holders prevent redistributions away from them, it must be the case that the post-recapitalization value of senior debt is not decreased by the addition of the project. (The recapitalization may be thought of as the repurchase and reissuance of senior debt). Thus, the face value of the senior debt, X, must be reset such that $B_A(t_0) = B(t_0)$. X_A may be obtained implicity from relation (4).

$$V_A(t_0) - B(t_0) = C[V_A(t_0), X_A, \tau] + [1 - e^{-\delta\tau}]V_A(t_0). \tag{4}$$

Relation (4) views the value of the combined claims of the subordinated debt holders, the new equity holders, and the old equity holders as a call on the firm at the new exercise price of the senior debt, plus a claim on future dividends.

The senior debt is less risky after the addition of the project and hence the face value of senior debt and its promised interest rate will decrease, $X_A < X$ and $k_B^A \equiv [1/\tau] \ln[X_A/B(t_0)] < k_B$. The magnitude of this decrease is tabulated in section V.

The after-project value of common stock (new plus old) may be expressed as the option to buy the firm at T for the combined exercise prices of the junior and the senior debt, $X_A + X_D$ (where X_D is the, as yet, unknown face value of the junior debt), plus a claim on future dividends. Thus,

$$S_A(t_0) = C[V_A(t_0), X_A + X_D, \tau] + [1 - e^{-\delta\tau}]V_A(t_0).^9 \quad (5)$$

The value of senior debt is held constant, and therefore $S_A(t)$ may be alternatively expressed (see relation (6)) as the value of old equity, plus the value of new equity, less the loss in value caused by the introduction of the project,

$$S_A(t_0) = S(t_0) + (1 - \gamma)I - (1 - \xi)I. \quad (6)$$

Equating relations (5) and (6) (remembering that $D(t_0) = \gamma I$), allows computation of $X_A + X_D$ (and hence X_D, as X_A is known). Once X_D is determined, the promised interest rate on the junior debt may be computed as $k_D \equiv (1/\tau)\ \ln(X_D/D(t_0))$. In the next section, k_D is compared with the market interest rate on junior debt that would prevail under a loan guarantee.

IV. The Introduction of a Loan Guarantee

Suppose that before financing the project the firm obtains a loan guarantee at zero cost for a fraction α, $0 \leqslant \alpha \leqslant 1$, of its obligation to subordinated debt (i.e. the face value).[10] (Maximum α for the Steel Program is 0.90). The value of this loan guarantee may be computed in stages. First, consider the value of the senior debt. In the paradigm of the paper, the value of senior debt (assuming that the firm recapitalizes in the manner described above) is *not* affected by the existence of a loan guarantee. Thus, $B_G(t_0) = B(t_0)$ where G signifies value under the loan guarantee. This result obtains because all distribution to subordinated debt and equity (dividends, interest and principal) are made after the firm has retired the senior debt. In a setting wherein the firm makes interest payments throughout the life of the firm, rather than just on the termination date, the existence of the guarantee could be of marginal benefit to senior debt holders. This follows because the guarantee decreases the size of the interest obligation to subordinated debt thereby increasing the likelihood that senior debt will be paid its interest obligation and that the terminal value of the firm will be such that senior debt holders recover their principal. Here the promised interest distributions to senior debt holders that would be necessary to maintain the value of senior debt at its

[9] One may legitimately wonder, as did members of the Subcommittee on Economic Stabilization of the Committee of Banking, Finance and Urban Affairs, why firms in need of loan guarantees are allowed to pay dividends. In fact the allowed dividends payment under a guarantee is a subject of negotiation between the firm and the guaranteeing agency. Of course, the model used here could accommodate a change in the proportional dividend that was allowed, and numerical methods (see footnote 6) could accommodate alternative dividend policies.

