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THOMAS HO and ANTHONY SAUNDERS*

ABSTRACT

Most models of bank failure have assumed that the path towards bankruptcy or insolvency is smooth and continuous. As a consequence a number of early-warning systems have been suggested in the banking and financial literature to aid regulators in the identification of potential problem banks. However, these systems may be of little use when the path towards failure is explosive, involving a sudden crash or catastrophe. This paper seeks to examine such cases by applying the theory of catastrophes to bank failure. A model is developed to show how the interaction between bank management, regulators and depositors can induce catastrophic failure. It is argued that there is a crucial relationship between the power of regulatory intervention and depositors confidence levels which is both necessary and sufficient for catastrophe to occur. It is also argued that catastrophe appears to be more likely for large money market banks rather than small banks. Finally, some suggestions are made for regulatory policy and for further research in the area.

I. Introduction

THE RECENT FAILURES OF large banks, such as the Franklin National Bank of New York and the U.S. National Bank of San Diego, has generated a considerable literature on the causes and effects of bank failure. A number of authors have been paticularly concerned with the development of early-warning systems to aid regulators in the identification of problem banks.[1] The major premise underlying most of these models is that bank failure, or the time path towards failure, is a continuous process so that it is possible to identify a potential failing bank from an analysis of ex-post balance-sheet data. As Pettway and Sinkey [10 p. 2] have stated: "Since the failure path tends to be a decaying one rather than an explosive one, identifying banks with financial difficulties is a first step toward achieving the failure prevention goal."

However, an interesting question which has yet to be examined is the circumstances under which the trend towards bank failure is discontinuous or explosive, involving a sudden crash or catastrophe. Although in the post-depression period such crashes have been relatively rare, particularly among larger banks, recent trends towards lower capital-deposit ratios, poorer quality loans and a reliance on borrowed funds makes an examination of catastrophe conditions increasingly pertinent. Especially as the unexpected failure of even a single large bank could adversely effect depositors and investors confidence as to the soundness of the banking system[2] and ultimately of the payments mechanism itself.

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[1] See for example, Altman [1], Martin [6], Pettway and Sinkey [10], and Sinkey [12] [13] [15].

[2] Murphy [8] in a recent article discusses the adverse impact on depositors and investors of the public disclosure of a bank as a problem bank.

The objective of this paper is to develop a model of bank failure based on the theory of catastrophes originally suggested by Thom [16] [17], and developed by Zeeman [19–24], in order to examine the conditions under which catastrophic bank failure might occur. Although the model is, by its very nature, descriptive and exploratory, an attempt is made to relate the dynamics of failure to the underlying interactions of relevant economic agents, namely bank management, bank relgulators and bank depositors.

In Section II of the paper, catastrophe theory is briefly described and some of its applications discussed. In Section III a formal model is developed that allows for catastrophic failure as a special case of bank failure. Finally, Section IV is a summary and conclusion containing some suggestions for possible extensions of the model.

II. Catastrophe Theory

In this section we present an outline of catastrophe theory, deferring a more rigorous mathematical discusion until Section III. Basically René Thom's [14] catastrophe theorem proves how, through the use of singularities of maps, continuous changes in system parameter(s) can cause catastrophic behavior in a state or dependent variable. While the proof of the theorem is fairly complex, its great advantage lies in its ability to describe elementary catastrophes by simple polynomial equations and the use of graphs that are visually informative. In general terms the simplest way to express the dynamics of the system under consideration is by a differential equation of the form

$$\dot{z} = f(z, a, b, c \ldots.) \tag{1}$$

where z is some state variable, $\dot{z}$ is its time derivative and a, b, and c are parameters that affect the behavior of z. In equilibrium $\dot{z}$ must be zero. However, interest lies in the adjustment path of z, that is, how z changes over time as the control variables change, in order to maintain long-run equilibrium.

Thom's theorem shows that catastrophe models have three important properties. The first is the property of divergence whereby small continuous changes in initial conditions (or parameters) can lead to large discontinuous (catastrophic) changes in state variables. This type of behavior contrasts with traditional, Hamiltonian, dynamic systems in which small changes in initial conditions result only in small changes in the state variable. The second is bifurcation (or asymmetries) in the behavior of z, as certain parameters increase or decrease, bifurcation implies that there will be a discontinuous jump in z at some value of parameter b when b is increasing that is different from the behavior of z when b is decreasing. As a result z will be multi-valued over a certain range of b. The third is the property of stability, the catastrophe condition is robust to marginal changes in the structural relationships underlying the system. Hence, although some relationships may (marginally) change, catastrophe will still occur, albeit, at different values of the control variables in the system.

It is illuminating to discuss a simple, more concrete, example of catastrophe, assuming only two control variables a and b, and one state variable z. Hence, suppose that

$$\dot{z} = f(z, a, b) \tag{2}$$

If, for the purposes of comparison, we were to graphically represent equation (2) to show the no catastrophe case, the equilibrium relationships between these variables might look as in Figure 1.

The surface of the plane in Figure 1 represents all values of a, b and z for which $\dot{z} = 0$, i.e. the surface is the locus of all equilibrium points. Consider point v on that surface. For any small changes in a or b there will be a slow continuous adjustment of $\dot{z}$ away from v until a new equilibrium z is reached. The flatness of the equilibrium surface guarantees this type of adjustment path, in that neither discontinuities nor bifurcation are present.

