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An Analysis of Risk and Return Characteristics of Corporate Bankruptcy Using Capital Market Data

JOSEPH AHARONY, CHARLES P. JONES and ITZHAK SWARY*

I. Introduction

THE ABILITY TO PREDICT corporate failure has drawn considerable attention from economic researchers. The importance of such efforts is obvious. Corporate failure is an indication of resource misallocation which is undesirable from a social point of view. Therefore, as Lev [10] points out:

> "An early warning signal of probable failure will enable both management and investors to take preventative measures; operating policy change, reorganization of financial structure, and even voluntary liquidation will usually shorten the length of time losses are incurred and thereby improve both private and social resource allocation."

Most of the empirical research in this area has used accounting data, comparing the ability of various financial ratios to predict corporate failure.[1] Typically, the time series behavior of various financial ratios for a sample of failed firms is compared with that of a control group of nonfailed firms. It seems safe to say that the various empirical models that have used financial ratios have been successful in discriminating between failing and nonfailing firms several years before bankruptcy actually occurred.

Regardless of the success of the accounting ratio models, they really have little or no definitive theoretical foundation. Instead, the financial ratios are simply utilized in various statistical procedures until they do, in fact, work. Perhaps because of this success, market-derived data have received little attention in studies of corporate bankruptcy.[2] However, market data can provide a satisfying

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[1] A partial list of these studies consists of Altman (1), Beaver (3), Meyer and Pifer (12), Deakin (6), Sinkey (14) and Altman, Haldeman, and Narayaman (2). For an excellent summary and critical evaluation of the literature on the prediction of corporate failure, see Lev (10), Chapter Nine.

[2] An early attempt to use market data to predict bankruptcy was made by Beaver (4) who used a sample of failed and nonfailed firms. The study utilized annual data for the period 1954–64. Beaver's results indicated that holding period returns (i.e., market data) predicted failure sooner than did individual financial ratios (although for two of the four ratios compared, the differences were relatively small). Beaver, in his concluding remarks, suggested that "more knowledge is needed about the non-ratio information investors rely upon" (p. 192).

theoretical basis for analyzing corporate bankruptcy in that investors' expectations should be reflected in market risk-return measures.

The major purpose of this study is to compare the characteristics of bankrupt and non-bankrupt firms, prior to actual bankruptcy, with respect to various risk and return measures suggested by the capital asset pricing model. In addition, the study suggests an approach to estimating the probability of corporate failure, using capital market data.

II. Source and Nature of Data

The sample consists of a group of 45 industrial companies that went bankrupt during 1970–78 and a group of 65 control firms.[3] To be included, bankrupt companies were required to have at least six years of daily rates of return prior to bankruptcy.[4] One or two control companies were matched to each bankrupt firm on the following basis: (1) the control company was in the same industry as its paired bankrupt firm; (2) it was as close (as we could find) in size to its paired bankrupt firm; (3) it was incorporated at roughly the same time as its paired bankrupt firm;[5] (4) daily rates of return for a calendar period identical to that of its paired bankrupt firm were available.

Weekly rates of return ($\tilde{R}_{jw}$) were computed for each firm in the sample and for the Market Index ($\tilde{R}_{mw}$) over the period for which daily data were available.[6] For the jth firm $\tilde{R}_{jw}$ and $\tilde{R}_{mw}$ were computed so that the same five trading days were always used. Furthermore, for any two firms with data for the same calendar period, weekly returns were computed using daily returns from identical periods of five consecutive calendar-days.

III. Measuring Shifts in Risk

The conventional measure of the total risk borne by common stockholders is the variability of the rates of return. For the purpose of this study, shifts in the variance of stock returns are used to measure changes in the risk of corporate failure.

Using the market model, the variance of the rates of return on the jth stock is composed of three different risk elements and is given by:

$$\mathrm{Var}(\tilde{R}_j) = \beta_j^2\,\mathrm{Var}(\tilde{R}_m) + \mathrm{Var}(\tilde{e}_j) \tag{1}$$

[3] The list of bankrupt firms was obtained from the Compustat Industrial Research File. This file consists of all the companies which were deleted, for various reasons, from the active Compustat files. All the bankrupt firms from this file, for which data requirements were fulfilled, are included in our sample.

[4] Daily rates of return were obtained from the tapes constructed by the Center for Research in Security Prices (CRSP) at the University of Chicago.

[5] These characteristics were checked in Moody's annual Industrial Manuals. Size refers to book value of total assets.

[6] $\tilde{R}_{jw}$ and $\tilde{R}_{mw}$ are holding period returns. The daily closing Standard and Poor's Industrial Common Stock Price Index was used as the Market Index. Weekly returns are the compounded returns for each five consecutive trading days.

where:

$\mathrm{Var}(\tilde{R}_j)$ = total variance of returns of security j;

$\mathrm{Var}(\tilde{R}_m)$ = variance of returns of the market portfolio of risky assets;

$\mathrm{Var}(\tilde{e}_j)$ = variance of the disturbance term of security j;

β_j = covariance $(\tilde{R}_j, \tilde{R}_m)$/Variance $(\tilde{R}_m)$; and Covariance $(\tilde{e}_j, \tilde{R}_m) = 0$.

The term $\mathrm{Var}(\tilde{R}_m)$ reflects market-wide variables, whereas β_j and $\mathrm{Var}(\tilde{e}_j)$ are affected by variables specific to security j.

The direction and significance of changes in the total variance of returns are of interest in examining the information concerning corporate bankruptcy that is contained in market data. However, it is important to examine whether any possible shift in the total risk is caused by firm-specific effects or is merely due to changes in market-wide factors. Therefore, each of the components of the total variance of returns is examined by employing both cross-sectional and time series analyses.

