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The Demand for Life Insurance: An Application of the Economics of Uncertainty

RITCHIE A. CAMPBELL

Abstract

Financial economists typically assume that capital income uncertainty, derived from investments in uncertain returned marketable securities, represents the major source of household consumption uncertainty. But, for many households, if not most, labor income uncertainty dominates capital income uncertainty.

This study analyzes households optimal reactions to labor income (human capital) uncertainty that is derived from the possibility of their wage earners' non-survival. By introducing a risk resolution mechanism—an insurance market—and allowing for the possibility that future tastes may be state-dependent, simple demand-for-insurance equations are mathematically derived to explicitly describe households optimal responses to human capital uncertainty.

THIS PAPER ANALYZES A problem of consumer choice in an environment of uncertainty. In dealing with consumer choice, one has a rather wide latitude in the specification of the assumed source of uncertainty and the object of consumer choice. For example, as Merton [20] points out, the most common sources of consumer uncertainty might include:

- uncertainty about the future inflows of household income derived from investing in marketable assets (uncertain capital income),
- uncertainty about the future inflows of household income derived from the wage earner's labor input (uncertain labor income),
- uncertainty about the future age of death,
- uncertainty about the future investment opportunity set,
- uncertainty about future relative prices of consumption goods, the types of consumption goods available in the future, and future tastes.

While various combinations of these sources of uncertainty have been analyzed in the past, the investigation of household capital income uncertainty, derived from investments in uncertain returned marketable securities, has received the

Assistant Professor of Finance, Graduate School of Business, University of Washington*

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widest interest among financial economists. But, it might be argued that for many households, if not the most, labor income dominates capital income as a source of household consumption. Moreover, common to every household is the inescapable prospect of its wage earner's pre-retirement death and the associated loss of labor income (consumption) to the surviving dependents of the household. Accordingly, this study will analyze households' optimal reactions to labor income (human capital) uncertainty that is derived from the possibility of their wage earner's pre-retirement death. By introducing a risk resolution mechanism—an insurance market—households' optimal response to human capital uncertainty will be explicitly solved under the further condition that future tastes are possibly state-dependent.

This analysis deviates somewhat from past insurance research. Rather than focusing on comparative statics to analyze the probable effect that various economic parameters have on the demand for insurance, this paper starts with a simple statistical description of the household life problem, approximates the household's objective function by a Taylor Series, and maximizes expected utility. Using this approach, some surprisingly simple micro and macro demand-for-insurance equations are developed.

Although this analysis concentrates on a household's one-period problem, the conditions under which this special case characterizes the more general lifetime paradigm are discussed in Section V. In Section I the life problem is specified. Section II introduces an insurance market, and Section III develops the micro demand-for-insurance equation. Section IV focuses on economic interpretations of these demand equations, and conclusions developed from this study are contained in Section VI.

I. The Life Problem

The spirit of the life insurance problem is contained in a simple environment where the sole source of uncertainty is the age of death of the household's primary wage earner, and household choice is limited to the insurance-consumption decision.[1] Although households may be characterized by any number of non-wage earners (dependents), including zero, they are assumed to have only one primary wage earner who is currently age x and who plans to retire at age R; where $x < R$.[2] Because it is assumed that the wage earner's labor income ceases

[1] This limitation of household choice may be somewhat restrictive because the insurance decision may be interrelated with other basic economic choices made by the household. Richard [24] illustrates a case where a human capital certainty equivalent exists that is independent of risky market opportunities and preferences. It is doubtful, however, that separation obtains in more general cases.

[2] In this environment, if the wage earner's age of death were known with certainty, the household's future labor income would be certain and the household would be viewed as choosing a once-and-for-all, maximum utility, intertemporal consumption plan subject to a conservation constraint that $C^* = W_x + H_x$; where C^* represents household consumption over time, stated in present dollars; W_x represents the household's current accumulated marketable assets; and H_x (household human capital) is defined as the wage earner's future labor income, less his self-maintenance, stated in current dollars.

In a Fisherian world of perfect capital markets, household resources can be frictionlessly transformed over time by borrowing or lending to obtain the most desired pattern of intertemporal after retirement, this paper assumes that household human capital uncertainty is derived solely from the uncertainty associated with the survival of its wage earner between the ages of x and R.[3] Therefore, although the present value of the household's share of its wage earner's future labor income, conditional on survival to retirement, is assumed to be calculable and designated by ${}_RH_x$, due to mortality uncertainty, households have an inherent risk of receiving ${}_RH_x$.[4]

In this environment of uncertainty, the household is viewed as making sequential intertemporal consumption plans where future revisions may be necessary.[5] At the beginning of the planning period of duration Δt (which is defined in fractional units of a year) an insurance decision is made and the outcome is immediately revealed. Subsequently, an intertemporal consumption plan is formulated and time of Δt elapses. At the end of this period, another insurance decision is made, the consequences revealed, and the consumption plan reformulated if necessary. If the wage earner's nonsurvival is revealed, no subsequent insurance decisions are made and the final intertemporal consumption plan is chosen by the household.

More formally, there are two mutually exclusive mortality states in the life problem. $S_{x+\Delta t} = 1$ signifies that the currently aged-x wage earner will survive the Δt period, and $S_{x+\Delta t} = 0$ signifies the event of nonsurvival during the time period

consumption. Human capital, however, is not perfectly marketable. And, the existence of an insurance market does not increase the marketability of human capital. Insurance markets enable a household to transform its human capital over specific states-of-nature, but it does not enable a household to perfectly exchange human capital over time. Indeed, if human capital were perfectly marketable, individuals, acting on their own, would circumvent the insurance market completely by buying and selling their most preferred human capital claims.

