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Corporate Debt and Corporate Taxes: An Extension

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I. Introduction

IN THE SEPTEMBER 1979 issue of this journal, Bierman-Oldfield [1] show that the Modigliani-Miller Proposition I with corporate taxation is unaltered by the existence of risky debt. Bierman-Oldfield state on page 954 that the analysis implies the expected returns on debt, unlevered equity, and levered equity are determined simultaneously and linked together by the capital asset pricing equilibrium.

The purpose of this extension is to examine the exact linkage of the aforementioned expectations resulting from pure capital structure rearrangements. Hamada [5], by integrating Modigliani-Miller Proposition I with the standard capital asset pricing model, has shown the exact linkage between the return expectations of unlevered equity and levered equity. The Hamada result assumes returns on corporate debt are risk free.

An interesting, and not widely recognized, result is that the correlation between the levered firm and the market portfolio, a primary input to the measurement of diversification relative to the market, can be directly influenced by the financing decision of the firm.[1] This conclusion will obtain specifically by allowing for risky corporate debt. When considering that individual investors typically hold undiversified portfolios, the result takes on an added dimension [e.g., 2].

In Section II the equilibrium relationship of the return expectations is developed, and extended in Section III to demonstrate the effect of the financing decision on firm diversification relative to the market. The extension is concluded and implications are summarized in Section IV. The Bierman-Oldfield results are assumed throughout.

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[1] It has generally been acknowledged that diversification is related only to the investment decision (i.e., asset allocation) of the firm. For example, Rubinstein [11, p. 177] states "... that since the correlation coefficient is invariant to changes in financial leverage, the full impact of financial risk is absorbed by the standard deviation". Rubinstein [11] and Weston-Copeland [12] present proofs that the invariance is implicit from risk free corporate debt. Additionally, both have derived the Modigliani-Miller proposition with allowance for risky corporate debt. All represent timely proofs because Hsia [8] has recently shown the integration of the Modigliani-Miller propositions and the capital asset pricing model, with the Black-Scholes option pricing model requires the existence of risky corporate debt.

II. Equilibrium Relationships of Return Expectations

In this section the exact linkage between the expected returns on debt, unlevered equity, and levered equity is examined and discussed. The approach is consistent with specifying equity and debt claims as being priced via the standard asset pricing model.[2]

The following notation is used throughout the analysis. The notation is consistent with that employed by Bierman-Oldfield. Subscripts u and L refer to the unlevered and levered versions of the firm.

$E(\tilde{Z})$: expectations operator of random variable $\tilde{Z}$;
$\sigma\{Y,\tilde{Z}\}$: covariance operator of random variables Y and $\tilde{Z}$;
$\text{corr}\{Y,\tilde{Z}\}$: correlation operator of random variables Y and $\tilde{Z}$;
ρ_u, ρ_L, r : random equity returns and random return on corporate debt;
ρ_m, r_F : random market return and risk free rate;
S_u, S_L, D : market values of unlevered equity, levered equity and corporate debt;
X_u, X_L : earnings before tax of the unlevered and levered firms;
T : corporate tax rate;
λ : the market price of risk.

The expected return of the unlevered firm is given by

$$E(\rho_u) = E\{(X_u)(1 - T)/S_u\} = r_F + \lambda\sigma\{\rho_u, \rho_m\}. \tag{1}$$

The expected return of the levered firm is given by

$$E(\rho_L) = E\{(X_u - rD)(1 - T)/S_L\} = r_F + \lambda\sigma\{\rho_L, \rho_m\}. \tag{2}$$

Solving expressions (1) and (2) for $(1 - T)E\{X_u\}$:

$$(1 - T)E\{X_u\} = S_u r_F + S_u\lambda\sigma\{\rho_u, \rho_m\}, \tag{1a}$$

and

$$(1 - T)E\{X_u\} = S_L r_F + S_L\lambda\sigma\{\rho_L, \rho_m\} + D(1 - T)\{r_F + \lambda\sigma\{r, \rho_m\}\}. \tag{2a}$$

Equating (1a) and (2a), and simplifying by utilizing the Bierman-Oldfield Proposition I result with risky debt:

$$S_L\lambda\sigma\{\rho_L, \rho_m\} = S_u\lambda\sigma\{\rho_u, \rho_m\} - D(1 - T)\lambda\sigma\{r, \rho_m\}. \tag{3}$$