To describe catastrophe we can use an example from Zeeman [18]. Let a and b be two conflicting control variables which both affect human or animal behavior. Rage and fear, for example, when positive, are two conflicting control variables affecting an individual's level of aggression. Each has the tendency to pull the individual in different behavioral directions. If only rage is present it will induce the individual to strike an attacking pose, if only fear is present it will cause the individual to cower and perhaps flee. However, when both factors are present,

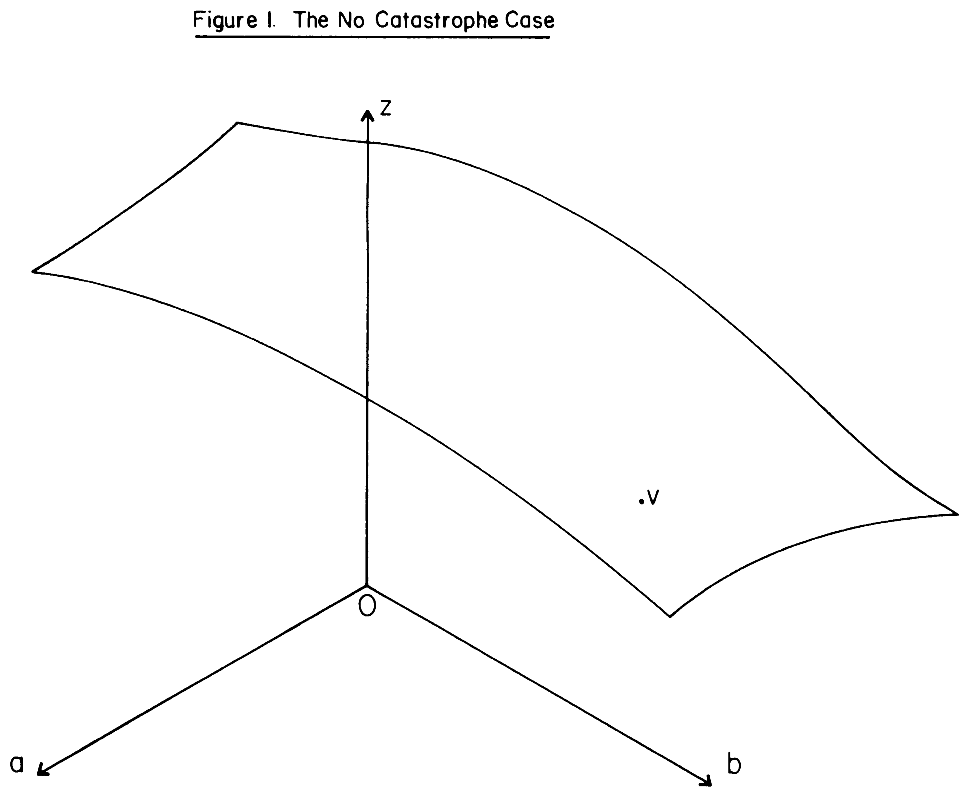

Figure 1. The no catastrophe case

one slowly increasing relative to the other, sudden or catastrophic changes in behavior are often observed. An example would be a sudden switch from attack to defense as fear begins to gradually dominate rage. Such behavior is shown graphically in Figure 2.

In this example part of the equilibrium surface is folded and looks like an overhanging cliff. Because of its shape, this type of surface is called the cusp catastrophe. With only two control variables this behavior represents the "worst" that can happen locally. This is perhaps the most powerful aspect of Thom's theorem for no matter what the precise functional form (or forms) that the behavioral function takes in the "real" world, the cusp catastrophe is the worst possible outcome. To see how catstrophe occurs consider point 1 in Figure 2, where there is a high level of a (rage) relative to b (fear), causing a high level of z (aggression). As we increase the level of fear, keeping rage more or less constant, aggression will gradually fall until we get to point 2, the edge of the overhanging cliff. Any marginal increase in fear beyond this point will lead to a large, discontinuous, fall in aggression to the bottom of the cliff, point 3. This fall in z is the catastrophic behavior discussed above, i.e. the sudden change from attack

Figure 2 The Cusp Catastrophe

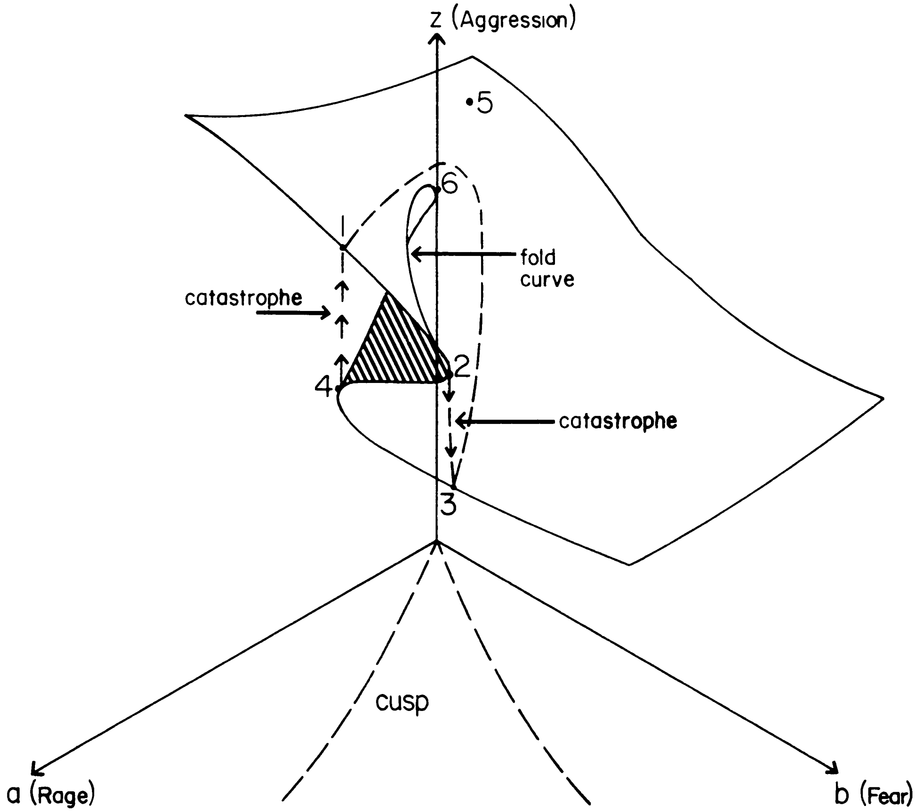

Figure 2. The cusp catastrophe

to defense as fear gradually dominates rage. In Zeeman's [24, p. 8] terminology b is the splitting factor, since it splits the behavioral pattern into two extremes and makes intermediate behavior inaccessible. Note that for values of a and b in the cusp, that is between the dotted lines bounded by the axes in Figure 2, z is multi-valued with one unstable equilibrium value of z bounded by two stable equilibria.