In the simple economy posed in this paper, capital markets are abstracted from, and because human capital is not perfectly marketable, an additional constraint is imposed on the household's intertemporal consumption plan: future consumption is constrained rather directly by the time profile of future earned income. Regardless of this additional constraint, however, it must still be true that the present value of the chosen (but perhaps restricted) intertemporal consumption plan equals the present value of the household's total resources, $W_x + {}_RH_x$.

[3] More generally, human capital uncertainty might be derived from two broad sources: the economic uncertainty associated with labor markets and the economic uncertainty associated with the wage earner's pre-retirement death. Although this paper assumes labor market certainty, the assumption might be less restrictive than it appears. Evidence by Fama and Schwert [11] indicated that the volatility of per capita income is of smaller order than most well-diversified portfolios. Additionally, the U.S. Bureau of the Census [5] found a rather stable relationship between average annual changes in income over time and the age and education of the wage earner. Thus labor income, conditional on wage earner survival, might be assumed to be relatively certain.

[4] Because it is assumed that the household's share of its wage earner's future labor income is assumed to be certain in the event that the wage earner survives until retirement, these certain (conditional-on-survival) future cash flows could be converted to their present value equivalence by applying a series of riskfree discount rates.

It is important to note that the calculation of this present value should not connote that future human capital is marketable—no such assumption is made—the present value calculation renders a convenient way to summarize the time profile of future, conditional-on-survival, human capital. This summary value represents the household's maximum loss, summarized in present dollars, of human capital in the event that its primary wage earner does not survive the immediate period.

[5] More specifically, this paper assumes timeless gambles, not temporal gambles. For differences in timeless versus temporal gambles see Dreze and Modigliani [6].

of length Δt. The mathematical description of this two-point state distribution over time is a jump process.[6]

The probabilities associated with these mortality states during duration Δt may be summarized by the following transition matrix:[7]

Initial States (at age x)	Final States (at age $x + \Delta t$)	
	$S_{x+\Delta t} = 0$ (Nonsurvival)	$S_{x+\Delta t} = 1$ (Survival)
(Nonsurvival) $S_x = 0$	$\Pr[\tilde{S}_{x+\Delta t} = 0 \mid S_x = 0] = 1$	$\Pr[\tilde{S}_{x+\Delta t} = 1 \mid S_x = 0] = 0$
(Survival) $S_x = 1$	$\Pr[\tilde{S}_{x+\Delta t} = 0 \mid S_x = 1] = q_x \Delta t$	$\Pr[\tilde{S}_{x+\Delta t} = 1 \mid S_x = 1] = 1 - q_x \Delta t$

where $\tilde{S}_{x+\Delta t}$ represents the uncertain mortality state during the future period Δt for an individual currently age x, and q_x represents the probability of the wage earner's death during the age interval of x to $x + 1$ (conditional on his previous survival to age x). This annual mortality rate is assumed to be a step function over time but constant throughout any particular age interval.

Consistent with the properties of a jump process, it is natural to define a state movement or displacement term, $\Delta\tilde{S}$, which is stated in terms of movements away from state one during duration Δt; that is $\Delta S \equiv S_{x+\Delta t} - 1$. Because the mortality distribution is a two-point distribution, there will either be no movement during duration Δt, $(\Delta\tilde{S} = 0)$; or there will be a radical change, $(\Delta\tilde{S} = -1)$. Therefore, state movement is dichotomic; it follows a Bernoulli process over time.

Using the transition matrix and the simpler notation $\tilde{S}$ for the more complicated notation $\tilde{S}_{x+\Delta t}$, an uninsured household's intertemporal consumption possibilities may be described by

$$W(\tilde{S}) = W_x + {}_RH_x + \Delta H(\tilde{S}); \tag{1}$$

where $W(\tilde{S})$ represents the household's uncertain total resources over the next period of duration Δt; W_x represents the household's (whose wage earner is currently age x) current marketable assets at the beginning of the period; ${}_RH_x$ represents the household's future time profile of human capital, conditional on its wage earner's survival from age x to retirement age R, summarized in present

[6] As first brought to my attention by Stephen Ross, Brownian motion, used rather extensively in the literature to describe a household's stochastic wealth process associated with the portfolio problem (c.f. Merton [18, 20], Friend and Blume [12], Ross [25], and Landskroner [14]), is ill-suited to describe the stochastic wealth process associated with the life problem. In the event that the wage earner dies before reaching retirement, the household's intertemporal consumption plan might have to be drastically changed. This phenomenon is best described by a jump process because a jump process implies that, in a small interval of time, there is either no change or there is a potentially radical change. In contrast to a jump process, Brownian motion can only describe small changes for small time intervals. (See, for example, Cox and Miller [3].)

[7] A "joint life" insurance problem could be analyzed by allowing households to have more than one wage earner and by incorporating their mortality information in an extended transition matrix.

dollars; and $\Delta H(\tilde{S})$ represents a potential consumption displacement term relevant for the next time period, Δt: it has values $\Delta H(\tilde{S} = 1) = 0$ with probability $(1 - q_x\Delta t)$, and $\Delta H(\tilde{S} = 0) = -{}_RH_x$ with probability $q_x\Delta t$ if life insurance is not purchased during period Δt.

