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Deposit Rate-Setting, Risk Aversion, and the Theory of Depository Financial Intermediaries

C. W. SEALEY, JR.

IT IS WIDELY RECOGNIZED that the theory of the depository financial intermediary is not well developed.[1] There are at least two areas, however, where a thorough understanding of intermediary behavior is essential. First, financial intermediaries are among the most heavily regulated firms in the economy. The effectiveness of this regulation is largely dependent on an understanding of the behavioral characteristics of these intermediaries. Second, the new view of money supply determination, which is gaining renewed interest in the literature, emphasizes the role of intermediary decisions in determining the money supply. According to this view, an understanding of intermediary behavior is essential to understanding money supply movements.

Two divergent approaches have been employed in the literature to model the depository financial intermediary. A number of writers e.g., Hart and Jaffee [6], Hyman [9], Kane and Malkiel [11], Parkin [18], and Pyle [24], have adopted the Markowitz-Tobin portfolio theory as their analytical apparatus. The principal advantage of this approch is the explicit treatment of uncertainty which has long played a prominent role in discussions of intermediary behavior. This approach, however, omits two key aspects of the behavior of depository financial intermediaries. First, it is assumed that asset and deposit markets are perfectly competitive so that quantity-setting is the relevant behavioral mode in both markets. This assumption is not applicable to deposit markets since such markets are virtually always highly concentrated where intermediaries set rates and face random deposit levels. The effect of deposit quantity-setting behavior is that liquidity considerations, which have also played a prominent role in discussions of intermediary behavior, are ignored.[2] Second, the approach ignores the resource costs incurred in intermediary operations.

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[1] Hereafter, the terms "intermediary" or "financial intermediary" refer to "depository financial intermediary".

[2] In a recent paper Mason [16] makes essentially the same argument that the assumption of quantity-setting behavior is not appropriate for depository intermediaries since it leads to the neglect of liquidity considerations. The shortcomings of quantity-setting behavior are also noted by Hart and Jaffee [6]. As they point out, however, rate-setting behavior cannot be adequately treated within a portfolio model.

Klein [12, 13] was among the first to question the applicability of the portfolio approach to intermediary behavior. Klein bases his criticism on the existence of imperfectly competitive markets and shows that some basic theorems of portfolio theory are not applicable under imperfect market structures. The neglect of resource costs has also been questioned (e.g., see [19, 29, 31]) since diversification of intermediary assets and liabilities may result from cost differentials as well as from more traditional portfolio considerations. As an alternative, a number of writers have developed models that closely resemble traditional theory-of-the-firm models. These models allow the inclusion of more realistic market and cost conditions along with the more appropriate behavioral mode of deposit rate-setting. Unfortunately, only certainty and/or linear risk preferences have been considered in these models. Hence, whereas the portfolio-theoretic models present numerous implications concerning the influence of nonlinear risk preferences on intermediary behavior, the firm-theoretic models as developed to date are silent on this account.

In light of previous work, the purpose of this paper is to develop a model of intermediary behavior that integrates the risk considerations of the portfolio-theoretic approach with the market conditions, cost considerations, and deposit rate-setting behavioral mode of the firm-theoretic approach. The comparative static properties of the model are investigated to determine the influence of risk aversion on optimal intermediary decisions. The results of the model show (i) how cost, liquidity and risk conditions jointly determine intermediary decisions; (ii) that the effects of risk aversion on optimal intermediary decisions are critically dependent on the liquidity and cost conditions facing the financial intermediary; (iii) that many of the conclusions of portfolio-theoretic models, which ignore deposit rate-setting behavior and resource costs, cannot be generalized to models that explicitly take these factors into consideration; and (iv) that the model can be used to provide alternative explanations for some recent empirical evidence concerning intermediary behavior.

The paper is organized as follows. An uncertainty model of a depository financial intermediary is presented in Section I. In Sections II and III, the nonsimultaneous effects of risk aversion on loan and deposit rate decisions, respectively, are investigated. In Section IV, the simultaneous determination of loan and deposit rate decisions under uncertainty is examined. Section V investigates some of the implications of the model for the theory of financial intermediation. Section VI contains the conclusion.

I. The Model and Its Solution

A. The Behavioral Mode With Uncertain Deposit Supplies

Uncertainty has been introduced into models of intermediary behavior in a number of ways, the principal ones being random loan and security returns, and random deposit supplies. Although uncertainty about loan and security returns are important in determining the behavior of the depository financial intermediary, random deposit supplies are at the heart of the liquidity problem at this type of institution.