A further interesting implication of the cusp catastrophe is the implied asymmetries of behavior. Suppose we reach equilibrium at or near point 3 in Figure 2, and there is a small decrease in b (fear), aggression will not jump straight back to point 2, but will slowly increase until it reaches point 4, before jumping up to the top of the cliff again (point 1). Hence, there are different catastrophe points for increases and decreases in b.

Finally, it should be noted that it is only for values of a and b within the cusp that behavior will be discontinuous. Consider point 5 that sits back on the surface away from the cliff-edge outside the area of the cusp, any small changes in b, holding a constant, will result in only small continuous adjustments to behavior. This result also implies that points on the edge of the surface, such as 3, have two possible paths back to point 1. The fast path involves decreasing b, keeping a constant, until we get to the bottom of the cliff at 4, then jumping up to the top to point 1. The slow path would involve following a route around the origin of the cusp (point 6) as indicated by the heavily dotted line. Since this path is continuous, involving no jumps, it will tend to take longer to reach point 1, the top of the cliff.

With more control variables, and hence dimensions, the number of possible outcomes increases and the problem becomes more complex. For example, in four dimensions (three control variables) there are three additional catastrophes that are possible, the swallow-tail, elliptic umbilic and hyperbolic umbilic and in five dimensions, seven additional catastrophes.[3] Hence, it is usual to keep the number of control variables to a maximum of five, since with more than five the classification of catastrophes becomes infinite. Nevertheless, it is generally agreed[4] that for applications of the theory, restricting the problem to small dimensions makes case classification far more concrete and important. Consequently, in the model of bank failure developed in Section III below we constrain ourselves to three dimensions, one state and two control variables.

III. A Catastrophe Model of Bank Failure

The model developed below, while obviously simplified, attempts to retain, as far as possible, current banking-regulatory institutional arrangements. In so doing we aim to show how the interaction of the relevant economic agents: bank management, regulators, and depositors can produce a catastrophic jump in the probability of bank failure.

The model's assumptions are delineated below for each agent:

(i) *The Bank*

(a) Parameter α defines the bank's risk-class (the higher α the higher the

[3] See Zeeman [24].

[4] Zeeman [24].

risk-class) and is a decision parameter for the bank. Where risk-class (α) is characterized by the variability of cash flows from the bank's asset portfolio.

(*b*) High α banks are large banks and are members of the Federal Reserve System and FDIC, whereas low α banks are small banks and are members of the FDIC only. (Hence high α banks have access to the discount window, while low α banks do not. However, all banks are members of the FDIC).

(ii) *Depositors*

(*c*) Depositors are either fully insured by the FDIC or are only partially insured.[5]

(*d*) Depositors are risk-averse expected utility maximizers, maximizing return per unit of risk or minimizing risk per unit of return.

(*e*) Partially insured depositors can be identified as corporate and institutional depositors (i.e. those at high α banks) while fully insured depositors can be identified as retail depositors (those at low α banks).

(*f*) Partially insured depositors will be more sensitive to changes in the probability of bank failure than fully insured depositors.

(iii) *Regulators*

(*g*) Regulators are defined as the Federal Reserve and the FDIC.[6] The Federal Reserve (hereafter the Fed) provides member banks with access to the discount window, and is prepared to extend loans to member banks over and above normal discount window facilities.

(*h*) If failure occurs the FDIC will either act as receiver and pay-off depositors, or seek a restructuring of the bank through merger or acquisition (the assumption method), both entail positive costs to the FDIC.[7]

(*i*) To avoid such costs the FDIC will increasingly intervene in the bank's management as the probability of failure rises.

With these assumptions, the determinants of bank failure are developed in two steps. The first step, Model A, ignores regulators and analyzes the interactions of bank management and depositors decisions on the probability of failure. The second step, Model B, includes the influence of regulators (the Fed and the FDIC) on the probability of failure.

[5] Currently, the first $40,000 of each depositors funds are fully insured, this was increased from $20,000 after the failure of the Franklin National Bank. The FDIC [5] estimates that while 99% of all depositors are fully insured, the 1% partially insured depositors hold 38% of total bank deposits.

[6] For ease of exposition the role of the Comptroller of the Currency in regulating banks is not directly considered. This is not too costly since the type of supervisory actions carried out by the Comptrollers Office, such as balance-sheet examinations, are very similar to those conducted by the FDIC. Indeed, there tends to be a considerable overlap of power in this area (See Reed *et al* [11] pp. 30–31).

[7] Although the pay-off method is obviously more costly. In certain extreme cases a third possibility is present in that the FDIC may actually organize a Deposit Insurance National Bank to run the bank for up to two years. This has been done when a community would be without banking services if the bank closed.

Model A: No Regulators

Let

$\tilde{Y}$ = The bank's net cash inflows from asset interest and liquidations (i.e. all asset income)

$\tilde{D}$ = The bank's net deposit inflows (outflows)

$\tilde{Z} = \tilde{D} + \tilde{Y}$ = The bank's net cash inflow

Suppose that

$$\tilde{Z} = \tilde{D} + \tilde{Y} < 0. \tag{4}$$

That is, suppose net deposit outflows are greater than the bank's net cash inflows derivable from its asset structure, so that the bank's first line of defense, its asset cash flow (see [11]), is insufficient to offset deposit outflows. To prevent failure the bank will have to use funds from its capital reserves (K) as a secondary line of defense to offset or meet the net cash short-fall (see Votja [18]). Consequently, failure will only occur if condition (5) holds, i.e.

$$\tilde{Z} = \tilde{D} + \tilde{Y} \leq -K \tag{5}$$

In other words, failure occurs when net negative cash flow is greater than the banks capital funds.