[10] Actual programs guarantee either the initial contribution of the subordinated debt holders (γI) or the initial contribution plus interest payments. The latter assumption is used here.

pre-project value would be reduced first by the existence of junior debt and then again by the existence of a loan guarantee.[11]

Shareholders receive dividends plus all value that remains after both classes of debt have been redeemed. Hence,

$$S_G(t_0) = C[V_A(t_0), X_A + X_D^G, \tau] + [1 - e^{-\delta\tau}]V_A(t_0), \tag{7}$$

where X_D^G is the face value of the guaranteed subordinated debt and is yet to be determined. The value of the guaranteed subordinated debt plus the common stock may be dichotomized into a riskless and a risky component as illustrated in relation (8),

$$D(t_0) + S_G(t_0) = \alpha X_D^G e^{-r\tau} + [C[V_A(t_0), X_A + \alpha X_D^G, \tau] + [1 - e^{-\delta\tau}]V_A(t_0)]. \tag{8}$$

The first term on the RHS of relation (8) is the present value of the guaranteed (and therefore riskless) portion of the subordinated debt. It is the guaranteed payment (αX_D^G) discounted at the riskless rate of interest. The second term is the value of the risky portion of subordinated debt plus the value of equity; it is the value that remains after the senior debt and the riskless portion of subordinated debt have been paid, plus the value of the proportional dividend stream. A competitive loan market implies that $D(t_0) = \gamma I$; thus, combining relations (7) and (8) allows computation of the appropriate face value of the subordinated debt (X_D^G) and hence the promised interest rate on subordinated debt (k_D^G) under the loan guarantee. Putting the pieces of the firm back together yields the value of the debt and equity of the firm under the guarantee as $V_G(t_0) \equiv B(t_0) + S_G(t_0) + D(t_0)$, and hence the value of the guarantee as, $G[\cdot] = V_G(t_0) - V_A(t_0)$.

$G[\cdot]$ represents the theoretical value of the guarantee and would be the cost to the government if it were to buy the guarantee in a competitive market. $G[\cdot]$ is value transferred from the government to the firm. By construction, $G[\cdot]$ accrues entirely to the pre-existing equity holders and thus the net loss (or gain) to the pre-existing equity holders, assuming that the firm finances the project and recapitalizes, is $G[\cdot] - (1 - \xi)I$.[12]

V. Model Estimates

The parameters for the following tables were chosen to bracket historical market averages ($r = 0.08$, $\sigma^2 = 0.04$ for equity on a yearly basis, Debt/Total Value =

[11] Note that under the steel program not only will senior debt holders benefit by the existence of junior debt, but, in addition, it is typically the case that since the senior debt holders have an on-going relationship with the firm, they will be chosen to extend the loan that is guaranteed by the government. Furthermore, senior debtholders often provide most of the data used by the EDA in its determination of whether or not to guarantee a particular loan.

Failure of the firm to recapitalize as suggested above increases the value of senior debt. If the senior and junior debt holders are one in the same, the increase in value caused by a failure to recapitalize can be viewed as decreasing the risk associated with the junior debt. When combined with a loan guarantee the junior debt may become effectively riskless. If this were to happen it would violate a provision of the Steel Program which requires that junior debt holders bear some risk.

[12] It is of course true that this value is derived in a partial equilibrium framework—the creation of the guarantee and the funding of the loan are assumed to not alter equilibrium prices.

0.25) while allowing for meaningful deviations. The yield on equity was assumed constant at 5% and hence the payout to equity as a fraction of the value of the firm (δ) is an increasing function of the value of equity relative to the value of the firm (i.e. $\delta = 0.05(S/V)$). Throughout this section the initial value of the firm is $V = \$100$; the percentage guarantee (for Tables I and II) is the maximum allowed under the steel program, $\alpha = 90\%$; the percentage of equity financing in the project is the minimum required under the steel program, $(1 - \gamma) = 15\%$; and for Tables I and II the present value profitability index of the project is unity, $\xi = 1.0$.