For each bankrupt company in the sample, the risk measures from equation (1) are estimated for each week prior to that particular company's bankruptcy week (event-related week) using the current and previous 103 weeks (2 years) of data. Estimates of β_j and $\mathrm{Var}(\tilde{e}_j)$ are derived from ordinary least squares regressions of weekly firm returns $(\tilde{R}_{jw})$ on weekly market returns $(\tilde{R}_{mw})$ using the following market model:

$$\tilde{R}_{jw} = \infty_{jw} + \beta_{jw} \cdot \tilde{R}_{mw} + \tilde{e}_{jw} \tag{2}$$

where

$$w = W,\ W\text{-}1, \ldots,\ W\text{-}103, \text{ and } W = \text{bankruptcy week.}$$

$\mathrm{Var}(\tilde{R}_j)$ and $\mathrm{Var}(\tilde{R}_m)$ are estimated using $\tilde{R}_{jw}$ and $\tilde{R}_{mw}$ from the same time period. As a result, time series estimates of the various risk measures are obtained for each firm, using 104 weekly observations at different (moving) points relative to the corresponding bankruptcy week.[7] The same procedure is also utilized for each control company using data for exactly the same time period as that of its paired bankrupt company; therefore, the estimates obtained for each pair of companies correspond exactly to the same calendar period.

Obviously, the calendar dates of bankruptcy differ among firms. Our interest, however, is in analyzing the statistical properties of each risk measure at various points in time (weeks) relative to the respective bankruptcy dates. Accordingly, for each event-related week, the sample mean and standard deviation of each risk measure are calculated across the bankruptcy group and across the control group.

[7] If NW is the total number of weekly rates of return available for a given company, then (NW-103) estimates for each risk measure are obtained for that company. It should be noted that beyond 202 weeks prior to bankruptcy the number of companies in each group is decreasing since not all companies in the sample have a full ten years of data. For example, in week 301 there are 52 control firms and 38 bankrupt firms; in week 400 there are 36 control firms and 26 bankrupt firms. However, the number of firms available each week always consists of matched pairs.

A. Cross-Sectional Analysis

The null hypothesis of the cross-sectional analysis is that the means of the total variance ($\mathrm{Var}(\tilde{R})$) for the two groups, bankrupt and control, are equal at any given point in time relative to the event week. Table I presents the sample means and standard deviations for each risk measure, for each of the two groups, at various periods (weeks) ranging from 1 week before to 390 weeks before bankruptcy. In addition, a t-statistic (for the differences in two population means) is presented for each week.[8]

Table I indicates that the sample mean of the total variance ($\mathrm{Var}(\tilde{R})$) for the bankruptcy group is significantly larger than that of the control group for every number of weeks measured prior to the bankruptcy week. Furthermore, the level of statistical significance increases as the bankruptcy week is approached. The results further indicate that this significant difference is attributable solely to differences between the groups with respect to the sample means of $\mathrm{Var}(\tilde{e})$ and not β's, for which there are non-significant differences.[9] This implies that the cross-sectional differences in the total variance are caused on average, by differences in firm-specific variables such as production-investment activities, leverage positions and the quality of management, and not by market-wide factors.

B. Time-Series Analysis

To test for possible changes in risk over time, a time-series analysis of the statistical properties of each risk measure for each group is conducted. Again, comparisons were made at various points preceding bankruptcy. Specifically, we compare the sample means of a given risk measure between two points in time for each group.

To portray the relationship over time between the control group's risk measures and the bankrupt group's risk measures, the time-series of the sample means of the betas, total variances, and residual variances for both subsamples are shown in panels (a), (b), and (c) of Figure 1, respectively. Panel (a), showing the sample means of betas from 437 weeks before bankruptcy up to bankruptcy, indicates that the two groups had very similar and flat patterns. Panels (b) and (c) indicate that starting about four years before bankruptcy, the sample means of both $\mathrm{Var}(\tilde{R})$ and $\mathrm{Var}(\tilde{e})$ for the bankruptcy group diverge from those of the control group in an increasing pattern as the bankruptcy date approaches.

Table II shows the results for various selected pairs of weeks relative to the bankruptcy week.[10] The results in Panel A for the bankruptcy group reveal a highly significant increase in the sample mean of the total variance between 226 weeks before bankruptcy and 120 weeks before bankruptcy. Similar results are obtained for any pair of weeks which are closer to the bankruptcy week. This increase in risk is due primarily to the increase in the sample mean of $\mathrm{Var}(\tilde{e})$, and

[8] For detailed calculation of this t-statistic see, for example, Chou (5) pp. 330–333.

[9] The distribution of $\mathrm{Var}(\tilde{R}_m)$ is approximately identical for the two groups in any given week and is therefore omitted from Table I.

[10] Each pair of weeks compared are 106 weeks apart. Since each risk measure was estimated using 104 observations, this procedure avoids over-lapping weekly returns.