Dividing each term in (5) by the level of deposits we derive

$$\tilde{z} = \tilde{d} + \tilde{y} \leq k \tag{6}$$

where $k = -K/D$, small letters show the relationships in per dollar terms. As defined above the probability of bank failure (F) will depend on the probability that $\tilde{d} + \tilde{y} \leq k$ or,

$$F = \text{probability of failure} = \text{prob. } (\tilde{d} + \tilde{y} \leq k) \tag{7}$$

or more generally,

$$F = h(d, y, k). \tag{8}$$

Using our assumptions, delineated above, we can separate out the effects of a change in the bank's decision parameters (α and k) on F into a direct (or bank) effect and an indirect (or depositors) effect. The bank effect measures the direct impact of a change in either decision parameter on F. If the bank chooses to move to a higher risk-class my making, for example, riskier loans, this implies from assumption (a) that the variance of its $\tilde{y}$ will increase. As a result, it seems reasonable to expect that, *ceteris paribus*, F will also increase. Moreover, since the risk-class parameter α fully describes the characteristics of y we can, without loss of generality, substitute α for y in equation (8). The effects on F of the bank increasing its k (reducing its capital deposits ratio) is also clear from equation (7). Any increase in k will lead, *ceteris paribus*, to a higher probability of bank failure (F).

Next, consider the indirect or depositors effect. In assumption (e) above we argued that depositors can be divided into different clienteles, those who held deposits at low α banks (retail depositors) and those who held deposits at high α

banks (corporate depositors). As a result parameter α will also be a determinant of the bank's $\tilde{d}$. Moreover, in a world of no regulation, and assuming no private deposit insurance, all deposits are uninsured. Hence as a bank alters its α or k and F changes, depositors can be expected to react to such changes. For example, since a rise in F increases the risk-return trade-off facing depositors, they are likely to increase their deposit withdrawals. Consequently, d in equation (8) will also be a function of F, hence

$$d = d(\alpha, F) \tag{9}$$

Substituting equation (9) into equation (8) we derive

$$F = h(d(\alpha, F), \alpha, k) \tag{10}$$

It is possible to further simplify (10) by either assuming that the direct (bank) and indirect (depositors) effects on F are additively separable or to employ a Taylor series approximation to separate out the two effects. This is shown in (11) below

$$F = f(\alpha, k) + d(\alpha, F) \tag{11}$$

where $f(\cdot)$ is the direct (or bank) effect and $d(\cdot)$ is the depositors (indirect or reaction) effect.

Model B: Regulators Present

To investigate the effects of the Fed and FDIC on F we have to consider their relationship to the $d(\cdot)$ and $f(\cdot)$ functions in equation (11) above. First, consider the effects of deposit insurance on $d(\cdot)$. As implied by assumption (b) all banks are members of the FDIC. However, smaller depositors, those generally equated with low α banks, are fully insured, while large corporate depositors, those who bank mostly with large (high α) banks, are only partially insured. This suggests that the effect of deposit insurance will be to make large depositors relatively more sensitive to changes in F than small depositors, when compared to the pre-insurance situation. If $\dot{F}_1 = 0$ in figure 3 below was the equilibrium hyperplane between the bank's control parameters α and k and the level of F before deposit insurance, the effect of deposit insurance is likely to tilt the hyper-plane surface to $\dot{F}_2$, making any increase in α by a large bank relatively more expensive in terms of depositor withdrawals (and therefore F) than for a small bank. Consequently, the presence of deposit insurance will increase the relative sensitivity of large depositors to changes in α and F in the $d(\cdot)$ function.

However, the effect of deposit insurance on $d(\cdot)$ is likely to be partially offset by the effect FDIC regulators have on the bank's management, i.e. on the $f(\cdot)$ function. It was assumed (assumptions (h) and (i)) that because of the costs to the agency and to society of allowing a bank to fail it would be reasonable to expect that intervention by the FDIC is an increasing function of F, so that those banks with high α's and k's, and therefore $f(\cdot)$, are likely to face increased regulatory intervention by the FDIC. This intervention may in practice take many forms, including bank inspections, moral suasion and balance-sheet restructuring (see Mayne [7]). Whatever the type or types of intervention assumed, each

Figure 3. The Effect of Deposit Insurance on F

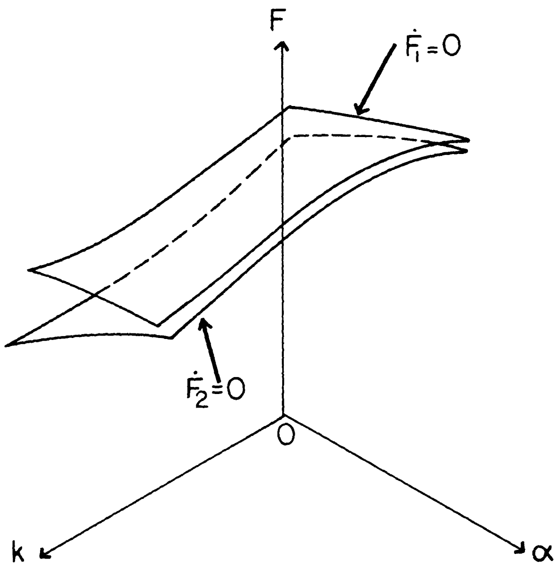

Figure 3. The effect of deposit insurance on F

will work to constrain the banks choice over α and k and tend to reduce $f(\cdot)$ for the bank.

If we call the FDIC intervention function g[8], where

$$g = g(F), \qquad \frac{\partial g}{\partial F} > 0, \tag{12}$$

then g will enter directly into the banks $f(\cdot)$ function as a constraint on its choice over α and k. Thus,

$$F = d(\alpha, F) + f(\alpha, k, g(F)) \tag{13}$$

Again assuming either a Taylor series approximation, or that the FDIC intervention function is additively separable from the banks $f(\cdot)$ function, we derive

$$F = d(\alpha, F) + f(\alpha, k) - g(F), \tag{14}$$

the negative sign on the $g(\cdot)$ function indicating that regulatory intervention is likely to reduce the overall probability of bank failure. Moreover, since regulatory intervention is positively related to F, this will tend to flatten the F surface, offsetting the effect deposit insurance has on the slope of the equilibrium hyperplane.