Randon deposit supply can be specified through the following implicit supply function:

$$\Phi(D^s, R_D, \mu) = 0 \tag{1}$$

where D^s and R_D are the quantity supplied and interest rate on a homogeneous type of deposit, respectively. The supply function in (1) contains a random element, μ, that is not known *ex ante* but has a known subjective probability distribution. It is assumed that, for any μ, the relationship between D^s and R_D is positive. The random variable μ is assumed to vary over the range $\alpha \leq \mu < \infty$.[3]

When deposit supplies are random, two modes of behavior are possible in the deposit market. First, the intermediary may be a quantity-setter and face random deposit rates. This mode of behavior has been investigated by writers such as Hart and Jaffee [6], Parkin [18], and Pyle [24]. Second, the intermediary may demonstrate deposit rate-setting behavior and face a random deposit level. Broaddus [3, 4], James and Brophy [10], Klein [12], Miller [17] and Sealey [28] among others have adopted some variation in this behavioral mode. Although, as Pyle [25] points out, there are some instances in which the intermediary may demonstrate quantity-setting behavior, e.g., when determining purchases of Federal funds, Eurodollars, and certificates of deposit, the most interesting mode for a depository financial intermediary is deposit rate-setting behavior. Thus, assuming the function Φ has continuous partial derivatives, the deposit supply function in (1) can be restated for the rate-setting intermediary as

$$D^s = D(R_D, \mu), \qquad (\partial D/\partial R_D) > 0, \qquad (\partial D/\partial \mu) > 0. \tag{2}$$

B. The Intermediary's Objective Function

In addition to random deposit supplies, uncertainty is also assumed to enter the model through stochastic loan rates. The intermediary is assumed to operate in a perfectly competitive loan market. The random return on loans is denoted by R_L which has a known subjective density and range $0 \leq R_L < \infty$.

To develop a formal model, assume the intermediary faces a single period planning horizon and that the balance sheet takes the following simplified form:

$$L = D^p + Z \tag{3}$$

where $L \equiv$ loans, $Z \equiv$ a composite variable that measures the difference between money market borrowing and lending, and $D^p \equiv$ the quantity of deposits purchased.[4]

At the beginning of the planning period, denoted by $t - n$, $n > 0$, the intermediary makes plans to extend a specified quantity of loans and pay a specified rate of return on deposits. The optimal value of L and R_D chosen at $t - n$ are assumed fixed over the remainder of the period. At time t, when the decisions are actually implemented, the quantity of deposits attracted and the

[3] The lower limit of α for the random variable μ must be such at $D^s \geqslant 0$.

[4] For purposes of simplicity, security holdings are ignored. It should be apparent in what follows that this abstraction does not affect the basic conclusions of the paper.

rate of return on loans becomes known and remains constant over the period. Once the deposit uncertainty is resolved, adjustments in the *ex post* decision variable Z takes place (either money market borrowing or lending) and the balance sheet is then fixed for the duration of the period.[5]

Finally, since deposits are stochastic, the *ex post* liquidity adjustments will influence the intermediary's *ex ante* evaluation of profit. The *ex post* liquidity costs are as follows:

$$l(Z) = \begin{cases} p_1 Z & \text{if } Z < 0 \\ 0 & \text{if } Z = 0 \\ p_2 Z & \text{if } Z > 0 \end{cases} \tag{4}$$

where p_1 and p_2 are certain *ex post* lending and borrowing rates, respectively. Initially, to avoid introducing discontinuity into the objective function, it is assumed that $p_1 = p_2 \equiv p$. This assumption is relaxed in a later section of the paper.

The intermediary's profit function (Π) may now be stated as follows:

$$\Pi = R_L L - R_D D^p - l(Z) - C_D(D^p) - C_L(L) \tag{5}$$

where $C_D(D^p)$ and $C_L(L)$ denote the resource costs of servicing deposits and loans, respectively.[6] It is assumed that $C_L'(L) > 0$, $C_L''(L) > 0$, $C_D'(D) > 0$, and $C_D''(D) > 0$.[7]

Following a number of previous writers [e.g., 9, 11, 18, 24, 30] the objective of the intermediary is assumed to be the maximization of expected utility of profit. The utility function is denoted by $U(\Pi)$ where $U'(\Pi) > 0$ and $U''(\Pi) \gtreqless 0$ depending on risk preferences. The relevant maximization problem is

$$\underset{L, R_D}{\text{Max}}\, E\left\{\underset{D^p, Z}{\text{Max}}[U(R_L L - R_D D^p - l(Z) - C_D(D^p) - C_L(L))]\right\} \tag{6}$$

where E is the expectations operator. The objective function in (6) is optimized subject to constraints (1) and (3) in addition to the following:

$$D^p \leq D^s. \tag{7}$$

The inequality in (7) reflects the fact that the quantity of deposits purchased may be, at least in principle, less than as well as equal to the quantity supplied. Thus, D^p, like Z, is an *ex post* control in (6).

It is possible to state (6) in a more familiar form by first noting that the

[5] A similar analytical framework is used by Pringle [21, 22] in another context.

[6] It is assumed that the production functions and thus cost functions associated with servicing loans and deposits are separable. This assumption is frequently used in the literature. See, for example, [2] and [29].

[7] Although there is wide spread evidence of economies of scale at financial intermediaries, these economies can only be present in the long-run, i.e., when all inputs can be varied. In any given short-run situation, intermediaries like other firms have some inputs that are fixed and others that can be varied. As a result, it is reasonable to believe that the short-run production law of diminishing marginal productivity applies to intermediary production as it does to other production processes. Thus, the marginal resource costs of servicing loans and deposits should be increasing in L and D. A more thorough discussion of production characteristics of intermediaries can be found in [29].

constraint in (7) will be met with equality since depository financial intermediaries agree in practice to accept, at any given deposit rate, whatever quantity of deposits is forthcoming. As a result, $D^p = D^s \equiv D$ so that the supply equation in (2) can be substituted into (3) and (6) for D^p. In addition, (3) can be solved for the *ex post* control, Z, in terms of the *ex ante* controls, L and R_D, and substituted into (4) and (6). After the relevant substitutions, the objective function becomes

$$\max_{L,R_D} E\{U[R_L L - R_D D(R_D, \mu) - p[L - D(R_D, \mu)] - C_D[D(R_D, \mu)] - C_L(L)]\}. \tag{8}$$

The objective function is now stated in terms of *ex ante* decision variables and parameters of the model.