Equation (1) indicates that if the wage earner's non-survival state is revealed for the Δt period, the household will have to drastically modify its previously formed intertemporal consumption plan. Therefore, at the beginning of the time period (for small valued q_x's) each uninsured household may be viewed as facing a gamble that gives them a large chance of no consumption displacement and a small chance of a possibly large consumption displacement.

II. Insurance Markets

If an insurance contract is purchased at the beginning of the Δt-period, before the mortality state is revealed, the household's potential consumption displacement is modified. Although the household's consumption plan decreases by an amount equal to the insurance premium regardless of its wage earner's survival, *INS* dollars of insurance proceeds will be received by the insured household if its wage earner's nonsurvival state is revealed. The total premium charged at the beginning of the period by the insurance firm for insuring *INS* dollars of Δt-period human capital insurance is denoted by *PREM* and determined by

$$PREM_x = (INS)(q_x\Delta t)(1 + \lambda), \tag{2}$$

where λ, the insurance firm's "loading charge", is a fractional amount of the total premium charge that is, as perceived by the household, in excess of the true expected loss, $(INS)(q_x\Delta t)$. Because of the law of large numbers, it is assumed that insurance firms obtain unbiased estimates of q_x. Individual households, however, may form biased estimates. Therefore, λ simultaneously defines the actual amount of the premium charge that is in excess of the expected loss, plus an error term that captures the household's possible misestimation of q_x.

Using equations (1) and (2), an insured household's state-dependent consumption possibilities may now be described by

$$W(\tilde{S},INS) = W_x + {}_RH_x - (INS)(q_x)(1 + \lambda)\Delta t + \Delta H(\tilde{S},INS), \tag{3}$$

where

$$\Delta H(\tilde{S} \mid S = 1) = 0, \tag{3a}$$

and

$$\Delta H(\tilde{S} \mid S = 0) = INS - {}_RH_x. \tag{3b}$$

These equations indicate that contracting in insurance markets enables a household to modify its state-consumption position by foregoing (with certainty) current premium dollars in order to receive potential state-0 insurance dollars.[8] Relative to the no-insurance case, the extent of consumption modification is uniquely determined by the *INS* dollars of contracted insurance.[9]

III. Optimal Life Insurance

Mathematical Derivation of the Optimal Amount of Life Insurance

Comparing equations (3a) and (3b), if complete insurance is purchased at the beginning of the period ($INS = {}_RH_x$), household economic risk is completely eliminated. That is, regardless of what state occurs, the household's previously selected intertemporal consumption plan is completely supportable.[10] But, should households fully insurance and obtain economic certainty? If not, what insurance contract, *INS*, will be demanded? In the usual formulation of expected utility, household choices are assumed to be consistent with von Neumann-Morgenstern (VNM) preference axioms [28]. The general household problem is to choose that series of actions that lead to maximum multiperiod expected utility, subject to the condition that those actions are feasible. In Section V of this paper, the household's multiperiod insurance problem is analyzed, and conditions are specified that must obtain before one can assume that a single-period objective function incorporates the more difficult problem that would have to be solved in a multiperiod setting. For now, however, it might be easier to assume that the household is only interested in one future planning period of small length Δt. In this case, the single-period objective function, for any pre-retirement period, Δt, may be written as

$$\max_{INS} E\{U[W(\tilde{S}, INS), S]\}, \tag{4}$$

where *INS* represents a feasible insurance contract, $0 \leq INS \leq {}_RH_x$; E is the statistical expectations operator defined over the mortality states, $\tilde{S}$; $W(\tilde{S}, INS)$ is defined by equations (3), (3a), and (3b); and $U[W(\tilde{S}, INS), S]$ represents the household's state-dependent VNM preference function for conditional terminal wealth or future aggregate consumption.[11]

[8] Equivalently, equations (3), (3a), and (3b) may be interpreted within the framework of, say, fire insurance. Reinterpret H as the fair market value of a house. In the event of no fire, (3a), no insurance proceeds are received. In the event that a fire occurs and destroys the house, (3b), the insured receives INS dollars of contracted insurance but he loses the opportunity to sell the house for H dollars. Regardless of the state that occurs (fire or no fire) the insurance premium is foregone.

[9] For another demonstration of state-wealth modifications associated with insurance markets, see Rothchild and Stiglitz [26].

[10] Geometrically, in two-state space, full insurance can be represented by a point in the 45° ray. Being on this line implies that the economic outcome is invariant to the specific state that occurs—consumption certainty obtains.

[11] A consumer's tastes for aggregate consumption dollars (terminal wealth) are assumed to be derived from his tastes for consumption goods. Because it is further assumed that individuals choose among probability distributions of future consumption bundles on the basis of *VNM* expected utility, it follows that their preferences for probability distributions of consumption dollars must also satisfy these axioms. (See Fama [9].)

The state-dependent preference function is defined over the range of conditional wealth and at least three continuous derivatives are assumed to exist. Moreover, $U[\cdot]$ is posited to be strictly concave over W[12] which implies that households prefer more wealth (consumption) to less, and they are risk averse.[13] And, because household preference functions are assumed to be state-dependent, the shape of the utility function may change depending upon the state circumstances of the household.