The role of the FDIC is only one aspect of regulatory intervention; we also

[8] Since all banks are covered by deposit insurance in the model α will not be a determining variable for g unless it is assumed that at any perceived level of F the FDIC reacts differently to high α and low α banks. Nevertheless explicitly entering α into equation (12) will have little substantive effect on the model's results and implications.

have to consider the impact of the Federal Reserve's discount window on $f(\cdot)$ and $d(\cdot)$. We aim to show below that (a) even in the most favorable situation with a continuous supply of discount window loans (i.e. the Fed imposes no ceiling on such loans) a cusp catastrophe can still occur,[9] (b) that since large (high α) banks have access to the window, high α banks are more likely to be prone to catastrophic jumps in F than small banks, and (c) the reasons for (b) can at least be partially attributed to the well known "moral hazard" problem of insurance.

The discount window provides "last resort" loans to high α (i.e. member) banks. Therefore, compared to the situation in Model A, the large bank has an additional source of cash flow to offset deposit outflows. If the volume of discount window loans per unit of deposits is called w, then the condition for bank failure specified in equation (7) becomes

$$\tilde{d} + \tilde{y} + \tilde{w} \leq k \tag{15}$$

and equation (14) becomes

$$F = d(\alpha, F) + f(\alpha, k, w) - g(F) \tag{16}$$

where $\dfrac{\partial f}{\partial w} < 0$.

There are two important variables impacting on w. The first is α, which determines whether or not the bank is a member of the Federal Reserve System. The second is the bank's probability of failure; the higher F the more likely it is that a member bank will use the discount window. For example, if the bank experiences deposit outflows that cannot be matched by net cash flow from assets, the bank will need to resort increasingly to the discount window. Thus,

$$F = d(\alpha, F) + f(\alpha, k, w(\alpha, F)) - g(F). \tag{17}$$

If we also invoke the additivity (or Taylor series) assumption to separate out the effect of $w(\cdot)$ we derive

$$F = d(\alpha, F) + f(\alpha, k) - w(\alpha, F) - g(F) \tag{18}$$

where $w(\cdot)$ is the discount window function. Since $w(\cdot)$ and $g(\cdot)$ together define the effects of regulators on $f(\cdot)$, the bank's decision set, and since both functions enter equation (18) in a negative fashion, it is useful to aggregate them into a single regulatory function called $G(\cdot)$; thus

$$F = d(\alpha, F) + f(\alpha, k) - G(\alpha, F) \tag{19}$$

where $G(\cdot)$ represents the combined effects of the Fed and the FDIC on the bank, and equation (19) describes the general relationships or interactions among the bank, its depositors and regulators in determining the equilibrium level of F.

However, it is possible to be more precise, and derive the mathematical conditions which are both necessary and sufficient to produce a catastrophic jump in F. These conditions are derived in equations (20)—(26) below. This is

[9] It might be noted that the Franklin National Bank had a continuous supply of loans from the Fed right up until its closure in October 1974 (see [4] p. 4).

followed by an examination of the economic validity of these conditions. Thom [16] has shown that for catastrophe to occur, equation (19) would have to be equivalent to a cubic polynomial in F.

The first step in deriving the necessary and sufficient conditions for catastrophe to occur is to transform F by any monotonic function[10] such that

$$0 < F \tag{20}$$

We also assume that

$$G(\alpha, 0) = d(\alpha, 0) = \frac{\partial G}{\partial F}(\alpha, 0) = \frac{\partial d}{\partial F}(\alpha, 0) = 0. \tag{21}$$

That is, when $F = 0$, a small increase in F will, *ceteris paribus*, have no effect on G and d. This seems a reasonable assumption to make and implies that the d and G functions pass through their respective origins as functions of F.[11]

Now from the Malgrange preparation theorem we can express $G(\cdot)$ and $d(\cdot)$ as[12]

$$G(\alpha, F) = g_2(\alpha)F^2 + g_3(\alpha)F^3 \tag{22}$$

$$d(\alpha, F) = d_2(\alpha)F^2 + d_3(\alpha)F^3. \tag{23}$$

If we simplify (22) and (23) by assuming that g_2, g_3, d_2, and d_3 are linear in α then

$$G(\alpha, F) = g_2\alpha F^2 + g_3\alpha F^3 \tag{24}$$

$$d(\alpha, F) = d_2\alpha F^2 + d_3\alpha F^3 \tag{25}$$

where d_2, d_3, g_2, and g_3 are constants such that

$$g_2, g_3, d_2, d_3 > 0$$

since we require $\partial G/\partial F$ and $\partial d/\partial F$ to be positive. Substituting (24) and (25) into (19) we derive the cubic polynomial

$$0 = f(\alpha, k) - \alpha(g_3 - d_3)F^3 + \alpha(d_2 - g_2)F^2 - F. \tag{26}$$

Condition: Now the cusp catastrophe will occur *if and only if* the following two conditions hold

$$g_3 > d_3 \tag{27}$$

$$d_2 > g_2. \tag{28}$$

That is, when the second-order depositor withdrawal reaction to an increase in F is greater than the second-order regulatory reaction to an increase in F but the third-order regulatory reaction is greater than the third-order depositor reaction,

[10] For example, transform by $\tan^{-1} x$, $x > 0$; where before $0 \le F \le 1$.

[11] Strictly speaking we mean $\lim_{F \to 0} G = D = \frac{\partial G}{\partial F} = \frac{\partial d}{\partial F} = 0$.

[12] The Malgrange preparation theorem implies that $G = g_2(\alpha)F^2 + g_3(\alpha)F^3 + g(\alpha, F)F^4$ and $D = d_2(\alpha)F^2 + d_3(\alpha)F^3 + d(\alpha, F)F^4$. But Thom's theorem implies that $g(\alpha, F) = d(\alpha, F)$. Hence, here, we only consider the relevant terms.

