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Nontransferable Interest-Bearing National Debt

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IN EARLIER PAPERS, BRYANT and Wallace [4] and [5], the inefficiency of interest-bearing national debt is studied. In those papers, examples are given in which default-free government bonds do not dominate fiat money because bonds are issued only in large denominations. A costly technology for subdividing debt is assumed. This note extends the previous analysis by considering bonds which do not dominate money because the bonds are nontransferable. Unlike in the previous papers, interest payments do not reflect a resource cost so inefficiency is not at issue. Moreover, the implied money and bond demand functions are more conventional. In particular, the model may better capture liquidity preference, and bonds play a valuable role in the economy. It is assumed that transferable assets are subject to an uninsurable stochastic loss, while the nontransferable bonds are not. As a result, the optimal ratio of bonds to money plus bonds is not zero. However, at the optimal ratio of bonds to money plus bonds, the nominal rate of interest on bonds is zero. This result that the optimal nominal rate of interest on bonds is zero can be overturned by assuming that the costs to government of servicing money and bonds are not the same.

The Model

The model employed in our analysis is a version of the Samuelson [6] pure consumption-loans model, as in Bryant and Wallace [4] and [5]. The pure consumption-loans model is used because it is a coherent, tractable, and well-known model of fiat money.[1] N individuals are born each period, and they live three periods. In their first period of life these individuals are endowed with $k > 0$ units of the single transferable, but nonstorable consumption good, while in their second two periods of life individuals are endowed with nothing. Their common increasing, strictly concave utility functions have as arguments the individual's consumption of the consumption good in his three periods of life. There exist M units of fiat money which the young get in exchange for the consumption good. In addition, there are B units of bonds which the young get from the government in exchange for money. The government costlessly services

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[1] For a defense of the use of the pure consumption-loans model as the model of fiat money, see Bryant [3] and Wallace [7]. In any case, our results hinge on the "wedge" between money and bonds, not on the particular model of money used.

money and bond holdings. A unit of bonds is a certain claim to a unit of money two periods hence.

Money is subject to a random loss. In addition to the honest individuals, there exists a population of thieves. Thieves are without endowment and hold no bonds. They just steal money and trade it for the consumption good in the same period. Each period, half the middle-aged honest individuals lose 100 δ, $1 > \delta > 0$, percent of their individual money holdings to the thieves, and half the honest old lose 100 δ percent of their money holdings to the thieves. Each individual is equally likely to win (not lose) or lose in each period of life. Therefore, each individual at birth faces four possible, equally likely, outcomes to money holding. Moreover, the losses are not insurable.[2] The losses occur before individuals acquire the consumption good in the second and third periods of life. Bonds are not transferable and are not subject to this random loss. However, when the government pays off on a bond, that money is included in the stock of money of the recipient and is subject to the same percentage loss.

Let U be the individual's utility function, C_1, C_2, C_3, his consumption of the consumption good in his three periods of life, m his money holdings at the beginning of his second period, m' his money holdings at the beginning of his third period, b his bond holdings, i the state of his random drawing on money in his middle age (1 "loser," 2 "winner"), and i' the state of his random drawing on money in his old age (1 "loser," 2 "winner"). Let P be the goods price of money and S be the money price of bonds. For simplicity we will consider only the stationary solution where P and S are constant through time (one can impose utility functions that ensure that this is the only monetary equilibrium, see Bryant [2]). In order to hold the stock of money and bonds fixed, we assume the artificial device that interest payments are financed by a costless system of equal lump-sum taxes on the young. Interest payments are the government's only expenditure, and there are no other taxes. The government has no assets. To ensure internal solutions, we assume that $U_1(0, C_2, C_3) = U_2(C_1, 0, C_3) = U_3(C_1, C_2, 0) = \infty$.

The young individual's problem can be displayed as:

$$\max_{m,b,m'(i)} EU[C_1, C_2(i), C_3(i, i')]$$

subject to

$$\begin{aligned}
C_1 &= k - Pm - PSb - P(1-S)B/N \geq 0 \\
C_2(1) &= P(1-\delta)m - Pm'(1) \geq 0 \\
C_2(2) &= Pm - Pm'(2) \geq 0 \\
C_3(i, 1) &= P(1-\delta)[m'(i) + b] \geq 0 \\
C_3(i, 2) &= P[m'(i) + b] \geq 0 \\
m &\geq 0 \\
b &\geq 0
\end{aligned}$$

$m'(i)$ is a function with domain $\{1, 2\}$ and range $[0, \infty)$.

We are now ready for our central proposition concerning the issuance of money

[2] Because, for example, it's impossible to prove that one has been stolen from. A similar device appears in Bryant and Wallace [5].

and bonds. Let z be the ratio of bonds to money plus bonds after the young have made their purchases of bonds.

Theorem 1: Considering only the current young and future generations of honest individuals, the unique optimum ratio of bonds to money plus bonds, z^*, is characterized by nominal interest rates on bonds equal to zero, $S = 1$, and is the only ratio of bonds to money plus bonds with $S = 1$.