Using equations (3), (3a), (3b) and the transition matrix, the household's objection function, equation (4), may be written more explicitly as

$$\max_{INS} [1 - q_x\Delta t]V[W_x + {}_RH_x - INSq_x(1 + \lambda)\Delta t] + [q_x\Delta t]B[W_x + (INS)(1 - (q_x)(1 + \lambda)\Delta t)] \quad (5)$$

where the state-dependent preference function is assumed to be a utility function, $V[\ ]$, if the wage earner's survival state is revealed, or a bequest function, $B[\ ]$, if the mortality state is revealed. These functions are also assumed to be at least three times continuously differentiable, monotonically increasing, and strictly concave over their arguments.

Expanding the objective function around $W_x + {}_RH_x$ by a Taylor Series expansion, and taking the derivative with respect to *INS*, implies that the optimal dollar amount of insurance coverage for any household occurs when

$$\begin{aligned} &V'(W_x + {}_RH_x)[q_x(1 + \lambda)\Delta t] + 0(\Delta t^2) \\ &\quad = B'(W_x + {}_RH_x)[q_x\Delta t] + B''(W_x + {}_RH_x)[(INS - {}_RH_x)(q_x\Delta t)] + R_2. \quad (6) \end{aligned}$$

$V'(W_x + {}_RH_x)$, $B'(W_x + {}_RH_x)$ and $B''(W_x + {}_RH_x)$ represent, respectively, the first derivative of the utility function with respect to current conditional resources (conditional on wage earner survival to age R), and the first and second derivative of the bequest function with respect to current conditional resources.

Even though the second and higher derivatives of the utility function (may) exist, these derivatives will be multiplied by very small terms (Δt^2 and higher) that result from the Taylor expansion. Therefore, the values that they contribute to the left-hand side of equation (6) are of small order in comparison with the first term. As such, these terms are summarized by the symbol, $0(\Delta t^2)$. Note that, unlike the left-hand side, the right-hand side of equation (6) explicitly contains the second derivative of the bequest function. The second bequest derivative cannot be eliminated because it is multiplied by terms of order Δt; not by Δt^2. Higher order bequest derivatives, multiplied by the terms that result from the Taylor expansion, are summarized by the symbol R_2. This term is more carefully defined in the appendix and, as shown, although R_2 contains multiplications of

[12] Although strict concavity is not esential, it does imply that if an optimal solution exists, this solution is unique.

[13] As Arrow [1] states: Aversion towards risk can be most naturally illustrated by an individual who always finds the expected value of an uncertain prospect at least as desirable to the uncertain prospect. Arrow further argues that "... the risk aversion hypothesis owes much of its durability to its success in giving a qualitative explanation to otherwise puzzling examples of economic behavior. The most obvious is insurance." (Arrow [1], p. 91.)

terms of order Δt, as well as Δt^2 and higher, R_2 is relatively insignificant to the analysis. Therefore, because terms of order Δt^2 and higher are relatively insignificant for small duration planning periods, and because R_2 is itself relatively insignificant, the optimal insurance decision may be re-expressed by eliminating these terms.

By expressing the bequest function as a linear transformation of the utility function, that is letting $B(\ ) = kV(\ )$ for any scale k such that $0 < k < \infty$, the optimal insurance decision must satisfy equation (7).[14]

$$V'(W_x + {}_RH_x)[(1 + \lambda) - k] = V''(W_x + {}_RH_x)k[INS - {}_RH_x]. \tag{7}$$

The household's "intensity for bequests", $k = \dfrac{B(\ )}{V(\ )}$, measures the household's utility for all levels of supportable intertemporal consumption conditional on the wage earner's death relative to its utility for intertemporal consumption conditional on its wage earner's survival. The household's intensity for bequests is likely to be a function of the age and number of dependents in the household, their future need of economic support, the probability of their future deaths, and the psychological traits of the family: sense of moral responsibility, education and the like.

One might expect that a normal range of values for the intensity coefficient is positive but less than or equal to unity. Consider the case corresponding to $k > 1$: this implies that the household's pleasure (holding its income position constant) from intertemporal consumption has increased due to its wage earner's death. But this seems contrary to the psychological effects that one typically associates with a surviving household. In terms of the bottom part of the intensity range, assuming that $k > 0$ implies that some pleasure is gained from, say, charitable bequest giving even if one has no dependents to provide for. And, although some contradictory examples of human nature can be thought of, these assumptions seem reasonable. Concerning other researchers' assumptions of the magnitude of k: Mirman [21] and Hakannson [13] have assumed that the bequest function is exactly equal to the utility function ($k = 1$), and Yaari [29] and Merton [18] have assumed that k is some positive scale factor.

If a household's intensity for bequests equalled zero, its optimal amount of life insurance would equal zero. The household would not pay a positive price (Prem > 0) for a good from which no utility was derived. For households that have a positive intensity for bequests, equation (7) can be solved to determine their

[14] In terms of this approximation, it becomes more and more precise the greater is k, the smaller is λ, and the greater the relative risk aversion coefficient. Terry Barron evaluated this approximation for the natural log case and found that the approximation was rather good for $k \geq .8$ and $\lambda \leq .2$. Its precision increases even more if households conform to utility functions characterized by constant relative risk aversion where the coefficient is greater than 1.0. Moreover, it is exact for $k = 1$ and $\lambda = 0$. (See appendix)

optimal dollar amount of insurance. A household's optimal amount of human capital insurance equals

$$INS = {}_RH_x - b[W_x + {}_RH_x], \tag{8}$$

where,

$$b = \frac{1 + \lambda - k}{kR[W_x + {}_RH_x]} \tag{8a}$$

and,

$$R(W_x + {}_RH_x) = \left[\frac{V''(W_x + {}_RH_x)}{V'(W_x + {}_RH_x)}\right][W_x + {}_RH_x]$$

$$= A(W_x + {}_RH_x)[W_x + {}_RH_x]. \tag{8b}$$

That is, $R(\ )$ and $A(\ )$ represent Pratt-Arrow relative and absolute risk aversion coefficients.[15]

Equation (8) indicates that, conceptually, a household must solve a very easy problem in order to determine its optimal amount of one-period life insurance; it equals the household's human capital loss if the wage earner died during the period, less a proportion of its total conditional resources, $b(W_x + {}_RH_x)$. The factor of proportionality is determined by the risk and bequest characteristics of the household, as well as its perception of the insurance firm's loading charge.