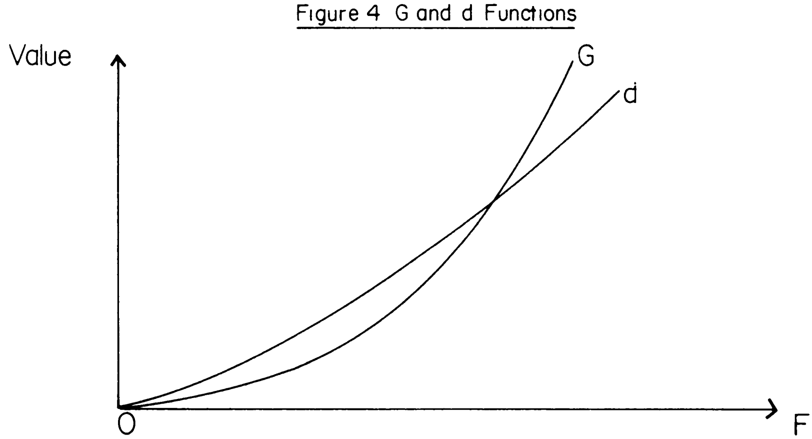

Figure 4. G and d Functions

catastrophe will occur. Figure 4 below shows functional shapes for the G and d functions that satisfy conditions (21), (27), and (28) which are both necessary and sufficient to induce a cusp catastrophe.

The graphical relationship implies that depositors withdrawals d dominate regulatory reaction G at low levels of F, but the greater curvature of $G(g_3 > d_3)$, ensures that G will dominate d at higher levels of F.

In order to understand the real world possibility and economic implications of these functional relationships consider the following example. A large (high α) bank is at an initial equilibrium point on the three dimensional hyperplane. Assume for the sake of exposition that the equilibrium F is near (or at) zero in figure 4. The bank's management then seeks to exploit some seemingly profitable opportunity which entails making some new riskier loans. As the bank makes these loans, its risk-class α rises, resulting in increasing $f(\cdot)$ and hence F in equation (19). This is the positive direct (or bank) effect on F. This increase in α will, as we have seen, also have an indirect effect on $d(\cdot)$ and hence F, since many depositors at large banks are only partially insured. That is, the rise in α increases the risk exposure large depositors have to bear inducing them to withdraw some of their deposits from the bank. Even if the interest rates (risk premia) on the bank's C.D.'s are fully adjusted to match the increase in risk borne by (risk-averse) depositors, leading to no loss in these deposits, the rates on time and savings deposits are regulated and sticky, and on demand deposits constrained to zero.[13]

Hence as a lower bound, a large bank is likely to begin losing some of its large demand and time deposit,[14] these deposit withdrawals adding to the direct (or

[13] However, it is possible that the bank could pay higher implicit interest on demand deposits by increasing its remission of service charges.

[14] Sinkey [14] has shown that in the Franklin National case the greatest run off in deposits that followed adverse publicity about its α occurred for demand deposits, losing 44.5% between December 1973 and June 1974. The loss in time and savings deposits over the same period was 33.2%. A similar type of depositor reaction was also identified in the USNB of San Diego case.

bank effect) on F. The deposit outflow function $d(\cdot)$ modelled in equations (21) to (26) above, is graphically represented by d in Figure 4 above. Initially the bank is likely to meet these induced deposit withdrawals by relying on the cash-flow from its asset portfolio, with only a minimal reliance on discount window loans. Moreover, at this stage the level of FDIC intervention is also likely to be small. Consequently it seems quite reasonable in terms of economic behavior for the regulatory function G to lie below d in the low range of F, as shown in Figure 4 (and implied by our catastrophe conditions).

Suppose the bank continues this process of making more and more risky loans. As it does this its α and hence $f(\cdot)$ will be increasing in a continuous fashion. The increase in α will induce further deposit withdrawals; the level of withdrawals being traced by the d function in Figure 4. While the bank can rely on its net cash flow from assets to meet the initial deposit withdrawals, it will find such sources increasingly insufficient as the level of deposit withdrawals increase. Consequently, the bank will have to turn more and more to discount window loans to meet its short-fall in funds. This will make G rise at a faster rate than before, and at a faster rate than deposit withdrawals. This rise in G is also likely to be enhanced by increased FDIC intervention as F rises. Eventually, if this process is allowed to continue, the G function will rise above the d function as shown in Figure 4. Hence the G and d functions derived in the equations (21)—(26) and graphed in Figure 4 seem reasonable descriptions of possible economic behavior.

However, it was proved in equations (21)—(26) that such shapes are both necessary and sufficient for a cusp catastrophe to occur, i.e. for a discrete or discontinuous jump in F. This is perhaps the most important implication of the catastrophe model, and one that cannot be overemphasized. For even if the Fed is willing to provide funds continuously to the bank through the discount window to offset continuous deposit withdrawals, the probability of failure can increase in a discontinuous fashion. This case is shown in Figure 5.

The bank was initially at some low F equilibrium point 1 in Figure 5. Points on the edge of the surface between 1 and 2 trace the equilibrium path followed by the bank as it increased its α. In this region we can think of a smooth and continuous process of adjustment in F as the bank increases α, depositors withdraw funds, and the bank offsets these withdrawals by discount window loans. However, if the bank increases its α beyond point 2 there is a catastrophic jump in the probability of failure to point 3, even though deposit withdrawals and government intervention are continuous. The cause of this jump simply being due to the differentials in the slope (g_2 and d_2) and rate of change of slope (g_3 and d_3) coefficients in the G and d functions. Thus, even if regulatory intervention or aid (G) is sufficient to outweigh depositor withdrawals (d) at high F's as shown in Figure 4, a catastrophic increase in the probability of failure can still occur. This has important policy implications since it suggests that it is not regulation *per se* that will prevent catastrophe but the rate of regulatory intervention and aid compared to the rate of deposit withdrawals over the whole range of F. Referring again to Figure 4, this would imply that if G were always above d for all F, and not just high F, a catastrophic jump in F could not occur. A high