An alternative view of deposit costs. In the above objective function, interest and resource costs of deposits are random since they are functions of the *ex post* level of deposits which is a random variable. The resource cost of loans is of course nonrandom since L is chosen with certainty. In contrast, the most common approach in the literature for the deposit rate-setting intermediary is to consider all deposit costs as nonrandom. This approach was apparently first used by Klein [12] but has subsequently been adopted by a number of other writers (see, e.g., [3, 4, 10, 15, 17]). This approach will hereafter be referred to as the Klein model.

Stated in terms of the model presented in this paper, the Klein model assumes that the deposit supply function at time $t - n$ is known with certainty and that interest and resource costs of deposits are related to that certain deposit supply function. The approach assumes, however, that deposits at time t are random which gives rise to the liquidity decision. Investigators adopting this approach give no indication that this treatment of costs is in any way restrictive or has any influence on intermediary decisions. In fact, for risk-neutral intermediaries, which they consider, the assumption of certain deposit costs is of little importance. However, under nonlinear risk preferences, as will be shown in later sections of this paper, the assumptions of the Klein model have profound influences on intermediary behavior.

It is unlikely that deposit supply can be observed with certainty at the beginning of the period and then be random at other times. If deposits are assumed to be *ex ante* random, then deposit rates must always be set prior to observing the actual quantity of deposits forthcoming. In this case, both interest and resource costs should be random. Nevertheless, since the Klein model is widely used, the effects of nonrandom interest and resource costs of deposits will be considered along with random deposit costs in the sections that follow.

C. Solution to the Model

The first-order conditions for the maximization of expected utility of profit are:

$$\frac{\partial}{\partial L} E[U(\Pi)] = E\{U'(\Pi)[R_L - p - C_L'(L)]\} = 0 \tag{9a}$$

$$\frac{\partial}{\partial R_D} E[U(\Pi)] = E\{U'(\Pi)[p(\partial D/\partial R_D) - R_D(\partial D/\partial R_D) - D(R_D, \mu) - C_D'[D(R_D, \mu)](\partial D/\partial R_D)]\} = 0. \tag{9b}$$

Sufficient conditions for an optimum are (i) that the utility function demonstrate risk aversion and that profit is concave,[8] or (ii) that the firm is risk-neutral and that profit is strictly concave. Based on rather general assumptions, it is reasonable to believe that profit is strictly concave at least in the short-run.

II. The Effects of Risk Aversion on Optimal Loan Decisions of the Financial Intermediary

The first-order conditions in (9) determine the optimal loan portfolio, deposit rate, and liquidity position of the financial intermediary. In this section, optimal loan decisions under risk aversion (denoted by $\hat{L}^a$) are compared with the loan decisions of the risk-neutral intermediary ($\hat{L}^n$) when the deposit rate is fixed. These nonsimultaneous results are obtained for two reasons. First, financial intermediaries frequently encounter situations where loan decisions must be made in the presence of fixed deposit rates. For example, in the United States, when deposit rate ceilings are binding, intermediaries must make asset decisions with deposit rates given. This behavioral mode has been modeled by Hyman [9] and Kane and Malkiel [11] among others. Second, these results are used in a later section when the simultaneous effects of risk aversion are analyzed.

To determine the effect of risk aversion on optimal loan decisions, (9a) can be rearranged as follows:

$$E[R_L] = C_L'(L) + p - \frac{\mathrm{Cov}(U'(\Pi), R_L)}{E[U'(\Pi)]} \tag{10}$$

where $\mathrm{Cov}(U'(\Pi), R_L)$ is the covariance between the marginal utility of profits and the marginal revenue of loans. Under risk neutrality, $U'(\Pi)$ is a constant so that the last term on the RHS of (10) vanishes. Thus, the risk-neutral intermediary will extend loans up to the point where the expected marginal revenue of loans equals the sum of the marginal resources and liquidity costs of loans.

When risk preferences are nonlinear, a convenient interpretation of the last term on the RHS of (10) can be provided. Let Δ denote the risk premium that yields the equality $U(E[\Pi] - \Delta) = E[U(\Pi)]$. It is easily verified that[9]

$$\frac{\partial \Delta}{\partial \hat{L}^a} = -\frac{\mathrm{Cov}(U'(\Pi), R_L)}{E[U'(\Pi)]}. \tag{11}$$

[8] The second-order condition may of course be met even if profit is convex providing the utility function is sufficiently risk averse.

[9] Equation (11) can be verified as follows. The risk premium (Δ) is defined to yield the equality $U(E[\Pi] - \Delta) = E[U'(\Pi)]$. Maximizing expected utility of profit and evaluating at $L = \hat{L}^a$ yields

$$\frac{\partial U(E[\Pi] - \Delta)}{\partial \hat{L}^a} = U'(E[\Pi] - \Delta)(E[R_L] - C_L'(\hat{L}^a) - p - \partial\Delta/\partial\hat{L}^a) = 0.$$

It is evident from (10) that

$$E[R_L] - C_L'(\hat{L}^a) - p = \frac{-\mathrm{Cov}(U'(\Pi), R_L)}{E[U'(\Pi)]}.$$

Imposing this condition on the first-order condition above and simplifying yields equation (11).