Most researchers seem to agree that the most plausible theoretical form of absolute risk aversion is a decreasing function of its argument.[16] Concerning the theoretical form of relative risk aversion, there appears to be a greater divergence of opinion. The most realistic form of household risk aversion, however, is ultimately an empirical issue, not a theoretical one. And, in this area, Friend and Blume [12] have concluded from their study of portfolio behavior of households that households appear to exhibit constant proportional risk averasion. The class of preference functions that exhibit constant relative risk aversion (and, therefore, linearly decreasing absolute risk aversion) are given by

$$V[W_x + {}_RH_x] = Ln[W_x + {}_RH_x], \qquad \text{and} \tag{9a}$$

$$V[W_x + {}_RH_x] = \frac{1}{1 - c}[W_x + {}_Rh_x]^{1-c}, \qquad \text{for} \quad c > 1.^{17} \tag{9b}$$

Using the more general class of utility function, (9b), $A[W_x + {}_RH_x]$ is equal to $c(W_x + {}_RH_x)^{-1}$, and $R[W_x + {}_RH_x]$ is equal to c. Substituting this information into

[15] Pratt [23] and Arrow [1].

[16] See, for example, Arrow [1].

[17] In terms of the household's portfolio problem, Friend and Blume [12] have estimated c to be greater than unity (ruling out the log function), and probably greater than 2. Therefore, their findings indicate that (9b) is a more accurate description of households than (9a).

equation (8a), the household's optimal demand for one-period human capital insurance may be rewritten as

$$INS = {}_{R}H_x\left[1 - \frac{1+\lambda-k}{kc}\right] - W_x\left[\frac{1+\lambda-k}{kc}\right]. \tag{10}$$

IV. Economic Interpretations of Households Optimal Demand for Life Insurance

As equation (10) indicates, households should view some proportion of their accumulated marketable assets, W, as a perfect substitute for human capital insurance. Moreover, the same portion of human capital should be self-insured by households.

The inclusion of c and k in the demand function correctly differentiate between a household's level of risk aversion and its intensity for bequests. One may have thought that because the bequest function was assumed to be a simple linear transformation of the utility function, which does not alter the Pratt-Arrow relative risk aversion coefficient, that the actual coefficient of transformation (the household's intensity for bequests) would be unimportant to the mathematical solution. As demand equations (8) and (10) indicate, this is not the case. The magnitude of the household's intensity for bequests is very important to the insurance decision. For example, it would be inappropriate to infer that a household is not risk averse if life insurance is not purchased, or only an insignificant amount is purchased; this demand posture might simply reflect an intensity for bequests that is equal to or close to zero.

It should also be noted from these insurance demand equations that if a risk averse household's intensity for bequests (k) is greater than $1 + \lambda$, as the household became more risk averse, it would demand less insurance. Because this contradicts the response to increasing risk aversion that one would expect, $(1 + \lambda)$ is probably greater than k for most households.

Thus, if $(1 + \lambda) > k$, if the household is risk averse, and it has a positive intensity for bequests, insurance demand equations (8) and (10) indicate that the household's demand for human capital insurance increases if either its psychological reaction to risk increases (as measured by its relative or absolute risk aversion coefficient), if its intensity for bequests increases, or if it perceives the insurance firm as charging a lower loading charge.

If households form upward biased estimates of the durational probability of their wage earner's death, their perception of the insurance firm's loading charge underestimates the true loading charge. And this downward biased estimate of the insurance firm's loading charge leads to an increased demand for insurance. It is difficult to say whether life insurance advertisements attempt to increase a household's psychological reaction to risk, whether they attempt to increase the household's intensity for bequests, or whether they lead to a reformulation of upward biased mortality rates. In any of these cases, however, the resulting change in insurance demand would, presumably, be positive.

As can be seen from equation (10), even if the household had a small amount of marketable assets to substitute for life insurance, complete household insurance ($INS = {}_RH_x$) would generally be a sub-optimal strategy. Although economic certainty can be obtained by purchasing complete insurance, this is not the household's most preferred strategy if

- $(1 + \lambda) > k$; for any $k > 0$ (insurance is a normal good).
- or, if it was assumed that $0 < k \leq 1$, then $\lambda > 0$ is a sufficient condition for risk averse households to optimally purchase less than complete insurance.

Therefore, if the household's intensity for bequests is positive, but less than or equal to unity, the presence of perceived non-actuarially neutral premium charges ($\lambda > 0$) is a sufficient condition for the household's most preferred insurance position to correspond to purchasing less than complete insurance; there is an optimal level of insurance that is less than complete insurance.

Contrastingly, all risk averse households would find it optimal to purchase complete insurance ($INS = {}_RH_x$) if the following conditions held:

(1) if the household's bequest function equalled its utility function ($k = 1$) as some researchers have assumed, and
(2) if insurance firms charged an actuarially neutral insurance premium,[18] and
(3) if households correctly perceived the premium charge to be an actuarially neutral amount.