In Table I, the columns labeled "G" display the dollar value of loan guarantees (by construction these numbers are identical to the percentage of the initial value of the firm represented by the guarantee). G is shown to be an increasing function of the size of the investment (I), of the standard deviation of the rate of return of the firm's assets (σ), of the leverage of the firm $(1 - S)$ and of the term of the loan (T). G ranges in value from close to zero to approximately 15% of the initial value of the firm. Note that the combination of assumptions that $\xi \equiv 1$ and that the firm recapitalizes when it takes on the project, implies that the value of equity held by pre-existing shareholders increases by $G[\cdot]$.

The columns labeled $(G/I)\%$ display the values of guarantees as percentages of the values of investments. $(G/I)\%$ is shown to be an increasing function of σ, T, and s and is a relatively constant function of I. $(G/I)\%$ ranges in value approximately zero to just over 66%.

Table II displays pre- and post-project equilibrium interest rates on senior debt (k_B and k_B^A) together with pre- and post-guarantee equilibrium interest rates on subordinated debt (k_D and k_D^G). k_B and k_B^A are increasing functions of T, σ, and s. By construction, k_B is independent of I, while the decrease in risk caused by the introduction of the project implies that k_B^A is a decreasing function of I. k_D and k_D^G increase with I and σ.

The provisions of corporate loan guarantee programs indicate that the designers were motivated by a desire to provide a subsidy to firms so that they will undertake projects that are otherwise not financially viable, and not by a desire to increase the value of equity. Not financially viable has been interpreted in this paper to mean projects with present value profitability indices of less than unity. It is therefore of interest to determine the implicit present value profitability index as a function of the percentage loan guarantee that would make a firm just indifferent to undertaking a project. Table III displays values of $\xi * 100$ (the present value profitability index) as a function of α (the fractional loan guarantee) for alternative parameters assuming investments with costs of \$10 and \$20 respectively. It illustrates that ξ is a decreasing function of α, a decreasing function of the variance of the rate of return of the firm's assets (σ^2), a decreasing function of the term (T), and tends to be a decreasing function of leverage (an increasing function of s).[13]

[13] Note that given an α it is possible to determine an implicit ξ. However, the reverse is not true because, for sufficiently small ξ, if the government is constrained to not guaranteeing more than the face value of the debt ($\alpha \leq 1$), there need not exist an equalibrating α. This fact, along with others, calls into question the usefulness of loan guarantees as opposed to direct subsidies to firms.

Table I

Absolute and Relative Values of Loan Guarantees

	$s = 0.85$		$s = 0.75$		$s = 0.65$	
I	G	$(G/I)\%$	G	$(G/I)\%$	G	$(G/I)\%$
			$T = 5$ years	$\sigma = 0.15$		
5	0.00	0.00	0.00	0.00	0.02	0.40
10	0.00	0.00	0.01	0.10	0.06	0.60
15	0.00	0.00	0.01	0.07	0.10	0.67
20	0.00	0.00	0.02	0.10	0.15	0.75
25	0.00	0.00	0.04	0.16	0.21	0.84
			$T = 5$ years	$\sigma = 0.25$		
5	0.01	0.20	0.13	2.60	0.40	8.00
10	0.05	0.50	0.30	3.00	0.79	7.90
15	0.11	0.73	0.50	3.00	1.21	8.07
20	0.20	1.00	0.73	3.65	1.66	8.30
25	0.32	1.28	0.99	3.96	2.10	8.40
			$T = 5$ years	$\sigma = 0.35$		
5	0.19	3.80	0.57	11.40	1.04	20.80
10	0.45	4.50	1.16	11.60	2.05	20.50
15	0.77	5.13	1.80	12.00	3.06	20.40
20	1.17	5.85	2.50	12.50	4.06	20.30
25	1.63	6.52	3.22	12.88	5.06	20.24
			$T = 10$ years	$\sigma = 0.15$		
5	0.01	0.20	0.13	2.60	0.41	8.20
10	0.05	0.50	0.29	2.90	0.83	8.30
15	0.11	0.73	0.51	3.40	1.27	8.47
20	0.19	0.95	0.74	3.70	1.74	8.70
25	0.32	1.28	1.04	4.16	2.23	8.92
			$T = 10$ years	$\sigma = 0.25$		
5	0.34	6.80	0.85	17.00	1.43	28.60
10	0.78	7.80	1.76	17.60	2.82	28.20
15	1.33	8.87	2.71	18.07	4.18	27.87
20	1.98	9.90	3.71	18.55	5.52	27.60
25	2.70	10.80	4.75	19.00	6.85	27.40
			$T = 10$ years	$\sigma = 0.35$		
5	1.05	21.00	1.78	35.60	2.39	47.80
10	2.21	22.10	3.54	35.40	4.67	46.70
15	3.48	23.20	5.31	35.40	6.88	45.87
20	4.82	24.10	7.08	35.40	9.03	45.15
25	6.26	25.04	8.86	33.04	11.13	44.52
			$T = 15$ years	$\sigma = 0.15$		
5	0.16	3.20	0.59	11.80	1.16	23.20
10	0.41	4.10	1.24	12.40	2.28	22.80
15	0.76	5.07	1.96	13.07	3.40	22.67
20	1.22	6.10	2.75	13.75	4.51	22.55
25	1.77	7.08	3.61	14.44	5.63	22.52