Table I

Sample Statistics of Risk Measures—Cross Sectional Analysis

Weeks Prior to Bankruptcy Week	VAR($\tilde{R}$) Bankruptcy Group # of Obs.	VAR($\tilde{R}$) Bankruptcy Group Mean	VAR($\tilde{R}$) Bankruptcy Group S.D.	VAR($\tilde{R}$) Control Group # of Obs.	VAR($\tilde{R}$) Control Group Mean	VAR($\tilde{R}$) Control Group S.D.	VAR($\tilde{R}$) t^a Value	VAR($\tilde{R}$) D.F.	VAR($\tilde{e}$) Bankruptcy Group Mean	VAR($\tilde{e}$) Bankruptcy Group S.D.	VAR($\tilde{e}$) Control Group Mean	VAR($\tilde{e}$) Control Group S.D.	VAR($\tilde{e}$) t Value	VAR($\tilde{e}$) D.F.	β Bankruptcy Group Mean	β Bankruptcy Group S.D.	β Control Group Mean	β Control Group S.D.	β t Value	β D.F.
1	45	.0117	.0067	65	.0046	.0027	6.75[b]	54	.0110	.0066	.0039	.0022	6.96[b]	51	1.012	.830	1.051	.617	−.27	76
20	44	.0101	.0062	65	.0045	.0028	5.68[b]	55	.0094	.0061	.0038	.0022	5.95[b]	51	1.024	.685	1.081	.589	−.45	83
52	45	.0091	.0065	65	.0046	.0028	4.35[b]	55	.0084	.0065	.0039	.0024	4.46[b]	52	1.121	.690	1.105	.568	.13	82
104	45	.0075	.0046	65	.0044	.0028	4.04[b]	66	.0067	.0045	.0037	.0026	4.02[b]	64	1.145	.719	1.066	.566	.61	80
156	45	.0065	.0035	65	.0040	.0026	4.07[b]	77	.0058	.0033	.0035	.0024	4.01[b]	76	1.222	.748	1.109	.604	.83	81
208	44	.0055	.0032	64	.0039	.0025	2.84[c]	77	.0049	.0030	.0034	.0023	2.84[b]	78	1.208	.672	1.144	.592	.51	85
260	40	.0054	.0033	57	.0038	.0025	2.58[d]	68	.0049	.0032	.0033	.0024	2.54[d]	67	1.207	.542	1.051	.532	1.41	83
312	37	.0052	.0039	52	.0035	.0029	2.20[e]	63	.0046	.0036	.0031	.0026	2.25[e]	62	1.188	.695	1.101	.584	.62	69
350	34	.0050	.0033	48	.0032	.0028	2.66[c]	63	.0043	.0028	.0027	.0024	2.79[c]	63	1.383	.811	1.149	.568	1.45	55
390	29	.0050	.0037	40	.0029	.0029	2.55[d]	51	.0044	.0036	.0025	.0026	2.48[d]	48	1.342	.580	.989	.638	2.39†	64

[a] The t-test assumes unequal population variances. (Similar results are obtained using a t test which assumes common population variance.)
[b] Significant at 0.05 percent level (one-tailed test).
[c] Significant at 0.5 percent level (one-tailed test).
[d] Significant at 1.0 percent level (one-tailed test).
[e] Significant at 2.5 percent level (one-tailed test).

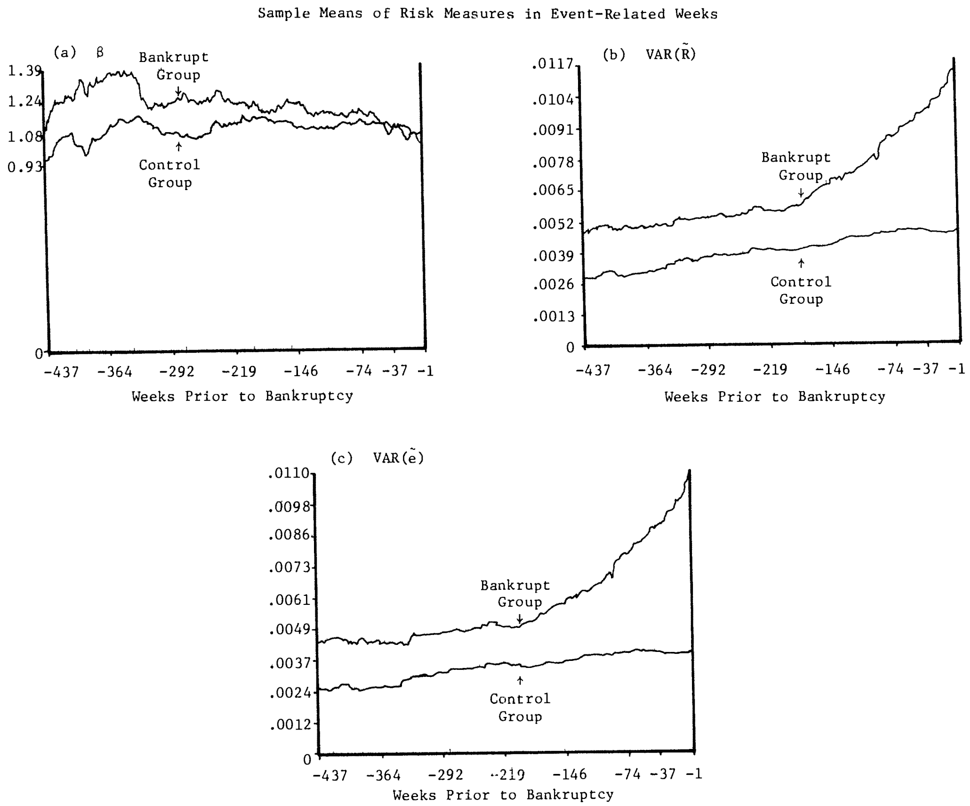


Figure 1. Sample Means of Risk Measures in Event-Related Weeks

in some segments of time, to some increase in the sample mean of $\mathrm{Var}(\tilde{R}_m)$. No significant shifts in $\bar{\beta}$ are observed over time.

The results in Panel *B* reveal no significant shift over time in any of the risk measures for the control group.