Under these conditions, accumulated marketable assets would not be used as a substitute for life insurance, nor would households find it optimal to self-insure any proportion of human capital.

As a final economic interpretation of insurance demand equation (10), households should probably hold decreasing (term) insurance over its lifecycle because, as the household becomes older, human capital potential decreases and, presumably, accumulated marketable assets increases. Because a whole life (ordinary) insurance contract may be viewed as providing decreasing term insurance and increasing accumulated savings over a policy-holder's lifetime,[19] the whole life policy might be interpreted as a reflection of rational consumer lifecycle needs.

Proportional Insurance Demand

Rather than determining the dollar amount of demanded insurance, the optimal proportion of human capital that should be insured can be easily found by dividing both sides of equation (10) by human capital. That is, letting $\Omega = \dfrac{INS}{{}_RH_x}$,

[18] Abstracting from transaction costs, to assume that the insurance firm's loading charge is zero is to assume that the risk impact of each insured hazard to the insurance firm is zero. This condition would hold if all insured hazards were independent and the number of the insured hazards was "large". Equivalently, but less satisfactory, one could assume that the insurance firm was risk neutral.

[19] A whole life contract may be conceptualized as having two components: an investment (savings) component, and a pure insurance component. The investment component is a manifestation of the level premium method: in the early years of the contract a policyholder pays a higher annual premium than what is actuarially called for, enabling a surplus, or reserve, to accumulate at a compounded rate of interest. While accumulated savings is increasing over the life of the policy-holder, the pure insurance component is decreasing. See, for example, McGill [16].

the optimal proportion of future human capital that households should currently insure equals

$$\Omega = 1 - \left[\frac{1 + \lambda - k}{kc}\right]\left[\frac{W_x}{{}_RH_x} + 1\right]. \tag{11}$$

As this equation indicates, even though households appeared not to differentiate between future human capital and current marketable assets in the determination of their optimal dollar amount of insurance in equations (8) and (10) (both are used as partial substitutes for life insurance, and their factors of proportional substitutability are precisely the same), households are not indifferent between the composition of their total maximum resources. For any positive k and c where $(1 + \lambda)$ is greater than k, the greater the household's accumulated marketable assets relative to its human capital, the smaller the proportion of human capital that will be insured. Again, viewed in a lifecycle context, this equation indicates that as the household becomes older (accumulated marketable assets increases and human capital decreases), the proportion of human capital that is insured should decrease even if the household's relative risk aversion is constant.

In terms of the household's intensity for bequests or risk aversion, the greater these factors, the greater the proportion of human capital that will be insured. And the greater the household's perception of the insurance firm's loading charge, the smaller the proportion of demanded human capital insurance.

V. Multiperiod Utility Analysis

Review

This analysis has been concerned with economic agents' maximization of their expected utility of one-period terminal wealth. Fama [8], in a widely quoted article, presents what is commonly regarded as a justification of the appropriateness of single-period models in a multiperiod world. As originally conceived by Mossin [22], Samuelson [27], Merton [17], and Fama [8]; and later clarified and extended by Fama and McBeth [10], Merton [18], Long [15], Elton and Gruber [7], and Blume [2], the more general utility of lifetime consumption problem was formulated as a backwards optimization dynamic programming problem. Fama found that the utility functions derived from this dynamic programming problem were state dependent. Removing the state-dependency, Fama argued, made the more general derived utility function resemble a one-period problem utility function.[20] In this case, the one-period objective function incorporated all the relevant choice variables contained in the multiperiod case.

[20] Fama [8] argues that state-dependencies will not exist if the following conditions hold:

1. The agent's tastes for particular consumption goods and services are independent of the state-of-the-world.
2. The agent acts as if future consumption opportunities (in terms of what goods will be available and the prices of goods) are known at the beginning of any previous period.
3. The consumer acts as if the distribution of the stochastic argument in terminal wealth formulation is known at the beginning of any previous period.

Application to the One-Period Insurance Problem

Blume [2] has shown that the state variables which must be included in the derived utility function at any period t are only those conditioning variables whose certainty is resolved at time t + 1. Conditioning variables whose uncertainty is resolved at later periods are irrelevant to period t's derived utility function. Therefore, a two-period insurance problem should contain all the relevancies of a longer duration multiperiod problem.

It is assumed that households evaluate aggregate dollar (consumption) time profiles by a multiperiod utility function, V, defined over time-period 1 wealth and time-period 2 wealth. As of time 0, future wealth is uncertain due to the mortality distribution of the household's wage earner. Mathematically, for any household i, multiperiod insurance decisions are assumed to be made so as to

$$\max_{u_0, u_1} E\{V[W_1(\tilde{S}_1, \Omega_0),\ W_2(\tilde{S}_2, \Omega_1)]\}, \tag{12}$$

subject to the constraint that the insurance decision is feasible. The tildes in equation (12) denote uncertain aggregate consumption due to the time state mortality index which contains information on the survival ($S_t = 1$) or death ($S_t = 0$) of the wage earner during time period t. Given its initial endowment of marketable assets and its share of the wage earner's one-period labor wages (contracted for at the beginning of the period), $W_t(\tilde{S}_t,\ \Omega_{t-1})$ represents the household's end-of-period wealth that is uniquely determined by the insurance decision made at the beginning of the period, $t - 1$, and the mortality state that transpires during the period, $S_t = \{0, 1\}$.