Dividing (3) by $S_L\lambda\sigma\{\rho_m, \rho_m\}$ and employing systematic standard deviation notation (β)

$$\beta_L = \beta_u\{S_u/S_L\} - \beta_{\text{Debt}}\{(1 - T)D/S_L\}, \tag{4}$$

or with the Bierman-Oldfield Proposition I result as

[2] As Bierman-Oldfield point out, by allowing for default risk the distributions of return are non-symmetric; thus the assumption of quadratic utility is required. Gonzalez *et alia* [4] discuss the inadequacies of both assumptions with regard to pure capital structure rearrangements.

$$\beta_L = \beta_u\{1 + (1 - T)D/S_L\} - \beta_{\text{Debt}}\{(1 - T)D/S_L\}. \tag{4a}$$

The standard Hamada [5] formulations with risk free corporate debt can be expressed as

$$\beta_L = \beta_u\{S_u/S_L\}, \tag{5}$$

or as

$$\beta_L = \beta_u\{1 + (1 - T)D/S_L\}. \tag{5a}$$

Relationships (4) and (4a) define the exact linkage between the expectations of debt, levered equity, and unlevered equity in an asset pricing equilibrium as referred to by Bierman-Oldfield.[3] The systematic standard deviation of debt has entered the Hamada [5] formulation.[4] Logically, the systematic standard deviation of debt is less than the unlevered systematic standard deviation in a market dominated by risk averse investors. This is evident for the risk of debt cannot in equilibrium exceed the riskiness of an unlevered firm. The partial derivative of (4) or (4a) with respect to a positive systematic standard deviation of debt is negative. This implies that with risky corporate debt the systematic standard deviation of equity decreases from the level implied by risk free debt. The decrease occurs essentially because the risk in net operating income is now shared by both debt and equity claimants.[5]

III. Risky Debt and Firm Diversification

It is generally acknowledged that pure capital structure rearrangements affect systematic standard deviation through changes in total risk. This is consistent with the standard Hamada formulation assuming risk free debt. The implication is that the correlation of levered equity with the market is equal to the correlation of unlevered equity with the market. Firm diversification is independent of the financing decision. With risky corporate debt, this implication does not necessarily follow.

The equilibrium relationship between the correlation of the levered equity, unlevered equity, and debt, respectively with the market portfolio is determined by rearranging relationship (4) by multiplying by $\sqrt{\sigma\{\rho_m, \rho_m\}}$ and noting

[3] There are naturally other methods of arriving at (4) and (4a). See Haugen-Pappas [6] for a derivation assuming no corporate taxation. Imai-Rubinstein [9], in a reply to [6], discuss the circularity in derivations of internally consistent models. Implicit in relationship (4) and (4a) is the assumption that the cost of debt is independent of the debt ratio of the firm. Both Krainer [10] and Haugen-Senbet [7] present interesting theoretical arguments based on the rationality of investors to support this assumption.

[4] The primary theoretical problem often associated with evaluating debt via the asset pricing model is that assumption of returns' independence breaks down. Additionally, as developed by Boquist *et alia* [3], the systematic standard deviation of debt is temporally unstable. This is evident because of its relationship to the temporally dependent product of (1) the duration of the bond and (2) the covariance of changes in yield to maturity with the market portfolio. Fn. 2's caveat also applies.

[5] This is precisely the point referred to by Rubinstein [11, p. 177, fn. 19].

$$\sqrt{\sigma\{X_u, X_u\}} = (S_u/(1-T))\sqrt{\sigma\{\rho_u, \rho_u\}},\ \sqrt{\sigma\{X_L, X_L\}} = (S_L/(1-T))\sqrt{\sigma\{\rho_L, \rho_L\}}:$$

$$\begin{aligned}\text{corr}\ \{\rho_L, \rho_m\} = \quad & \text{corr}\ \{\rho_u, \rho_m\}\ \left[\sqrt{\sigma\{X_u, X_u\}/\sigma\{X_L, X_L\}}\right] \\ & -\text{corr}\ \{r, \rho_m\}\ \left[\sqrt{\sigma\{r, r\}/\sigma\{X_L, X_L\}}\right]\ \text{(D)}.\end{aligned} \tag{6}$$

The correlation between the return on levered equity and market portfolio is directly related to the financing decision of the firm which is measured by the dollar value of risky debt. The investment decision directly affects (1) the correlation of unlevered equity and market portfolio and (2) the standard deviation of net operating income.[6]