Thus the risk premium is an increasing (decreasing) function of loan extensions as $\mathrm{Cov}(U'(\Pi), R_L) < 0$ (>0).

PROPOSITION 1. *Assuming R_D fixed and R_L and μ independently distributed, $\hat{L}^a < \hat{L}^n$. In this case, the risk of intermediary profit is an increasing function of loan extensions.*

Proof: The validity of this proposition is dependent on $\mathrm{Cov}(U'(\Pi), R_L) < 0$ which in turn requires that $\partial U'(\Pi)/\partial R_L = U''(\Pi)(\partial\Pi/\partial R_L) < 0$. Since $U''(\Pi) < 0$ by assumption, $\mathrm{Cov}(U'(\Pi), R_L) < 0$ if $\partial\Pi/\partial R_L > 0$. When R_L and μ are independent, $\partial\Pi/\partial R_L > 0$ for $L > 0$; therefore, the covariance is negative. From (10) it is evident that the intermediary stops short of equating expected marginal revenue and marginal cost of loans. Leland's [14] *Principle of Increasing Uncertainty* implies, under the above conditions, that the risk of profit is increasing in L. Increasing risk is defined in this case in terms of stochastic dominance.

Assuming the random loan rate is the result of loan default, it is reasonable to assume that the loan rate and deposit level are positively correlated ($\rho_{\mu R_L} > 0$). Positive correlation can easily be justified on the basis of business cycle movements in loan defaults and deposits.

PROPOSITION 2. *For μ and R_L joint normally distributed with $\rho_{\mu R_L} > 0$ and R_D fixed at its optimal risk neutral level ($\hat{R}^n_D$), $\hat{L}^a < \hat{L}^n$.*

Proof: This proposition requires that $\mathrm{Cov}(U'(\Pi^n), R_L) < 0$ where Π^n is profit evaluated at $R_D = \hat{R}^n_D$. Using a Taylor series expansion of $U'(\Pi^n)$ about $E[\Pi^n]$, this covariance can be written as

$$\mathrm{Cov}(U'(\Pi^n), R_L) = U''(\Pi_C)\mathrm{Cov}(\Pi^n, R_L) \tag{12}$$

where Π_C is a constant which exists from the law of the mean. It has been shown by Rubinstein [26] that, when R_L and μ are joint normal and the functions D and C_D are at least once differentiable, (12) can be rewritten as

$$\begin{aligned}&\mathrm{Cov}(U'(\Pi^n), R_L)\\ &= U''(\Pi_C)\{L\ \mathrm{Var}(R_L) + E[(p - \hat{R}^n_D\\ &\qquad - C'_D[D(\hat{R}^n_D, \mu)])(\partial D/\partial\mu)]\}\mathrm{Cov}(R_L, \mu).\end{aligned} \tag{13}$$

Clearly, $\mathrm{Cov}(R_L, \mu) > 0$ for $\rho_{\mu R_L} > 0$ and, by assumption, $U''(\Pi_C) < 0$, $(\partial D/\partial\mu) > 0$ and $C'_D(D) > 0$.

To sign the expectation term in (13), note that, at the risk-neutral optimum,

$$\begin{aligned}&E[\partial\Pi^n/\partial R_D]\\ &\quad = E\{[p - \hat{R}^n_D - C'_D[D(\hat{R}^n_D, \mu)] - D(\hat{R}^n_D, \mu)(\partial R_D/\partial D)](\partial D/\partial R_D)\} = 0.\end{aligned} \tag{14}$$

As a result,

$$E\{p - \hat{R}^n_D - C'[D(\hat{R}^n_D, \mu)]\} > 0. \tag{15}$$

Thus for $L > 0$, (13) is negative indicating $\hat{L}^a < \hat{L}^n$.

Corollary 1. For R_D arbitrarily fixed and R_L and μ jointly dependent, it is possible for $\hat{L}^a > \hat{L}^n$.

Corollary 2. For nonrandom deposit costs (i.e., the assumptions of the Klein model), (12) is negative for $\rho_{\mu R_L} > 0$ implying that $\hat{L}^a < \hat{L}^n$.

The former corollary follows from the fact that the covariance in (12) cannot in general be signed for an arbitrary deposit rate and joint distribution of R_L and μ.

Comment: The results presented above can be given an intuitive interpretation. For example, when R_L and μ are independently distributed, the risk or dispersion of profit is increasing in L. Thus, the risk-averse intermediary optimally reduces loans below the risk-neutral level trading a reduction in expected profit for a reduction in the dispersion of profit. A further reduction in L may result when R_L and μ are jointly dependent with $\rho_{\mu R_L} > 0$. For example, if $(\partial\Pi/\partial\mu) > 0$ as is the case when $R_D = \hat{R}_D^n$, then, for a given level of expected profit, the probability of realizing extreme profit levels is increased so that risk aversion leads to even smaller loan extensions.