Using equation (12), the two-period insurance problem facing a household at time 0 may be written as

$$\max_{u_0, u_1} \textstyle\sum_{S_1=0}^{1} \sum_{S_2=0}^{1} V[W_1(S_1, \Omega_0),\ W_2(S_2, \Omega_1)]f(\tilde{S}_1,\ \tilde{S}_2), \tag{13}$$

where $f(\tilde{S}_1,\ \tilde{S}_2)$ represents the joint probability density function of the wage earner's mortality distribution for periods 1 and 2.

Following Blume [2] and recognizing that the joint mortality density function may be written as

$$f(\tilde{S}_1, \tilde{S}_2) = f(\tilde{S}_2 \mid \tilde{S}_1)f(\tilde{S}_1), \tag{14}$$

equation (13) may be rewritten as

$$\max_{u_0} \textstyle\sum_{S_1=0}^{1} [\max\limits_{\Omega_1} \sum_{S_2=0}^{1} V[W_1(S_1, \Omega_0),\ W_2(S_2, \Omega_1)]f(\tilde{S}_2 \mid \tilde{S}_1)]f(\tilde{S}_1). \tag{15}$$

The term in brackets may be redefined by what is commonly called a derived utility function, U_0. Requiring the household to make an optimal insurance decision at period 2 regardless of what transpires during period 1, U_0 will have as arguments $W_1(S_1,\ \Omega_0)$ and the state mortality distribution $\tilde{S}_1$ (which conditions period two's decision). Therefore, equation (15) may be written as

$$\max_{u_0} \textstyle\sum_{S_1=0}^{1}\ U_0[W_1(S_1, \Omega_0),\ S_1]f(\tilde{S}_1). \tag{16}$$

Equation (16) represents the maximum expected utility of two-period uncertain wealth if the household arrives at period 0 with an initial endowment, W_0, finds that the mortality index is S_0, and makes optimal insurance decisions at time 0 and time 1. Equation (16) also contains all the information the objective function formulated in this study contains. Thus although the specific insurance solution, derived from a more complicated multiperiod model, might differ from the simpler one-period solution, the method of solution would be exactly the same. More generally, the utility functions used in this paper can now be viewed as derived utility functions—derived from a more general multiperiod function.

Limitations

An obvious limitation to the broader appeal of the previous one-period analysis concerns inflation. If households, rather than evaluating their nominal amount of wealth, evaluate real wealth levels, then the state variable contained in the previous analysis would have to convey information about future prices in addition to mortality information.[21] And since no inflation information was assumed in this paper, the claim of reconciling the one-period insurance problem with the multiperiod insurance problem would be incorrect: the one-period objective function in this paper would not be representative of the multiperiod utility function that households would use. Following Merton [19], Long [15], and Blume [2], it would appear that the proper modeling of uncertain inflation might cause an inclusion of some "hedge" component to the previously derived insurance equations.

A subtler issue that conditions the reconciliation between a one-period and multiperiod insurance objective functions centers around an insurance contract provision called the renewability clause. Term insurance (life insurance for a limited time) may contain, but need not contain, an option of renewing the contract for successive periods without evidence by the policyholder of his insurability. Although this analysis has assumed that the renewability clause exists, the lack of its existence would eliminate the generalness of the one-period problem because, under this circumstance, insurance would represent an asset whose future availabilty is dependent upon past decisions.[22]

Therefore, modeling the demand for term insurance that included a factor to account for the renewability provision or the proper accounting for inflation in a multiperiod setting would necessitate a change in the variables of household

[21] Blume [2].

[22] An important result of Fama's earlier work is that, while it is generally true that the utility function derived by dynamic programming could have any form, if the utility for multiperiod consumption is assumed to be monotonically increasing and strictly concave over the lifetime consumption vector, then the derived utility function is also monotonically increasing and strictly concave in its arguments. As Elton and Gruber [7] discuss, the intuition of this property follows by noting that the derived utility function represents a maximum expected utility function which involves taking a weighted average of assumed concave functions. And, the sum of concave functions is itself concave.

Blume [2] points out that implicit in this important proof is the assumption that agents cannot control the state variables. If an agent's prior decision alters future opportunities, Blume notes that Fama's proof will break down.

choice and the objective function would have to be multiperiod. The solution to these problems would result in additional terms not contained in the equations developed in this paper.

VI. Conclusions

Past theoretical research in the demand for life insurance have relied heavily on comparative statics to ascertain optimal household responses to various economic parameters. This methodology, however, does not enable one to generate explicit demand equations. This paper developed explicit micro demand-for-life-insurance equations. Macro demand equations can be found by aggregating these individual household equations.

In evaluating these demand functions, it was found that optimal human capital insurance corresponds to levels that are lower than complete insurance for most reasonable assumptions. That is, not only does a portion of currently accumulated household wealth act as a substitute for insurance, there is also a portion of future human capital that households should self-insure. Additionally, this paper identified the differences between a household's intensity for bequests and its level of risk aversion. It is very important that one does not co-mingle these terms, or confound them—they are separately important in the life insurance case.

It was also found that households are not indifferent between the composition of their total resources (accumulated marketable assets and human capital) in determining the optimal proportion of human capital that is insured. Viewed in a lifecycle context, it was noted that as households become older the proportion of human capital that is insured should probably decrease even if their level of relative risk aversion stayed constant.