Table I—*continued*

	T = 15 years			$\sigma = 0.25$		
	$s = 0.85$		$s = 0.75$		$s = 0.65$	
I	G	(G/I)%	G	(G/I)%	G	(G/I)%
5	1.00	20.00	1.77	35.40	2.44	48.80
10	2.14	21.40	3.55	35.50	4.75	47.50
15	3.40	22.67	5.33	35.53	6.98	46.53
20	4.77	23.85	7.12	35.60	9.14	45.70
25	6.22	24.88	8.93	35.72	11.24	44.96
			T = 15 years		$\sigma = 0.35$	
5	2.03	40.60	2.80	56.00	3.31	66.20
10	4.12	41.20	5.53	55.30	6.49	64.90
15	6.27	41.80	8.20	54.67	9.57	63.80
20	8.47	42.35	10.83	54.15	12.55	62.75
25	10.70	42.80	13.43	53.72	15.45	61.80

Table II

Interest Rates on Senior Debt With and Without the Project and on Junior Debt With and Without the Guarantee

		T = 5 years			$\sigma = 0.15$				
	$k_B = 8.01$		$s = 0.85$	$k_B = 8.01$		$s = 0.75$	$k_B = 8.02$		$s = 0.65$
I	k_B^A	k_D	k_D^G	k_B^A	k_D	k_D^G	k_B^A	k_D	k_D^G
5	8.01	8.00	8.00	8.01	8.00	8.00	8.00	8.16	8.04
10	8.01	8.00	8.00	8.01	8.00	8.00	8.00	8.16	8.04
15	8.01	8.00	8.00	8.01	8.02	8.01	8.00	8.22	8.04
20	8.01	8.00	8.00	8.01	8.04	8.01	8.00	8.24	8.06
25	8.01	8.00	8.00	8.01	8.06	8.02	8.00	8.29	8.07
	$k_B = 8.01$		T = 5 years	$k_B = 8.11$		$\sigma = 0.25$	$k_B = 8.44$		
5	8.01	8.10	8.04	8.09	8.74	8.13	8.37	10.25	8.22
10	8.01	8.16	8.04	8.07	8.86	8.13	8.30	10.36	8.30
15	8.01	8.24	8.07	8.06	8.99	8.16	8.25	10.46	8.33
20	8.01	8.33	8.08	8.05	9.12	8.19	8.21	10.60	8.35
25	8.01	8.43	8.11	8.04	9.27	8.23	8.18	10.72	8.39
	$k_B = 8.20$		T = 5 years	$k_B = 8.83$		$\sigma = 0.35$	$k_B = 9.89$		
5	8.17	9.09	8.13	8.73	11.20	8.30	9.67	14.41	8.57
10	8.15	9.32	8.17	8.63	11.50	8.39	9.48	14.54	8.61
15	8.12	9.58	8.24	8.56	11.79	8.45	9.32	14.73	8.66
20	8.11	9.87	8.30	8.49	12.05	8.48	9.18	14.93	8.68
25	8.08	10.14	8.36	8.44	12.32	8.53	9.06	15.13	8.71
	$k_B = 8.00$		T = 10 years	$k_B = 8.04$		$\sigma = 0.15$	$k_B = 8.20$		
5	8.00	8.04	8.01	8.04	8.36	8.04	8.16	9.18	8.13
10	8.00	8.06	8.01	8.02	8.44	8.07	8.13	9.25	8.16
15	8.00	8.11	8.02	8.02	8.51	8.08	8.10	9.32	8.18
20	8.00	8.16	8.04	8.02	8.60	8.12	8.08	9.41	8.20
25	8.00	8.23	8.07	8.01	8.69	8.14	8.07	9.50	8.22