These results provide important implications for comparisons of the risk characteristics of non-bankrupt and bankrupt firms prior to bankruptcy. Comparing the bankrupt sample to the control sample, there are substantial differences in the behavior of the mean variances, both total and firm-specific, roughly four years before formal bankruptcy is announced. This is true on both a cross-sectional and a time-series basis. Furthermore, the important component of risk that distinguishes between the two groups is the unsystematic risk. Systematic risk (beta) is not a useful indicator of firm deterioration over time. Thus, in assessing any information concerning corporate bankruptcy that may be contained in capital market data, the measure of risk used is very important.[11]

[11] These findings correspond with the recent contention of Levy (11) that variance plays "a crucial role in the risk measure of stocks." Levy's assertion is based on a relaxation of the perfect market assumption such that investors hold only a few stocks in their portfolios. Based on this analysis, Levy concludes that, "in an imperfect market, β_j plays no role, or at least a negligible role in price determination" and "for most securities, which are not held by many investors we would expect that the variance would provide a better explanation for price behavior" (p. 657).

Table II

Sample Statistics of Risk Measures—Time Series Analysis

Weeks Prior to Bankruptcy Week	VAR($\tilde{R}$)						VAR($\tilde{e}$)						β						VAR($\tilde{R}_m$)					
	A[a]		B[b]				A		B				A		B				A		B			
	Mean	S.D.	Mean	S.D.	t[c] Value	D.F.	Mean	S.D.	Mean	S.D.	t Value	D.F.	Mean	S.D.	Mean	S.D.	t Value	D.F.	Mean	S.D.	Mean	S.D.	t Value	D.F.
A. Bankruptcy Group[d]																								
1–107	.0117	.0067	.0074	.0045	3.61[f]	76	.0110	.0066	.0066	.0044	3.73[f]	76	1.012	.830	1.154	.710	−.87	86	.0006	.0003	.0006	.0004	−.03	88
7–113	.0110	.0064	.0072	.0044	3.25[g]	78	.0103	.0062	.0065	.0043	3.37[g]	79	1.027	.793	1.155	.679	−.82	86	.0006	.0003	.0006	.0003	.28	88
20–126	.0101	.0062	.0070	.0042	2.78[g]	75	.0095	.0061	.0063	.0041	2.87[g]	75	1.024	.685	1.139	.675	−.80	87	.0006	.0003	.0005	.0003	.61	86
52–158	.0091	.0065	.0065	.0035	2.38[h]	67	.0084	.0065	.0058	.0033	2.42[h]	65	1.121	.690	1.219	.738	−.65	88	.0006	.0004	.0005	.0003	2.18[i]	80
78–184	.0085	.0059	.0057	.0031	2.86[g]	66	.0078	.0058	.0051	.0029	2.73[g]	65	1.154	.736	1.172	.689	−.12	88	.0006	.0004	.0004	.0001	4.29[f]	51
104–210	.0075	.0046	.0055	.0032	2.35[h]	78	.0067	.0045	.0049	.0030	2.21[i]	77	1.145	.720	1.214	.670	−.46	87	.0006	.0004	.0003	.0001	4.61[f]	49
120–226	.0070	.0041	.0055	.0034	1.83[j]	85	.0063	.0041	.0050	.0033	1.66[j]	84	1.154	.667	1.231	.595	−.58	87	.0006	.0003	.0003	.0001	4.41[f]	51
155–262	.0065	.0035	.0053	.0033	1.59	83	.0058	.0033	.0048	.0032	1.39	83	1.222	.748	1.219	.529	.02	79	.0005	.0003	.0003	.0001	3.40[g]	56
208–314	.0055	.0032	.0052	.0039	.36	69	.0049	.0030	.0046	.0036	.39	70	1.208	.672	1.191	.691	.11	76	.0003	.0001	.0003	.0001	.55	75
B. Control Group[e]																								
1–107	.0046	.0027	.0044	.0028	.32	128	.0039	.0022	.0038	.0026	.27	125	1.051	.617	1.080	.570	−.27	108	.0006	.0003	.0006	.0004	−.01	128
7–113	.0045	.0028	.0044	.0028	.21	128	.0038	.0022	.0037	.0026	.15	124	1.038	.626	1.080	.571	−.40	127	.0006	.0004	.0006	.0003	.31	128
20–126	.0045	.0028	.0044	.0028	.19	128	.0038	.0022	.0038	.0026	.01	125	1.081	.589	1.082	.585	−.02	128	.0006	.0003	.0005	.0003	.60	128
52–158	.0046	.0028	.0040	.0026	1.21	128	.0039	.0024	.0035	.0024	1.00	128	1.105	.568	1.121	.599	−.16	128	.0006	.0003	.0005	.0003	2.65[g]	118
78–184	.0045	.0029	.0039	.0023	1.38	120	.0039	.0026	.0034	.0021	1.20	120	1.095	.581	1.145	.592	−.48	127	.0006	.0004	.0004	.0001	4.88[g]	73
104–210	.0044	.0028	.0038	.0024	1.19	125	.0037	.0026	.0034	.0023	.86	126	1.066	.566	1.123	.580	−.56	127	.0006	.0004	.0003	.0001	5.51[f]	72
120–226	.0044	.0028	.0039	.0027	1.00	127	.0037	.0026	.0034	.0026	.66	127	1.078	.578	1.106	.577	−.28	127	.0006	.0003	.0003	.0001	5.25[f]	75
155–262	.0040	.0026	.0038	.0025	.56	120	.0035	.0024	.0033	.0024	.32	120	1.109	.604	1.045	.531	.62	120	.0005	.0003	.0003	.0001	4.07[f]	82
208–314	.0039	.0025	.0036	.0030	.56	99	.0034	.0023	.0031	.0027	.62	102	1.144	.592	1.119	.594	.22	109	.0003	.0001	.0003	.0001	.44	107

[a] Statistics for the week closer to bankruptcy.

[b] Statistics for the week further away from bankruptcy.

[c] The t-test assumes unequal population variances.