In terms of aggregate insurance demand in the economy, there appears to be an incentive for insurance firms to undertake insurance campaigns that result in household formulation of upward biased mortality estimates. Additionally, advertising may be able to impact a household's intensity for bequests or its level of risk aversion, and, hence, aggregate demanded insurance.

Although the developed insurance equations can be interpreted as derived (from the more general multiperiod analysis) utility functions, the inclusion of inflation or the renewability feature in term insurance would result in additional terms in these rather simple equations; a more robust objection function and choice variables would be necessary to analyze these cases.

APPENDIX

The objective of this appendix is to demonstrate that the remainder term, R_2, in equation (6) is relatively insignificant to the analysis; it may be ignored as a first approximation. By expanding equation (6) around $W_x + {}_RH_x$, and taking the derivatives with respect to INS, it can be seen that the remainder term of the bequest function, R_2, has terms of order Δt as well as terms of order Δt^2 and higher; that is:

$$R_2 = q_x \Delta t[½!) B^3(W_x + {}_RH_x)(INS - {}_RH_x)^2$$
$$+ (⅓!) B^4(W_x + {}_RH_x)(INS - {}_RH_x)^3$$
$$+ \cdots\cdots\cdots\cdots$$
$$+ [(1/k - 1)!) B^k(W_x + {}_RH_x)(INS - {}_RH_x)^{k-1}] + 0(\Delta t^2) \qquad \text{(A.1)}$$

where $B^n(W_x + {}_RH_x)$, $n = \{3, 4, \ldots, k\}$, represents the household's nth bequest derivative with respect to current maximum resources, $W_x + {}_RH_x$; and $0\ (\Delta t^2)$ is read: plus terms of order (Δt^2) and higher. Because Δt is stated as fractional units of one year, terms of order Δt^2 and higher are relatively insignificant to the analysis. Thus, $0\ (\Delta t^2)$ in the previous equation may be ignored for short planning periods.

Suppose that the household's utility function approximates a generally accepted constant relative risk aversion function of the form:

$$B(X) = (1 - \gamma)^{-1} X^{(1-\gamma)}, \quad \text{for } \gamma > 0 \quad \text{and} \quad \gamma \neq 1.$$

For a function of this form, all odd bequest derivatives are positive and all even bequest derivatives are negative. Therefore, regardless of the magnitudes of the bequest derivatives, there will be some off-setting when the terms in R_2 are aggregated. The magnitude of the nth bequest derivative in R_2 may be written as

$$B^n(W_x + {}_RH_x) = [\textstyle\prod_{\tau=0}^{n-2} = (\gamma + \tau)][W_x + {}_RH_x]^{-(\gamma+n-1)}$$

If n is even, the term is negative; and if n is odd, the term is positive. Substituting this into equation A.1 implies that

$$R_2 = q_x \Delta t \left[[½!\,[\textstyle\prod_{\tau=0}^{1} - (\gamma + \tau)] \dfrac{(INS - {}_RH_x)^2}{[W_x + H]^{\gamma+2}} \right.$$
$$+ ⅓!\,[\textstyle\prod_{\tau=0}^{2} - (\gamma + \tau)] \dfrac{(INS - {}_RH_x)^3}{[W_x + {}_RH_x]^{\gamma+3}}$$
$$+ \cdots\cdots\cdots\cdots$$
$$\left. + (1/k - 1)!\,[\textstyle\prod_{\tau=0}^{k-2} - (\gamma + \tau)] \dfrac{(INS - {}_RH_x)^{k-1}}{[W_x + {}_RH_x)^{\gamma+k-1}} + 0(\Delta t^2). \right]$$

Notice that if households fully insured, $INS = {}_RH_x$, all terms of order Δt in R_2 will equal zero.[23] Therefore, in terms of INS, R_2 has the greatest magnitude when $INS = 0$. In this case, the last term of order Δt would equal

$$\frac{1}{(k-1)!} \left[[\textstyle\prod_{\tau=0}^{k-2} - (\gamma + \tau)] \dfrac{(-{}_RH_x)^{k-1}}{[W_x + {}_RH_x]^{\gamma+k-1}} \right] q\ \Delta t.$$

[23] Because $INS = {}_RH_x$ when $k = 1$ and $\lambda = 0$, equation (7) becomes an exact solution if these relationships obtain.

The largest magnitude of this term occurs when $W_x = 0$. In this case, the largest possible value of the last term becomes

$$\frac{1}{(k-1)!}[\prod_{\tau=0}^{k-2} - (\gamma + \tau)][-{}_RH_x]^{-\gamma} q \,\Delta t.$$

And this equation indicates that for any reasonable magnitude of ${}_RH_x$, the maximum order of magnitude of any remainder term of order Δt in R_2 is very small. Moreover, the order of magnitude is inversely related to the magnitude of γ—the greater γ, the smaller these remainder terms. Friend and Blume [12] have empirically estimated γ to be greater than 1, and probably larger than 2. These estimates indicate that terms of order Δt, contained in R_2, are rather insignificant to the analysis and, as a first approximation, may be ignored.

In summary, even under the most likely of cases, when $INS = 0$ and $W_x = 0$, the magnitude of the remainder terms of order at Δt in R_2 are small. The greater INS, W_x, ${}_RH_x$, or γ; the smaller is the absolute value of the remainder terms. Moreover, if the bequest function has alternating signs (+, −, +) on its higher order derivatives, the remainder terms, regardless of their absolute magnitude, are offset when added together. Therefore, under most realistic cases, the order of magnitude of R_2 is likely to be very small relative to its first and second bequest derivative that appear in the solution.

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