Table II—*continued*

	T = 10 years			σ = 0.25					
	k_B = 8.20 s = 0.85			k_B = 8.11 s = 0.75			k_B = 9.47 s = 0.65		
I	k_B^A	k_D	k_D^G	k_B^A	k_D	k_D^G	k_B^A	k_D	k_D^G
5	8.17	8.96	8.10	8.63	9.97	8.25	9.30	12.73	8.40
10	8.15	9.10	8.16	8.56	10.14	8.28	9.16	12.85	8.41
15	8.14	9.42	8.19	8.50	10.32	8.31	9.04	13.01	8.43
20	8.12	9.64	8.23	8.44	11.50	8.34	8.94	13.16	8.44
25	8.10	9.88	8.26	8.40	10.68	8.36	8.84	13.31	8.46
	k_B = 9.09	T = 10 years		k_B = 10.42		σ = 0.35	k_B = 11.95		
5	9.01	11.27	8.28	10.23	14.18	8.46	11.63	17.36	8.63
10	8.93	11.70	8.34	10.06	14.52	8.50	11.34	17.55	8.63
15	8.85	12.12	8.38	9.91	14.84	8.51	11.11	17.76	8.63
20	8.79	12.52	8.42	9.78	15.16	8.53	10.89	17.98	8.63
25	8.74	12.92	8.44	9.66	15.49	8.55	10.70	18.20	8.64
	k_B = 8.04	T = 15 years		k_B = 8.23		σ = 0.15	k_B = 8.58		
5	8.03	8.28	8.04	8.19	9.16	8.12	8.49	10.47	8.22
10	8.02	8.40	8.07	8.16	9.29	8.15	8.42	10.57	8.24
15	8.02	8.54	8.09	8.13	9.44	8.17	8.35	10.69	8.25
20	8.01	8.67	8.12	8.11	9.61	8.20	8.30	10.84	8.27
25	8.01	8.84	8.15	8.10	9.78	8.22	8.26	10.98	8.29
	k_b = 8.63	T = 15 years		k_b = 9.49		σ = 0.25	k_B = 10.52		
5	8.56	10.12	8.20	9.35	12.18	8.33	10.28	14.52	8.43
10	8.51	10.42	8.24	9.23	12.47	8.34	10.08	14.71	8.43
15	8.47	10.75	8.27	9.13	12.75	8.35	9.90	14.90	8.43
20	8.43	11.09	8.28	9.04	13.04	8.37	9.74	15.13	8.43
25	8.40	11.43	8.31	8.96	13.34	8.37	9.61	15.36	8.44
	k_B = 10.16	T = 15 years		k_B = 11.89		σ = 0.35	k_B = 13.67		
5	10.02	13.08	8.35	11.63	16.19	8.48	13.28	19.41	8.56
10	9.90	13.58	8.38	11.42	16.00	8.48	12.92	19.83	8.56
15	9.79	14.08	8.40	11.21	17.01	8.49	12.62	19.90	8.56
20	9.70	14.61	8.42	11.03	17.42	8.49	12.35	22.22	8.56
25	9.60	15.13	8.43	10.87	17.85	8.49	12.11	20.55	8.56