[d] The number of observations 208 and 314 weeks before bankruptcy are 44 and 37, respectively.

[e] The number of observations 208 and 314 weeks before bankruptcy are 64 and 52, respectively.

[f] Significant at 0.05 percent level (one-tailed test).

[g] Significant at 0.5 percent level (one-tailed test).

[h] Significant at 1.0 percent level (one-tailed test).

[i] Significant at 2.5 percent level (one-tailed test).

[j] Significant at 5.0 percent level (one-tailed test).

IV. Bankruptcy and Abnormal Returns

Abnormal returns may also be used to determine how far in advance market participants can recognize firm deterioration. In addition, an examination of such data may indicate the nature of the adjustment process before bankruptcy occurs. If deterioration is neither expected nor materializes, ex ante and ex post rates of return should be equivalent (ceteris paribus). This is the case for solvent firms. If, however, deterioration is unexpected but eventually happens, ex post returns should be lower than ex ante returns. This possibility characterizes potential failing films. Finally, if at some point deterioration is expected, stock prices should adjust such that thereafter ex ante and ex post returns will be the same as far as deterioration is concerned.

If one controls for the "other factors," then the extent of the difference between ex post rates of return for failing and nonfailing firms may reveal the magnitude of the unexpected deterioration in solvency position, and how soon prior to formal bankruptcy such deterioration signals are observed by capital market participants.[12]

A. Construction of Control and Bankruptcy Portfolios with Identical Systematic Risk

In order to assess that portion of the difference between ex post returns for failed and nonfailed firms which reflects unexpected deterioration in solvency position, factors unrelated to this phenomenon must first be controlled. These factors are mainly attributable to economy-wide effects and, in fact, may have been accounted for, to a large extent, by constructing the control sample to consist of firms in the same industry, with similar asset sizes, etc., as the respective bankrupt companies. Any remaining differences in market-related risk between the two groups should be controlled using the procedure described below.[13]

The control group is ranked by increasing order of β and then divided into two equal sub-groups of high β's and low β's. Utilizing Equation (2), β is estimated for the 104 weeks immediately preceding the 312 closest observations to the respective bankruptcy week.[14] This procedure, based on the previous results, assures that the beta estimate for each control firm is from a point far enough from the point where capital market participants start to notice bankruptcy signals.

For each event-related week, W, portfolio betas are calculated for the low and high sub-control groups and for the bankruptcy group and are denoted by $\beta_{L,W}$, $\beta_{H,W}$, and $\beta_{B,W}$, respectively. The time-series measures of β, estimated in section

[12] A similar approach was taken by Beaver (4). The present paper, however, provides a more rigorous empirical analysis.

[13] A similar procedure was used by Warner (15), and others.

[14] For companies that do not have that many weekly observations before the respective bankruptcy, the earliest 104 observations available were used.

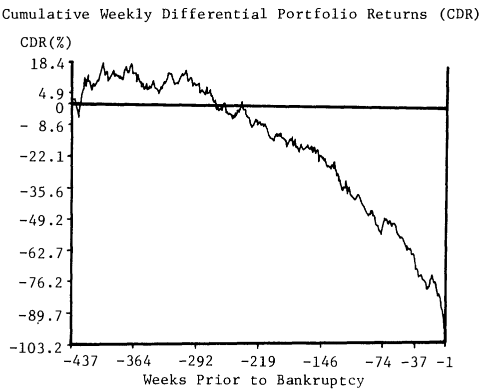


Figure 2. Cumulative Weekly Differential Portfolio Returns (CDR)

III for each company, are used for this purpose.[15] Similarly, using the individual companies' returns, $\tilde{R}_{jw}$, portfolio returns are calculated for each of the three groups and denoted by $\tilde{R}_{L,W}$, $\tilde{R}_{H,W}$, and $\tilde{R}_{B,W}$, respectively.

Finally, a unique set of weights summing to one is calculated for each event-related week, W:

$$\lambda_W \cdot \beta_{L,W} + (1 - \lambda_W) \cdot \beta_{H,W} = \beta_{B,W} \tag{3}$$

where λ_W is the weight assigned to $\beta_{L,W}$. These series of weights are used to construct control portfolios with systematic risk identical to the respective bankruptcy portfolios in various event-related weeks:

$$R_{C,W} = \lambda_W \cdot R_{L,W} + (1 - \lambda_W) \cdot R_{H,W} \tag{4}$$

B. Abnormal Returns and Deterioration Signals

Since the control portfolios are constructed to have risk equivalent to the bankruptcy portfolios, abnormal returns can be measured as the difference in returns between the two portfolios:

$$DR_W = R_{B,W} - R_{C,W} \tag{5}$$

If unexpected deterioration in the solvency position of bankrupt firms is sufficiently small in the periods preceding formal bankruptcy, then DR_W should be approximately zero. Otherwise, it is hypothesized that $DR_W < 0$.

Figure 2 shows the cumulative differential portfolio return from 437 weeks before bankruptcy up to bankruptcy.[16] As can be seen, it turns negative roughly 4.5 years before bankruptcy and declines sharply thereafter. Thus, on average,

[15] For example, $\beta_{l,W} = \frac{1}{N_{L,W}} \cdot \sum_{j=1}^{N_{L,W}} \beta_{jWL}$, where $N_{L,W}$ is equal to the number of firms in the low beta sub-control group in week W. The results indicate consistently that for every week W, $\beta_{L,W} < \beta_{B,W} \leq \beta_{H,W}$.

[16] That is, $CDR_m = \sum_{W=-437}^{m} DR_W$, where $m = -437, -436 \ldots, -1$, relative to the bankruptcy week.