For an arbitrarily fixed deposit rate, however $(\partial\Pi/\partial\mu)$ may actually be negative so that the positive correlation between R_L and μ partially offsets the variability of profit due to the random loan rate. In this case, the dispersion of profit is increasing in L but at a slower rate than the case where R_L and μ are independent. In the extreme, if $(\partial\Pi/\partial\mu)$ is sufficiently negative, the dispersion of profit may be decreasing in L and thus the intermediary would choose loan extensions greater than the expected profit maximizing level.

III. The Effects of Risk Aversion on Optimal Deposit Rate Decisions

In this section, the effects of risk aversion on optimal deposit rates are investigated when loans are fixed. Although this mode of behavior is less likely to be observed than that considered in the previous section, there may be instances where intermediaries have fixed loans and must set deposit rates. Stigum [30], for example, considers this behavioral mode for savings and loan associations. Moreover, the results of this section are also needed for the simultaneous analysis presented in the next section.

PROPOSITION 3. *Assuming L fixed and R_L and μ independently distributed,*

(3.1) $\hat{R}_D^a > \hat{R}_D^n$ for D additively separable in R_D and μ,

(3.2) otherwise, for more general functional forms for deposit supply, the deposit rate set under risk aversion may be less than, greater than, or equal to the rate set under risk neutrality.

Proof: The method of proof follows Leland [14] and involves evaluating $(\partial\Pi/\partial\mu)$ and $(\partial^2\Pi/\partial R_D\partial\mu)$ locally at $\hat{R}_D^n$. Note that the first-order condition (9b) evaluated at $R_D = \hat{R}_D^n$ was given in (14). For additively separable deposit uncertainty,

$$(\partial^2\Pi/\partial R_D\partial\mu) = -C_D''[D(\hat{R}_D^n, \mu)](\partial D/\partial R_D)(\partial D/\partial\mu) - \partial D(R_D^n, \mu)/\partial\mu < 0. \quad (16)$$

There is some $\mu = \mu^n$ such that, when (14) is evaluated at $\hat{R}_D^n$,

$$E[\partial\Pi(\hat{R}_D^n, \mu)/\partial R_D^n] = \{[p - \hat{R}_D^n - C_D'[D(\hat{R}_D^n, \mu^n] - D(\hat{R}_D^n, \mu^n)(\partial R_D/\partial D)]\cdot(\partial D/\partial R_D)\} = 0. \quad (17)$$

Therefore,

$$\partial\Pi(\hat{R}_D^n, \mu^n)/\partial\mu = \{p - \hat{R}_D^n - C_D'[D(\hat{R}_D^n, \mu^n)](\partial D/\partial\mu)\} > 0. \tag{18}$$

Using the conditions in (17) and (18), when $\mu > \mu^n$

$$\partial\Pi(\hat{R}_D^n, \mu^n)/\partial R_D > \partial\Pi(\hat{R}_D^n, \mu)/\partial R_D$$

and

$$U'[\Pi(\hat{R}_D^n, \mu^n)] > U'[\Pi(\hat{R}_D^n, \mu)].$$

Thus, when $\mu > \mu^n$

$$\{U'[\Pi(\hat{R}_D^n, \mu^n)] - U'[\Pi(\hat{R}_D^n, \mu)]\} \cdot [\partial\Pi(\hat{R}_D^n, \mu)/\partial R_D - \partial\Pi(\hat{R}_D^n, \mu^n)/\partial R_D] < 0. \tag{19}$$

The inequality in (19) also holds for $\mu < \mu^n$ and hence for the expectation which, after taking the expectation and simplifying, yields

$$E\{U'[\Pi(\hat{R}_D^n, \mu)](\partial\Pi(\hat{R}_D^n, \mu)/\partial R_D)\} > 0. \tag{20}$$

The first-order condition for the risk-averse intermediary evaluated at the risk-neutral deposit rate is positive implying $\hat{R}_D^a > \hat{R}_D^n$.

For the general case, the sign of $(\partial^2\Pi/\partial R_D\partial\mu)$ cannot be a priori determined. For example, an increase in the deposit rate increases revenues and therefore leads to more uncertainty of profit. This rate increase also increases costs which reduces the uncertainty of profit. The net effect of these two influences cannot in general be determined.

PROPOSITION 4. *Assuming L fixed and R_L and μ independently distributed, if the assumptions of the Klein model are adopted (i.e., all deposit costs are nonrandom),*

- (4.1) $\hat{R}_D^a = \hat{R}_D^n$ for D additively separable in R_D and μ,
- (4.2) $\hat{R}_D^a < \hat{R}_D^n$ for D multiplicative in R_D and μ,
- (4.3) otherwise, for more general forms of the deposit supply function, the deposit rate set under risk aversion may be less than, greater than, or equal to the risk neutral rate.

Proof: The condition in (20) is equivalent to the condition that $\mathrm{Cov}(U'(\Pi^n), \partial\Pi^n/\partial R_D) > 0$ where $\Pi^n \equiv \Pi(\hat{R}_D^n, \mu)$. For additively separable deposit uncertainty and nonrandom deposit costs, $(\partial^2\Pi^n/\partial R_D\partial\mu) = 0$ in which case (20) is zero which implies that the risk averse and risk neutral solutions are equivalent. For multiplicative deposit uncertainty, $(\partial^2\Pi^n/\partial R_D\partial\mu) > 0$ so that (20) is negative and $\hat{R}_D^a < \hat{R}_D^n$. Finally, it should be evident that the sign of $(\partial^2\Pi^n/\partial R_D\partial\mu)$ cannot in general be a priori determined and in fact may not be monotonic in μ.