VI. Summary

This paper used option pricing techniques to examine the properties of loan guarantees to corporations. Evidence concerning the value of guarantees was presented in Table I; the effect of the introduction of a subordinated and a loan guarantee on contract interest rates to senior and junior debt was displayed in Table II; and the implicit present value profitability index of projects as a function of the fraction of the loan that is guaranteed was computed in Table III. Using these tables, and tempering all comments with the caveat that one must be cognizant of the model being used, leads to the following conclusions. For firms with variances and capital structures approximating those of the market as a whole (σ = 0.15, s = 0.75) the costs of these guarantees are relatively small for five and ten year terms (4.16% or less of the value of the project) but nonnegligible

Table III

Implicit Present Value Profitability Index (ξ^* 100) of a Pr oject as a function of the Percentage Guarantee

α^*100	$\sigma = 0.15$, $s = 0.85$	$\sigma = 0.15$, $s = 0.75$	$\sigma = 0.15$, $s = 0.65$	$\sigma = 0.25$, $s = 0.85$	$\sigma = 0.25$, $s = 0.75$	$\sigma = 0.25$, $s = 0.65$	$\sigma = 0.35$, $s = 0.85$	$\sigma = 0.35$, $s = 0.75$	$\sigma = 0.35$, $s = 0.65$
$T = 5$ years $I = 10$									
100	99	99	98	98	97	91	94	88	77
80	99	99	99	99	97	94	97	89	81
60	99	99	99	99	98	95	97	92	84
40	99	99	99	99	98	97	98	95	91
20	99	99	99	99	98	98	99	97	95
$T = 10$ years $I = 10$									
100	98	97	91	91	80	69	75	61	48
80	99	97	92	94	84	72	80	66	53
60	99	98	95	95	88	78	84	72	59
40	99	98	97	97	92	84	89	80	69
20	99	99	98	98	97	92	94	88	81
$T = 15$ years $I = 10$									
100	95	86	73	75	61	47	55	41	31
80	97	89	78	80	66	53	59	45	34
60	97	92	83	84	72	59	66	50	39
40	98	95	88	91	78	67	75	59	47
20	99	97	94	95	88	80	84	72	61
$T = 5$ years $I = 20$									
100	99	99	99	98	95	90	93	85	77
80	99	99	99	99	97	92	95	89	80
60	99	99	99	99	98	95	97	92	85
40	99	99	99	99	99	97	98	95	91
20	99	99	99	99	99	98	99	98	95
$T = 10$ years $I = 20$									
100	98	95	89	88	78	67	72	59	48
80	99	97	92	91	83	73	77	65	53
60	99	98	95	95	88	78	83	71	60
40	99	99	97	97	92	85	88	78	69
20	99	99	98	98	96	92	95	88	81
$T = 25$ years $I = 20$									
100	92	83	73	73	59	48	53	41	31
80	95	88	78	77	64	52	58	45	34
60	97	91	83	83	70	59	64	50	39
40	98	95	89	88	78	67	72	58	47
20	99	98	95	95	88	80	84	72	61

for fifteen year terms (14.44% or less). The predicted interest savings on subordinated debt are, respectively, 55 and 155 bases points or less. As firm's move away from the market mold (especially as σ increases) the costs of guarantees and the interest savings increase dramatically. Similarly, implied profitability indices are close to one for firms with parameters approximating those of the market but change significantly as the firm's parameters deviates from those of the market (especially as σ increases).

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