Figure 5. A Catastrophe Jump in F

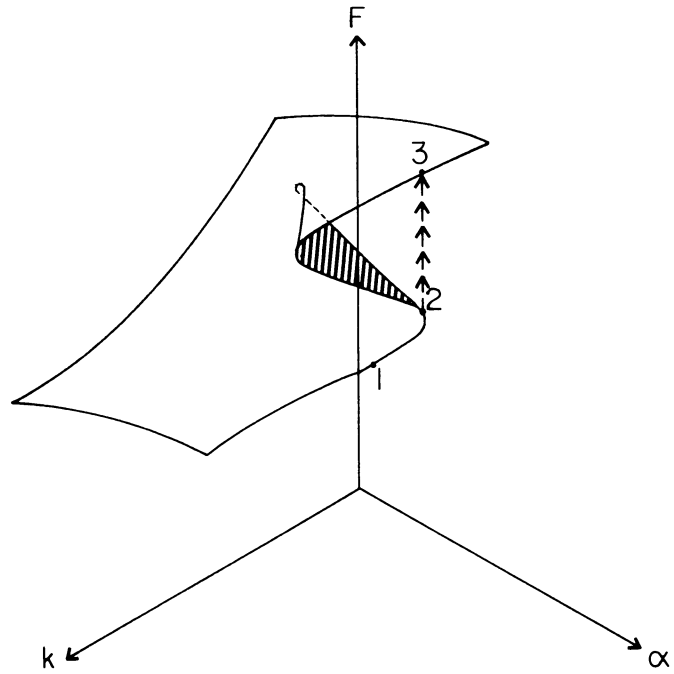

Figure 5. A catastrophic jump in F

level of regulatory intervention in banks with high F's may not be sufficient to prevent catastrophic increases in F.

Not only can we show that catastrophe will occur with these types of G and d functions, but it is also possible to derive the precise point at which catastrophe will occur in Figure 4, for each level of k assumed for the bank. Indeed for each k, catastrophe will occur at a different point in Figure 4.[15] Moreover, it is possible to derive both the origin and the whole area of the cusp, in which α and k combinations produce catastrophic jumps in F.

We can do this by assuming, without loss of generality, that

$$d_3 = g_2 = 0 \tag{29}$$

with conditions (21), (27) and (28) still holding. In this case (19) becomes

$$F = f(\alpha, k) - \alpha g F^3 + \alpha d F^2. \tag{30}$$

Rearranging (30), we find that,

$$\alpha g\left(F - \frac{d}{3g}\right)^3 + \left(1 - \frac{\alpha d^2}{3g}\right)\left(F - \frac{d}{3g}\right) = f(\alpha, k) - \frac{d}{3g} + \frac{2\alpha d^2}{27g^2} = h(\alpha, k, g, d) \tag{31}$$

[15] Rather than prove a specific case with some given k we proceed to the general case of deriving all α, k combinations that can produce catastrophic changes in F.

Since the RHS of (31) comprises of constants d and g plus the control variables α and k, we can again rearrange (31) as

$$F_1^3 + \left(1 - \frac{\alpha d^2}{3g}\right)F_1 = \frac{h(\alpha, k)}{g} = p(\alpha, k) \tag{32}$$

after rewriting $F - \frac{d}{3g}$ as F_1 in (32) above.

Using the inverse function theorem, since $\frac{\partial f}{\partial k} > 0$ implies $\frac{\partial p}{\partial k} > 0$, we can view, without loss of generality, p and c as the relevant independent control variables, where c is defined as

$$c = -\frac{(1 - \alpha d^2)}{\alpha g} \tag{33}$$

so that equation (32) becomes

$$F_1^3 - cF_1 = p \tag{34}$$

Differentiating equation (34) we get

$$3F_1^2 - c = 0 \tag{35}$$

and hence

$$c = 3F_1^2 \tag{36}$$

Substituting (36) into (34) and solving for p we find

$$p = -2F_1^3 \tag{37}$$

Using t as a dimensions parameter we can derive a "cusp line"

$$(p, c) = (-2t^3, + 3t^2) \tag{38}$$

with the origin of the cusp catastrophe line at $\alpha = \frac{3g}{d^2}$. To ensure that the cusp catastrophe occurs at $F > 0$ and $\alpha > 0$, we require $\frac{\mathrm{d}}{3g} > 0, \frac{\mathrm{d}^2}{3g} > 0$, or equivalently $d, g > 0$. This proves that in the general case, conditions (27) and (28) have to hold. These relationships are shown in Figure 6 below:

The next step is to examine why catastrophe is more likely to occur for large or high α banks. It was assumed above that the primary differences in the regulation of large compared to small banks were (*a*) large banks have access to the discount window (while small banks do not) and (*b*) many depositors in large banks are only partially covered by deposit insurance. Hence, in changing risk-classes large banks will be more susceptible to deposit withdrawals than small banks. The presence of (*a*), the discount window, creates a potential "moral hazard" problem for the Fed which is similar to that present in all forms of insurance (see Pauly [9] and Arrow [2]). A large bank joining the Federal Reserve System pays an entry-fee (reserve-requirements) in return for certain services

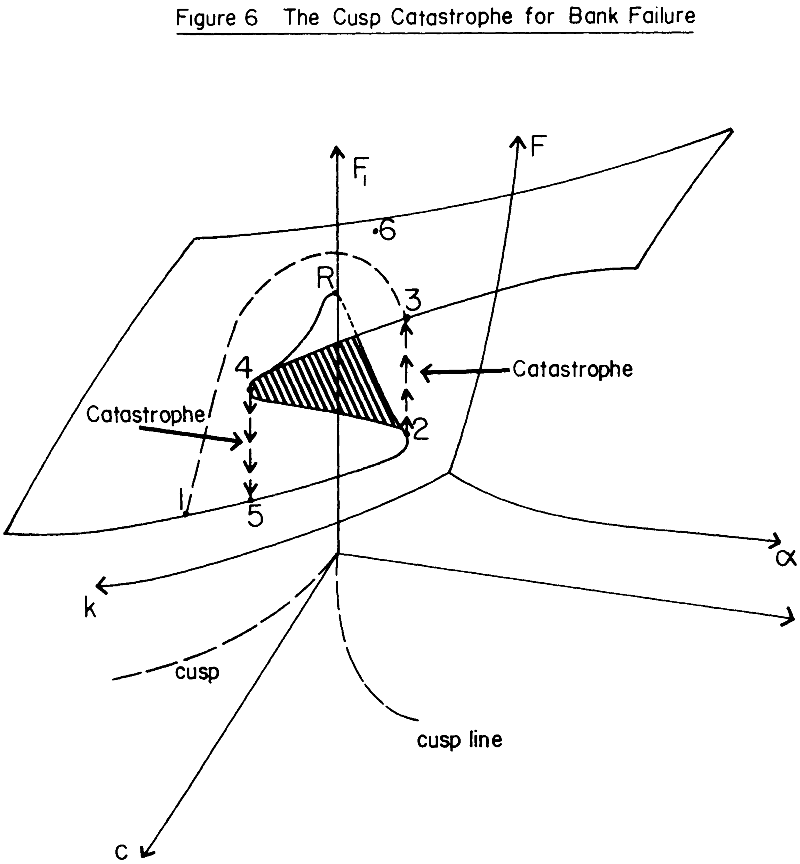

Figure 6. The cusp catastrophe for bank failure

provided by the Fed, the most important of which is the discount window. If the bank believes that the Fed will always be willing to meet its cash needs through the window at some price less than the return on risky loans, it may find it profitable to increase α and use discount loans to offset any deposit withdrawals that result.