PROPOSITION 5. *For L fixed and R_L and μ joint normal with $\rho_{\mu R_L} > 0$, $\hat{R}_D^a > \hat{R}_D^n$ for D additively separable in R_D and μ.*

Proof: Proposition 5 requires that the inequality in (20) hold for $\rho_{\mu R_L} > 0$ which implies that $\text{Cov}(U'(\Pi^n), \partial\Pi^n/\partial R_D) > 0$. Again, let $U'(\Pi^n)$ be represented by a Taylor's expansion about $E[\Pi^n]$ which yields

$$\text{Cov}(U'(\Pi^n), \partial\Pi^n/\partial R_D) = U''(\Pi_C)\text{Cov}(\Pi^n, \partial\Pi^n/\partial R_D)$$

$$= U''(\Pi_C)[L\ \text{Cov}(R_L, \partial\Pi^n/\partial R_D) + \text{Cov}(pD^n - \hat{R}_D^n D - C_D(D^n), \partial\Pi^n/\partial R_D)] \quad (21)$$

where $D^n \equiv D(R_D^n, \mu)$. With additively separable deposit uncertainty, (16) and (18) imply that the second covariance on the RHS of (21) is negative. Again, using Rubinstein's result,

$$\text{Cov}(R_L, \partial\Pi^n/\partial R_D) = E[\partial(\partial\Pi^n/\partial R_D)/\partial\mu]\text{Cov}(R_L, \mu). \quad (22)$$

The expectation term in (22) is negative from (16). Given $\rho_{\mu R_L} > 0$, (22) is negative and, for $U''(\Pi_C) < 0$ and $L > 0$, (21) is positive. Therefore, $\hat{R}_D^a > \hat{R}_D^n$.

Comment: The interpretation of the results concerning the determination of deposit rates is similar to the interpretation of the conclusions in the previous section although the results here are somewhat more restrictive. Depending on the assumptions concerning the functional form through which uncertainty is introduced and form of the joint distribution of R_L and μ, the uncertainty of profit may be increasing, decreasing, or invariant (at least locally around the risk neutral solution) in R_D in which case the risk averse intermediary sets a lower, higher, or equivalent deposit rate compared to the risk neutral intermediary.

IV. The Effects of Risk Aversion When Loan and Deposit Rate Decisions are Simultaneous

Based on the results of the previous two sections, it is now possible to determine the simultaneous influence of risk aversion on loan and deposit rate decisions. It is well known that, for small gambles, any risk-averse utility function can be approximated by a quadratic function of the form $U = \Pi - k\Pi^2$, $k > 0$. As k increases the Pratt-Arrow index of absolute risk aversion increases; therefore, dL/dk and dR_D/dk measure the changes in L and R_D in response to a small change in risk aversion.

To simplify notation, let $G \equiv E[U(\Pi)]$ so that $G_L = \partial E[U(\Pi)]/\partial L$, G_{LL} $\partial^2 E[U(\Pi)]/\partial L^2$ and so on. The signs of dL/dk and dR_D/dk can be determined by solving the following equations:

$$\begin{bmatrix} G_{LL} & G_{LR_D} \\ G_{LR_D} & G_{R_DR_D} \end{bmatrix} \begin{bmatrix} dL/dk \\ dR_D/dk \end{bmatrix} = -\begin{bmatrix} G_{Lk} \\ G_{R_Dk} \end{bmatrix}. \quad (23)$$

Solving (23) for dL/dk and dR_D/dk yields

$$dL/dk = \frac{G_{LR_D}G_{R_Dk} - G_{Lk}G_{R_DR_D}}{D} \quad (24)$$

$$dR_D/dk = \frac{G_{LR_D}G_{Lk} - G_{R_Dk}G_{LL}}{D}. \quad (25)$$

where $D = G_{LL}G_{R_DR_D} - G_{LR_D}^2 > 0$ from the second-order conditions.

PROPOSITION 6. *For a small increase in risk-aversion the simultaneous effects* *of risk aversion on L and R_D are consistent with the nonsimultaneous effects discussed in the previous two sections.*

Proof: To determine the influence of a small change in risk aversion on loan and deposit rate decisions, (24) and (25) are evaluated starting from the risk-neutral optimum of $L = \hat{L}^n$, $R_D = \hat{R}_D^n$, and $k = 0$. It can be shown that $G_{Lk} \gtreqless 0$ as $\hat{L}^a \gtreqless \hat{L}^n$ and $G_{R_Dk} \gtreqless 0$ as $\hat{R}_D^a \gtreqless \hat{R}_D^n$. The remaining sign of interest is that of G_{LR_D}. This term can be written as

$$G_{LR_D} = E[U'(\Pi)(\partial^2\Pi/\partial L\partial R_D) + U''(\Pi)(\partial\Pi/\partial L)(\partial\Pi/\partial R_D)]. \qquad (26)$$

It is easily verified from (9a) that $(\partial^2\Pi/\partial L\partial R_D) = 0$ and, when evaluating at $U''(\Pi) = -2k = 0$, the second term in (26) is zero as well. Since G_{LL} and $G_{R_DR_D}$ are negative from the second-order conditions, dL/dk and dR_D/dk have the same signs as G_{Lk} and G_{R_Dk}, respectively. Thus, the simultaneous results are consistent with the nonsimultaneous results of the previous two sections.