The standard deviation of the levered firm's earnings before tax (i.e., the denominator of (6)) is related to the variance of the risky component of net operating income, the variance of return on debt, and the correlation between the risky component of net operating income and risky debt. It is dependent upon both the investment and financing decisions of the firm. Specifically:

$$\sigma\{X_L, X_L\} = \sigma\{X_u, X_u\} + D^2\sigma\{r, r\} - 2D\sigma\{X_u, r\}. \tag{7}$$

The partial derivative of the levered correlation in (6) with respect to the dollar value of debt financing can be negative, neutral, or positive depending upon the variance-covariance structure. For example, assume in (6) debt is risky but uncorrelated with both the market and net operating income. If the net operating income of the firm is positively correlated with the market, the partial derivative is negative. Debt financing would, therefore, be increasing the noncompensated risk to the firm's undiversified investors while decreasing the percentage of market explained variation.[7]

Diversification by individual investors, a basic implication of the capital asset pricing model, is often not observed in reality. Consideration of unsystematic risk, influenced by the financing decision, then becomes relevant to the decision-making process of the firm.

IV. Conclusion

The purpose of this work was to extend the Bierman-Oldfield analysis by examining the exact linkage between expected returns on risky corporate debt, unlevered equity, and levered equity resulting from pure capital structure rearrangements. An interesting, and not widely recognized, result was that the correlation between the return of a levered firm and the market portfolio, a primary input to the measurement of diversification relative to the market, can be influenced by the financing decision of the firm. That is, pure capital structure

[6] The total risk in net operating income derives from the variability in price, cost, and demand; and their mutual microeconomic dependencies.

[7] A hypothetical application demonstrating the effect of pure capital structure rearrangements on firm diversification is available from the author. Changing capital structure, with the assumption of debt being uncorrelated with the market, affects correlation through changes in total risk and not covariance. Theoretically, in the single period asset pricing model it may be difficult to argue debt is uncorrelated with net operating income because the distribution of risky debt payments is affected by the distribution of end of period firm value, which is capitalized cash flow.

rearrangements were seen to directly affect the relative amount of diversifiable risk which investors are not compensated for in an efficient market characterized by risk aversion.

REFERENCES

1. H. Bierman and G. Oldfield, "Corporate Debt and Corporate Taxes," *Journal of Finance*, (September 1979), pp. 951–56.
2. M. Blume and I. Friend, "The Asset Structure of Individual Portfolios and Some Implications for Utility Functions," *Journal of Finance*, (May 1975), pp. 585–603.
3. J. Boquist, G. Racette and G. Schlarbaum, "Duration and Risk Assessments for Bonds and Common Stocks," *Journal of Finance*, (December 1975), pp. 1360–65.
4. N. Gonzalez, R. Litzenberger and J. Rolfo, "On Mean Variance Models of Capital Structure and the Absurdity of their Predictions," *Journal of Financial and Quantitative Analysis*, (June 1977), pp. 165–79.
5. R. Hamada, "The Effect of the Firm's Capital Structure on the Systematic Risk of Common Stocks," *Journal of Finance*, (May 1972), pp. 435–52.
6. R. Haugen and J. Pappas, "Equilibrium in the Pricing of Capital Assets, Risk-Bearing Debt Instruments and the Question of Optimal Capital Structure," *Journal of Financial and Quantitative Analysis*, (June 1971), pp. 943–54.
7. R. Haugen and L. Senbet, "The Insignificance of Bankruptcy Costs to the Theory of Optimal Capital Structure," *Journal of Finance*, (May 1978), pp. 383–93.
8. C. Hsia, "Coherence of the Modern Theory of Finance," Study Center in Managerial Economics and Finance, UCLA, Working Paper, 7–79.
9. Y. Imai and M. Rubinstein, "Equilibrium in the Pricing of Capital Assets Risk-Bearing Debt Instruments and the Question of Optimal Capital Structure: A Comment," *Journal of Financial and Quantitative Analysis*, (September 1972), pp. 2001–02.
10. R. Krainer, "Interest Rates, Leverage and Investor Rationality," *Journal of Financial and Quantitative Analysis*, (May 1973), pp. 1–15.
11. M. Rubinstein, "A Mean-Variance Synthesis of Corporate Financial Theory," *Journal of Finance*, (March 1973), pp. 167–81.
12. F. Weston and T. Copeland, *Financial Theory and Corporate Policy*. Addison-Wesley Publishing Company, Inc., 1979.