We have already shown above the conditions under which continuous discount loans to the bank, to offset the loss of deposits due to continuous increases in α, lead to catastrophic changes in F. Consequently, from a policy standpoint the Fed in order to eliminate or at least minimize the moral hazard problem (and hence the probability of catastrophe) might find it advantageous to set well defined limits on the amount of discount window loans available to any single bank, or at the very least to charge a truly "penal" rate on such loans. Both would act to inhibit a bank continuously increasing α.

By comparison with the above, a small bank is unlikely to face such temptations, as it does not have access to the discount window. Moreover, even if it chose to increase its α, its depositors are unlikely to be very responsive since most are fully covered by FDIC insurance.[16] As a result, catastrophe of the type considered in this paper is more likely to be a large rather than a small bank phenomena. Indeed, we can think of the small bank being at a point such as 6 in Figure 6, above the origin and outside the area of the cusp. Consequently increases in α in the vicinity of point 6 will only result in small and continuous increases in its F.

A further interesting implication of the model is derived if we examine what the model predicts will happen after a catastrophic jump in F. Because of the positive costs to the FDIC (as a public agency) if the bank were allowed to fail, it is likely to intervene to induce the bank's management to reduce α, perhaps by asking it to limit certain types of loans considered to be too risky. However, even if the bank's management complies and reduces α, F will not jump back from point 3 to 2 in Figure 6, but rather will fall very slowly, moving back along the "cliff-top" of the hyperplane's surface towards a point such as 4. That is, the catastrophe model implies that over a certain range of the bank's parameters (defined by the cusp), there will be asymmetric responses in F to increases and decreases in α. Such asymmetric behavior also has an economic explanation. In particular, since the jump in F radically alters the risk-return trade-off facing (risk-averse) depositors, it will take a large reduction in α before depositors regain their former level of confidence in the security of the bank. When they do regain their confidence, F will jump back to a point on the old equilibrium path, indicated by the movement from 4 to 5 in Figure 6.

The catastrophe model also suggests a way in which a large bank can increase α, without risking catastrophe. Suppose it is willing to increase its capital-deposits ratio (reduce k) at the same time as it increases α. An increase in the bank's capital affords large, partially insured depositors, an extra degree of protection to their deposits. Hence this will modify their propensity to withdraw funds as α increases. The dotted path between points 1 and 3 in Figure 6 suggests that there are a whole set of policies of increasing α and reducing k, which allow the bank to avoid the cusp and hence catastrophic jumps in F. Not only would such paths involve smooth and continuous changes in F, but they may well be predictable in the fashion of bank early-warning systems. As a result the model implies that catastrophe is more likely for those banks that take on more risky loans (increase α) without increasing their k.

A final implication of the model concerns the circumstances under which a micro-catastrophe can become a macro-catastrophe. So far we have been considering individual banks. However, to the extent that large banks compete in similar areas for similar types of deposit and draw their management from a similar pool of labor, we might reasonably expect them to adopt fairly homogeneous policies towards α and k. The more homogeneous large banks are as a group, the more likely it is that a micro-catastrophe will also be a macro-catastrophe. For example, if all large banks were completely homogeneous and sought to increase α (keeping k constant), all could arrive simultaneously at the catastrophe point. Clearly, in

[16] A further reason is that small banks are unlikely to be subject to the same adverse-publicity as large banks such as Franklin National.

this case, the social costs of such a combined increase in the probability of failure would be enormous.

IV. Conclusions

This paper has developed a catastrophe model of bank failure. We have attempted to show that the catastrophe model provides a number of new and useful insights into the causes of bank failure as well as having some interesting policy implications. It was proved that under certain reasonable behavioral conditions a catastrophic jump in the probability of bank failure F could occur, even if the Fed was willing to act as a continuous source of lender of last resort loans. The important relationship determining catastrophe appears to be the rate of regulatory intervention relative to the rate of deposit withdrawals over the whole range of F. In particular, it was shown that even if regulators intervened or heavily aided banks when they perceived their F's were very high, this was not sufficient to prevent catastrophic jumps in F.

The model also implied that large banks, those whose depositors are only partially insured and who have access to the discount window, are more susceptible to catastrophe than small banks. A possible "moral hazard" reason for this was suggested. It was also shown that the catastrophe model implied certain asymmetries in the behavior of F resulting from changes in the α and k parameters. The model implies that an increase in α that induced a catastrophic jump in F, would have a plateauing effect on depositors' risk perceptions as to the bank's safety. This might require a considerable reversal in the trend in α before their former level of confidence was restored, and F was to fall significantly. Finally, the model also suggests circumstances under which micro-catastrophe can become macro-catastrophe. In particular, the more homogeneous the large group of banks in terms of their decision parameters α and k, the more likely it is that a macro-catastrophe can occur.

Clearly, further extensions of the model are possible, such as experimenting with the variables included in each agents decision set, in an attempt to analyze other types of catastrophe. However, such experiments will be limited by the fact that the catastrophe set rapidly expands as additional parameters are added, so that it may be difficult to discover interesting and tractable examples. An alternative line of inquiry would be to develop two parameter catastrophe models to see what new insights are thrown on such classic finance problems as optimal capital structure and dividend policy. Indeed, catastrophe theory provides a whole new dimension for exploration and examination of such problems.

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