Corollary 1. Assuming R_L and μ independently distributed, and D additive in R_D and μ, for a small increase in risk aversion, $dL/dk > 0$ and $dR_D/dk > 0$.

Corollary 2. Assuming R_L and μ joint normal with $\rho_{\mu R_L} > 0$ and D additive in R_D and μ, for a small increase in risk aversion, $dL/dk < 0$ and $dR_D/dk > 0$.

The former corollary follows from Propositions 1, 3, and 6 and the latter from Propositions 2, 5, and 6.

To derive the above results, it was assumed that the *ex post* borrowing and lending rates are equal. This is a crucial assumption since Proposition 6 does not necessarily hold when $p_1 \neq p_2$ The influence of differential lending and borrowing rates can be analyzed by first noting that profit can be restated as follows:

$$\Pi = R_L L - R_D D(R_D, \mu) - p_1[L - D(R_L, \mu)] - C_L(L) - C_D[D(R_D, \mu)] \quad \text{for } D > L$$

and

$$\Pi^* = R_L L - R_D D(R_D, \mu) - p_2[L - D(R_L, \mu)] - C_L(L) - C_D[D(R_D, \mu)] \quad \text{for } D < L.$$

Assuming R_L and μ are independent with distributions $\phi(R_L)$ and $\theta(\mu)$, expected utility of profit can be written as follows:

$$E[U(\Pi)] = \int_{D_1}\int U(\Pi^*)\theta(\mu)\phi(R_L)\, d\mu\, dR_L + \int_{D_2}\int U(\Pi)\theta(\mu)\phi(R_L)\, d\mu\, dR_L \qquad (27)$$

where D_1 is the region $0 \leqslant R_L < \infty$ and $\alpha \leqslant \mu \leqslant D^{-1}(R_D, L)$ and D_2 is the region $0 \leqslant R_L < \infty$ and $D^{-1}(R_D, L) \leqslant \mu < \infty$. The bound $D^{-1}(R_D, L)$ on μ is the point below which *ex post* deposits are insufficient to cover loans and above which they are sufficient.

It can be shown, in a manner analogous to that used in the previous two sections, that the signs of G_{Lk} and G_{R_Dk} are determined in a manner similar to, and the signs are consistent with, the analysis in Sections II and III. To determine the sign of G_{LR_D}, note that the first-order condition is as follows:

$$G_L = \int_{D_1}\int U'(\Pi^*)(\partial\Pi^*/\partial L)\theta(\mu)\phi(R_L)\, d\mu\, dR_L + \int_{D_2}\int U'(\Pi)(\partial\Pi/\partial L)\theta(\mu)\phi(R_L)\, d\mu\, dR_L. \quad (28)$$

Now, differentiating (28) with respect to R_D, assuming U is quadratic, and evaluating at $L = \hat{L}^n$, $R_D = \hat{R}_D^n$, and $k = 0$ yields

$$G_{LR_D} = \int_0^\infty [U'(\Pi^*)(\partial\Pi^*/\partial L) - U'(\Pi)(\partial\Pi/\partial L)]|_{\mu=D^{-1}(R_D,L)}\phi(R_L)\, dR_L \quad (29)$$

which after simplification reduces to $(p_1 - p_2)$.

PROPOSITION 7. *For $p_1 \neq p_2$, contrary to Proposition 6, the simultaneous effects of risk aversion on L and R_D may not be consistent with the nonsimultaneous effects of risk aversion.*

To see the validity of this proposition, assume $p_1 < p_2$, $G_{Lk} < 0$, and $G_{R_Dk} > 0$. From (24) and (25), the signs of dL/dk and dR_D/dk are indeterminant.

V. Implications for the Theory of Financial Intermediation

The implications of the present model can be compared with those of Pyle [24] concerning conditions that are conducive to financial intermediation. Pyle [24, p. 737] addresses the following: "Under what circumstances would a firm be willing to sell a given deposit liability (e.g., savings deposits) and use the proceeds to purchase a given type of financial asset (e.g., home mortgages)?" The model developed by Pyle is based on hedging considerations and assumes quantity-setting behavior in both loan and deposit markets with random loan and deposit rates.[10] Among his major conclusions are that intermediation is more likely (i) the greater the correlation between loan and deposit rates and (ii) the greater the variance of deposit rates and the smaller the variance of loan rates.

Pyle's methodology is to evaluate $\partial E[U(\Pi)]/\partial L \equiv G_L$ at $L = 0$ and $D > 0$ and

[10] As Mason [16] points out, assumptions similar to those employed by Pyle effectively treat the intermediary as a mutual fund and not as a depository institution.