Table III

Percentage Weekly Average (DR) and Cumulative Weekly Differential (CDR) Portfolio Return for Selected Holding Periods Before Bankruptcy Week

Holding Period Prior to Bankruptcy Week	Cumulative Weekly Differential Portfolio Return (*CDR*) (%)	Mean of Weekly Differential Portfolio Return ($\overline{DR}$) (%)	Standard Deviation of Weekly Differential Portfolio Return (%)	*t*-Statistic	D.F.
−7 to −1	−22.96	−3.28	2.37	−3.66[b]	6
−20 to −1	−26.10	−1.30	2.45	−2.38[c]	19
−52 to −21	−21.73	−.68	1.38	−2.78[a]	31
−104 to −53	−16.01	−.31	1.44	−1.54	51
−156 to −53	−36.35	−.35	1.61	−2.21[c]	103
−208 to −105	−29.01	−.28	1.64	−1.74[d]	103
−312 to −209	−19.53	−.19	1.46	−1.32	103

[a] Significant at 0.5 percent level (one-tailed test).
[b] Significant at 1.0 percent level (one-tailed test).
[c] Significant at 2.5 percent level (one-tailed test).
[d] Significant at 5.0 percent level (one-tailed test).

the risk-adjusted returns from the control group exceed those from the bankrupt group for quite a long period of time. Given this overview of the results, it is of interest to test the hypothesis that the mean differential returns ($\overline{DR}$) over various time segments relative to the event week are significantly different from zero. For this purpose a *t*-statistic for differences in two population means with dependent samples is employed. The results are summarized in Table III and discussed below.[17]

The results indicate that in the various time segments within a four year period prior to bankruptcy, the mean return on the bankrupt portfolio is significantly less than that on the control portfolio. Furthermore, the mean weekly difference becomes increasingly negative as bankruptcy approaches. These results imply high unexpected deterioration, on average, for the bankrupt group, and that investors adjusted continuously for the declining solvency positions of these firms over about a four-year period. Moreover, there was a sharp decline in the differential portfolio return during the seven weeks prior to bankruptcy. This implies that even shortly before the event, the market did not fully expect that these firms would soon file for bankruptcy.

It could be argued that each bankrupt firm actually sustains a large stock price drop at only one point in time, when investors first observe substantial solvency deterioration; thereafter rates of return will not reflect unexpected solvency deterioration. If these large negative returns for the various firms occur at different times relative to the bankruptcy date, what appears as a gradual decrease in the cumulative mean differential portfolio return may actually be attributable to the fact that initial knowledge of solvency deterioration is randomly distributed across the sample over the four-year period prior to bankruptcy date. To gain more insight into this possibility, we revised the calculation of differential portfolio returns. Specifically, for each bankrupt and control firm we

[17] For detailed calculation of this *t*-statistic see, for example, Chou (5) pp. 333–336.

eliminated the most negative weekly return (and the corresponding β) in the 260 weeks prior to the respective bankruptcy date. Then we recalculated Equations (3), (4) and (5). Results obtained were similar to those presented in Figure 2 and Table III. This supports our previous contention that investors adjusted gradually to the declining solvency position of the bankrupt firms over approximately a four-year period.

V. Estimating the Probability of Failure and Bankruptcy Prediction

Differences in the various sample means, on both a cross-sectional and time-series basis, clearly indicate that for a period of roughly four years before bankruptcy, significant differences exist in the market-derived risk and return characteristics of failed firms as compared to solvent firms. This evidence, however, does not directly suggest the extent of predictive power embedded in capital market data. To gain some insight into the usefulness of such data for bankruptcy prediction purposes, an approach is suggested to estimate the probability of failure for each individual firm and then compare it to a probability of failure estimated for the respective industry as a whole. It should be emphasized that the implications from these estimates with respect to the predictive content of market data cannot be generalized because resource limitations prevent the representation of bankrupt and non-bankrupt firms in the sample from corresponding to their incidence in the population.[18]

A. Probability of Failure

At most, an investor in a company's common stock may lose his entire investment when bankruptcy occurs. Suppose an investor purchased common stock in period t at a price of P_t dollars per share. If bankruptcy occurs at period $t + 1$ and, as a result, the stock price drops to zero, the investor's return (assuming no dividends) is:

$$\tilde{R}_{t+1} = \frac{0 - P_t}{P_t} = -1. \tag{6}$$

Accordingly, the probability that a company will go bankrupt in the next period is defined as:

$$P_r(\tilde{R}_{t+1} = -1). \tag{7}$$

Assuming that security returns are distributed multi-variate normal with mean μ and standard deviation σ, the probability in (7) can be standardized and obtained from standard normal tables, as shown in (8):

$$P_r(\tilde{R}_{t+1} \leq -1) = P_r\left[\frac{\tilde{R}_{t+1} - \mu}{\sigma} \leq \frac{-1 - \mu}{\sigma}\right] = N\left[\frac{-1 - \mu}{\sigma}\right] = N(Z) \tag{8}$$

Given the previous results of the times-series pattern of the variance of returns,

[18] This shortcoming is common to most previous bankruptcy prediction studies. For criticism on this subject, see, for example, Foster (9, pp. 477–478) or Eisenbeis (7, pp. 889–890).

it is to be expected that Z would decrease (in absolute terms) for the distressed firms as the bankruptcy date approaches, but would remain unchanged and at a higher level (in absolute terms) over time for the control firms. Such results would be consistent with an increasing probability of failure for each distressed firm and an unchanged probability of failure (at a lower level) for the solvent firms.