Proof: First we will produce the optimal ratio of bonds to money plus bonds, then demonstrate that $S = 1$, $z = z^*$ is feasible, optimal, and unique.

Suppose that, unlike the above problem, individuals can, by themselves, costlessly convert money to bonds or the reverse one for one in their first period of existence. Then in equilibrium $S = 1$ for $1 > z \geq 0$, for if $S < 1$, individuals demand only bonds, and for $S > 1$, individuals demand only money. Because of the convexity of the individual's problem, he will create a unique ratio of bonds to money plus bonds at $S = 1$, $\bar{z}$, say. $z^* = \bar{z}$ is the optimal ratio.

Next we show that this solution $S = 1$, $z = \bar{z}$ is feasible. Consider now our original model with individuals constrained not to convert bonds to money or the reverse. If government imposes the same ratio of bonds to money plus bonds in the aggregate as created by the individual in the unconstrained case, $\bar{z}$, at $S = 1$ the constraint is not binding and there is no excess supply or demand for money and bonds. Therefore, $S = 1$ is the equilibrium at $z = \bar{z}$.

Now we show that $S = 1$, $z = \bar{z}$ is optimal. Welfare in the constrained case is, at best, no better than welfare in the unconstrained case because in the latter the individual's budget set contains all feasible allocations of a stationary monetary equilibrium; namely, the budget set in the constrained problem with $b = B/N$. Therefore, as the equilibrium at $S = 1$, $z = \bar{z}$ is optimal in the unconstrained case and is identical to the equilibrium at $S = 1$, $z = \bar{z}$ in the constrained case, $S = 1$, $z = \bar{z}$ is also optimal in the constrained problem.

Lastly we show uniqueness. By the convexity of the individual's problem in the constrained case, there is a unique ratio of bonds to money plus bonds that yields $S = 1$ and a unique optimum ratio of bonds to money plus bonds. Therefore, at $S = 1$ the unique optimum ratio $z^* = \bar{z}$ is achieved. Q.E.D.

It is not necessarily true that moving from $z = z' \neq z^*$ to $z = z^*$ is welfare improving. While the current young and future generations are made better off, the current middle-aged and old and current and future generations of thieves are worse off if the current young's demand for money falls. For example, some algebra reveals that if the utility function is additively separable moving from $S < 1$ to $S = 1$ is achieved by increasing the ratio of money to bonds, which increases the price level $(1/P)$. We see, then, that the price effects of open market operations found in Bryant and Wallace, [4] and [5], are not robust, but depend upon the details of transaction costs.

Embellishments

The technology of money and bond holding in the above model is easily modified to make it more "realistic." We could assume, for example, that bonds can be converted to money in either the second or third period, but that conversion of any amount entails a fixed or a proportional conversion cost. For the option of converting in the second period to be meaningful, it would be necessary to assume that the individual could convert after observing his loss of money and/or that bond-converted money is not subject to random loss. The fixed conversion cost can be viewed as a "trips to the bank" technology. One could also assume that the private sector has costly technologies for converting (risky or riskless) transferable assets into nontransferable assets and the reverse. While such modifications would change the optimal z, it would not affect the above theorem that the optimal z is characterized by $S = 1$. Our model does differ from the previous Bryant and Wallace, [4] and [5], models in that a positive nominal interest rate on bonds does not reflect a resource cost. However, addition of the above costly technologies adds this resource cost element to the model, with its implications for efficiency and the price level.

Notice that our model implies that a negative nominal interest rate on government bonds is a possible outcome. This is because the government bonds play a valuable role in the economy. We simply do not observe negative nominal interest rates on government bonds. One possible conclusion to draw is that such models in which government bonds play a valuable role are rejected. However, the private sector technologies suggested in the previous paragraph offer another possibility. If the private sector can costlessly convert money to bonds, but the reverse operation is costly, then only zero or positive nominal interest rates on bonds are possible. However, in such a world $z = 0$ is optimal, and we have reverted to a situation close to that of the Bryant and Wallace, [4] and [5], models.

What would alter our conclusion that optimal z implies zero nominal interest on bonds? The proof of the above theorem indicates how this can be done. In the proof optimal z is determined by examining the problem in which the individual has access to the costless technology of the government of exchanging bonds for money. In the original constrained problem this was replicated at $S = 1$. Suppose now that the government has costly technologies of servicing money and bonds. Similarly, assuming the cost functions are nicely shaped, the optimal z can be determined by including the marginal rate of transformation faced by the government in the individual's problem. If the optimal z is internal, $0 < z^* < 1$, this is replicated in the constrained problem by setting aggregate z such that the government's marginal return on money issue equals its marginal return on bond issue. The optimal equilibrium nominal rate of interest is positive if the implied marginal cost of bond servicing is less than the marginal cost of money servicing.

The observation that nominal interest rates are positive is consistent with optimality in a world where bonds are cheaper to service than is money. However, the actual cost of government servicing of money and bonds is small relative to the value of outstanding stocks so that only small deviations from $S = 1$ can be justified in this manner.[3]

[3] See, for example, [1] p. 502.

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