$\partial E[U(\Pi)]/\partial D \equiv G_D$ at $D = 0$ and $L > 0$. Assuming U strictly concave, necessary conditions for intermediation require the following:

$$G_L\,|_{\substack{L=0\\D>0}} > 0 \tag{30}$$

and

$$G_D\,|_{\substack{D=0\\L>0}} > 0 \tag{31}$$

The influence of increasing correlation between loan and deposit rates can be determined by differentiating (30) and (31) with respect to the correlation coefficient between the loan rate and deposit rate and evaluating at the appropriate values of L and D. For Pyle's model, such derivatives can be shown to be unambiguously positive indicating that the likelihood of the marginal utility of a change in loans and deposits being positive increases as the correlation coefficient increases. This result has intuitive appeal in that, when loan rates are low (high), intermediary revenue is low (high) but, if deposit rates are also on average low (high), then deposit costs are low (high) so that the overall variation in profit is reduced. Thus, large variations in profit are less likely to occur the greater the correlation between loan and deposit rates and, as a result, risk is reduced and hedging facilitated.

Given that increasing correlation between loan and deposit rates facilitates intermediation for the deposit quantity-setting intermediary, it is reasonable to ask whether or not increasing correlation between the deposit level and the loan rate will also facilitate hedging for the deposit rate-setting intermediary. In other words, can Pyle's results be generalized to the deposit rate-setting intermediary? To derive these results, (9a) and (9b) can be differentiated with respect to $\rho_{\mu R_L}$ and evaluated at $L = 0$, $R_D > 0$ and $L > 0$, $R_D = 0$, respectively. To examine the signs of these derivatives, let $p_1 = p_2 \equiv p$ and $U'(\Pi)$ be represented by a Taylor's expansion about $E[\Pi]$. The derivatives can then be written as follows:

$$G_{L\rho_{\mu R_L}}\,|_{\substack{L=0\\R_D>0}} = U''(\Pi_C)[\partial\, \mathrm{Cov}(\Pi, R_L)/\partial\rho_{\mu R_L}] \tag{32}$$

$$G_{L\rho_{\mu R_L}}\,|_{\substack{L>0\\R_D=0}} = U'(\Pi_C)[\partial\, \mathrm{Cov}(\Pi, \partial\Pi/\partial R_D)/\partial\rho_{\mu R_L}] \tag{33}$$

where all terms are defined earlier. It is evident from the discussions in the previous three sections that neither sign is a priori determinant.

In Pyle's model *ceteris paribus*, increasing correlation between loan and deposit rates unambiguously decreases the dispersion of profit so that the likelihood that the optimal values of L and D are positive is increased. However, when the intermediary sets deposit rates, it is not, in general, clear whether or not increasing correlation is desirable, i.e., whether or not dispersion of profit decreases as $\rho_{\mu R_L}$ increases. For example, if Klein's assumptions concerning deposit costs are adopted along with the assumption of additively separable deposit uncertainty, (32) is negative so that intermediation actually becomes less likely as $\rho_{\mu R_L}$ increases. On the other hand, if deposit costs are assumed to be random and deposit supply is multiplicative in R_D and μ, then the signs of (32) and (33) are indeterminant.

Although increasing correlation between loan and deposit rates is clearly conducive to intermediation when the intermediary sets loan and deposit quantities, when deposit rate-setting is considered, increasing correlation between the loan rate and the deposit level may or may not be conducive to intermediation. Furthermore, by differentiating (9a) and (9b) with respect to the variances of R_L and μ, it can be shown that the effects of these variables on intermediation are also indeterminant. The overall conclusion to be reached here is that many of the results concerning the theory of financial intermediation derives from models of quantity-setting behavior cannot be generalized to models of rate-setting behavior.

VI. Implications and Conclusion

A model of the depository financial intermediary under uncertainty has been presented. The distinguishing characteristic of the model is that deposit rate-setting behavior (and thus liquidity considerations), resource costs, and nonlinear risk preferences are simultaneously incorporated into the model. Based on a reasonable view of intermediary decision making and current institutional arrangements, it seems unlikely that the omission of any of the above aspects of intermediary behavior can be justified. More importantly, these considerations play a crucial role in determining optimal portfolio and deposit rate decisions under nonlinear risk preferences. Earlier models of intermediary behavior that ignore these considerations are incomplete and many of their implications cannot be extended to models based on more general assumptions. It was shown, for example, that many of Pyle's conclusions, reached within the confines of a quantity-setting model, cannot be generalized to the model presented in this paper. Moreover, neither Hyman's [9] conclusion that increases in risk aversion reduce the intermediary's scale of operation nor Pringle's [20] result that optimal loans are lower under risk aversion is consistent with a deposit rate-setting model.

The theoretical results can also be used to provide alternative explanations for some recent empirical findings. For example, Edwards [5] tested the applicability of the expense-preference theory to intermediary behavior. He finds support for the expense-preference hypothesis in that U.S. commercial banks hire more inputs than is consistent with profit maximizing behavior. In addition, Edwards argues that the fact that interest rates are higher in monopolistic banking markets but that profits are not, is consistent with expense-preference theory. Although expense preference may explain these findings, they are also consistent with the risk-averse model presented here. Monopolistic intermediaries may use their monopoly power, as the results of Heggestad and Mingo [7] suggest, to reduce risk rather than to increase profits. As shown in this paper, it is possible for an intermediary to reduce risk through greater scale of operation than would be consistent with expected profit maximization. In this case, the intermediary should have lower profits and should employ more resources than profit maximization would predict. Thus, Edwards' results may provide indirect evidence of the effects of risk aversion on intermediary behavior. Clearly, additional empirical research is required to determine the nature of the relationship between intermediary profits and market structure.

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