To estimate the probability of failure in the following quarter, quarterly rates of return are calculated. Specifically, for each firm j, every 13 consecutive weekly returns ($\tilde{R}_{jw}$) are compounded to obtain quarterly returns ($\tilde{R}_{jq}$) for each quarter q prior to the bankruptcy date. In order to estimate the mean ($\hat{\mu}_q$) and the standard deviation ($\hat{\sigma}_q$) of the quarterly rates of return for a given quarter q relative to the respective bankruptcy date, the current and previous seven quarterly rates of return are used. Accordingly, a series of moving eight-quarter means and standard deviations are estimated for each company. Finally, introducing $\hat{\mu}_q$ and $\hat{\sigma}_q$ into equation (8), the probability of failure is estimated for each firm in various quarters prior to the respective bankruptcy dates.

It should be noted that equity has limited liability, namely, stockholders' liability for the corporation's losses is limited to the value of their stocks that cannot drop below zero. This implies that μ and σ actually are estimated from sample observations drawn from a distribution that is truncated at the lower end. The approach used to correct this problem is discussed in Appendix A. This correction, however, provides results similar to those in Table IV below.

Table IV

Misclassification Percentages of Bankrupt and Control Firms in Quarters Relative to Bankruptcy Date

	$P_r(\tilde{R}_{t+1} \leq -0.6)$			$P_r(\tilde{R}_{t+1} \leq -0.7)$		
Quarters Prior to Bankruptcy Date	Estimated Type I Error (%)	Estimated Type II Error (%)	Estimated Total Misclassification (%)	Estimated Type I Error (%)	Estimated Type II Error (%)	Estimated Total Misclassification (%)
1	13.3	46.2	32.7	33.3	30.8	31.8
2	22.2	49.2	38.2	33.3	30.8	31.8
3	13.3	43.1	30.9	37.8	32.3	34.5
4	8.9	44.6	30.0	42.2	29.2	34.5
5	15.6	53.8	38.2	40.0	33.9	36.4
6	20.0	53.8	40.0	37.8	26.2	30.9
7	26.7	44.6	37.3	40.0	29.2	33.6
8	20.0	40.0	31.8	40.0	29.2	33.6
9	18.2	32.8	26.9	31.8	23.4	26.9
10	25.0	37.5	32.4	43.2	23.4	31.5
11	29.5	37.5	34.3	45.5	23.4	32.4
12	29.3	28.8	29.0	53.7	20.3	34.0
13	35.0	33.3	34.0	52.5	14.0	29.9
14	40.0	35.1	37.1	67.5	15.8	37.1
15	38.5	34.5	36.2	64.1	18.2	37.2
16	44.7	29.1	35.5	63.2	20.0	37.6
17	48.6	30.8	38.2	59.5	19.2	36.0

B. Bankruptcy Prediction

In order to use the probabilities estimated above for bankruptcy prediction, each probability is compared with an estimate of the actual probability of failure in the corresponding industry for the respective calendar quarter. As a quarterly industry probability of failure proxy we use the ratio of one-fourth of the actual annual number of failures in a given industry to the annual number of active corporations in that industry.

The number of failures by industry was obtained from *Dun's Statistical Review—Quarterly Failure Report*, published on a quarterly basis by Dun and Bradstreet, Inc. These numbers are based on court records of Chapter X and Chapter XI bankruptcy cases, and on information solicited from creditors. These reports were available to us for every year up to 1976. To convert these data into estimated failure rates, an appropriate base must be selected. The base selected was the number of corporations filing income tax returns by industry according to the annual reports of the *IRS Statistics of Income—Corporation Income Tax Returns*.[19,20]

In order to classify a firm as a potential failure in a given quarter prior to the respective bankruptcy date, the estimated probability of bankruptcy using market data is compared with the "actual" quarterly rate of bankruptcy for the corresponding industry and calendar quarter. If the former probability is larger than the latter, the firm is classified as a potential candidate for bankruptcy in the next quarter; otherwise it is classified as a nonfailed company. These classifications are then compared with the actual failure status of the firms.

Two types of errors can result in each event-related quarter. A Type I prediction error occurs when a firm that subsequently fails is predicted to be nonbankrupt. A Type II error occurs when a solvent firm is predicted to be bankrupt. Estimates of these two error types and the total misclassification (Type I plus Type II error) are calculated as percentages of the total number of bankrupt firms, control firms and the whole sample, respectively.

Given the previous results—that the deterioration of solvency among failing firms is a continuous process—the downward adjustment of respective stock prices should be gradual. Furthermore, in many cases formal bankruptcy occurs even before the stock price drops to zero. Therefore, an effective deterioration signal may be provided by a stock price drop that produces a large negative rate of return that is still higher than −100 percent. Since a priori knowledge is not available about such a critical value, we searched for that equity loss percentage that minimizes the total misclassification percentage. $R = -0.6$ and $R = -0.07$ (when used in Equations (7) and (8)) were found to provide the best cutoff points.

Table IV presents estimates of the percentage of errors made in classifying the sample by type of error at the two alternative cutoff points. Results indicate that the total misclassification ranges from about 27 to 38 percent over the 17 quarters

[19] A similar approach was used previously by Flath and Knoeber (8) for different purposes.

[20] These reports are available through 1975 and provide industry breakdowns similar to those provided in Dun and Bradstreet quarterly failure reports. In cases where data were needed for later years, we use as a proxy the data in the most recent reports available.

prior to bankruptcy for either cutoff point (−0.6 or −0.7). Assuming that a 60 percent equity loss is an effective deterioration signal, Type I errors are 20 percent or less in most of the nine quarters closest to the bankruptcy date. Type I errors increase as we move farther from the bankruptcy date. No clear-cut trend is observed for the higher level Type II errors.

Assuming a cutoff point of 70 percent equity loss, Type I errors range between 32 and 42 percent in the nine quarters closest to the bankruptcy date and are higher in earlier quarters. Type II errors, on the other hand, are lower than those observed under the −0.6 cutoff point and range between 14 and 34 percent in the 17 quarter period.

These results indicate that a solvency deterioration signal using capital market data is available some two years before the bankruptcy event. Although preliminary, such results show the potential usefulness of market data for the prediction of corporate bankruptcy.

VI. SUMMARY

The primary purpose of this paper has been to compare the characteristics of risk and return measures for a sample of bankrupt firms with the corresponding characteristics of a matched sample of nonbankrupt firms using capital market data. While much has been said about the use of accounting ratios for the analysis of corporate failure, little has been said about investors' analyses of failure as reflected in risk-return measures set in market data.

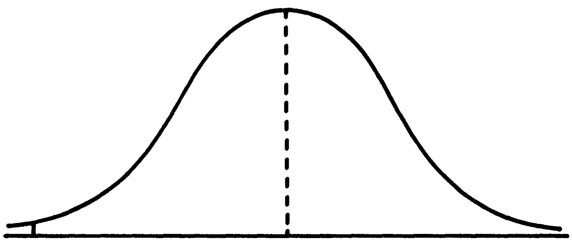
Using a sample of 45 industrial companies that went bankrupt during 1970–1978 and a control group of 65 firms, each of the components of the total risk borne by stockholders was examined in order to measure changes in the risk of failure. Extensive cross-sectional and time series analysis indicate that certain risk measures based on market data exhibit significantly different behavior between the two samples. Specifically, both the total variance and the firm-specific variance behave quite differently for the bankrupt and control groups as much as four years before bankruptcy. Systematic risk is not a useful indicator of the declining solvency position of firms over time.

A risk-controlled study of the returns from the two groups indicates a significant negative cumulative differential portfolio return starting roughly four years before bankruptcy. The unexpected deterioration in the bankrupt group was high, with investors having to continuously adjust for declining solvency over about a four year period. Investors were apparently surprised up to the time of the bankruptcy.

Finally, an approach was suggested to estimate the probability of failure for each company based on quarterly rates of return data. These probabilities were compared to estimates of the actual probability of failure in the corresponding industry for the respective calendar quarter. Equity loss percentages that minimize the total misclassifications of firms (both bankrupt and nonbankrupt) were determined. Although the results are interesting and suggest possibilities of using a methodology and data such as used here, they are preliminary and do not warrant conclusions about the predictive content of market data.

Appendix A. Estimating the Mean and Variance of a Normal Distribution from a Truncated Normal Distribution

Assume that $\tilde{R}$ is distributed normally with mean μ and variance σ^2. Given that equity has limited liability, sample observations of $\tilde{R}$ for estimating μ and σ^2 can be drawn only from the range $-1 \leq \tilde{R} \leq \infty$. In other words the sample observations are actually drawn from a normal distribution with a density *truncated* on the left at $a = -1$:



For securities that do not encounter solvency deterioration problems, the truncation phenomenon, although it exists, is negligible, since the truncation point is far from the mean and therefore, probably does not affect the estimation of μ and σ^2. For potential failing firms, however, it may cause a bias in the estimated parameters and hence affects the estimation of the probability of failure.

To correct for this potential bias we first calculate the mean and variance of the truncated distribution and then obtain μ and σ^2 for the whole distribution. The mean of a truncated normal distribution is:

$$E(\tilde{R} \mid \tilde{R} > a) = \int_a^{\infty} \tilde{R} \cdot \frac{(2\pi\sigma^2)^{-1/2} \cdot e^{-1/2(\tilde{R}-\mu)^2/\sigma^2}}{P_r(\tilde{R} > a)} \, d\tilde{R}$$

$$= \mu + \frac{\sigma}{\sqrt{2\pi}[1 - \phi(m)]} \cdot e^{-1/2m^2} \qquad \text{(A-1)}$$

where:

$\pi = 3.142$, $e = 2.178$, $m = \dfrac{a - \mu}{\sigma}$, $a = -1$, and

Φ = the cumulative standard normal distribution function.

The variance of a truncated normal distribution is:[21]

$$\mathrm{Var}(\tilde{R} \mid \tilde{R} > a) = E(\tilde{R}^2 \mid \tilde{R} > a) - [E(\tilde{R} \mid \tilde{R} > a)]^2$$

$$= \int_a^{\infty} \tilde{R}^2 \cdot \frac{(2\pi\sigma^2)^{-1/2} \cdot e^{-1/2(\tilde{R}-\mu)^2/\sigma^2}}{P_r(\tilde{R} > a)} \, d\tilde{R} - [E(\tilde{R} \mid \tilde{R} > a)]^2$$

$$= \frac{\sigma^2 \cdot e^{-1/2m^2}}{[1 - \Phi(m)] \cdot \sqrt{2\pi}} \cdot \left[m - \frac{e^{-1/2m^2}}{\sqrt{2\pi}[1 - \Phi(m)]} \right] + \sigma^2 \qquad \text{(A-2)}$$

[21] Detailed derivations of equations (A-1) and (A-2) are available from the authors upon request.

(A-1) and (A-2) are two non-linear equations with two unknowns, μ and σ. The left sides of these equations are the mean and variance of the truncated distribution, respectively, and are estimated from observed data. To solve for these unknowns, the Newton-Raphson method[22] of successive approximation is employed. The mean and variance estimates of the truncated distribution are used as initial points for the iteration procedure. The approximation is carried to the sixth decimal point. The μ and σ estimates for each quarter are then substituted into equation (8) to obtain an estimate for the probability of failure. This procedure resulted in higher probabilities. When these probabilities were used for classifying successful and unsuccessful predictions, however, the results were similar to those presented in Table 4.

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[22] For details see, for example, Saaty and Bram [13], pp. 56–60.