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# Co-Skewness and Capital Asset Pricing

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## I. Introduction

VIRTUALLY ALL OF THE early studies of the Sharpe-Lintner capital asset pricing model (CAPM) found the predicted linear relationship between return and the non-diversifiable risk of risky assets, generally represented by common stocks listed on the New York Stock Exchange (NYSE). However, they also found that this return-risk relationship seemed to imply for most periods a riskless market rate of return substantially above any reasonable measure of the actual risk-free rates of return. Recent papers point to a similar result if the market portfolio of risky assets is represented by an appropriately weighted portfolio of common stocks and bonds instead of common stocks alone.[1]

Thus, it is noteworthy that a study by Kraus and Litzenberger finds that a measure of co-skewness can be used as a supplement to the co-variance measure of risk to explain the returns on individual NYSE stocks, and in the process to explain the otherwise observed discrepancies between these returns and the returns on NYSE stocks as a whole.[2] In other words, they extend capital asset pricing theory to incorporate the effect of skewness in return distributions, making the assumption that investors have a preference for positive return skewness in their portfolios (and therefore positive or negative co-skewness in individual assets depending on the skewness in the market portfolio).[3] As a consequence Kraus and Litzenberger are apparently able to explain observed returns in the stock market without the substitution of a non-observable zero-beta construct for the risk-free rate. Kraus and Litzenberger assume that just as investors are averse to variance in their portfolios, and therefore beta in individual assets, they prefer positive skewness in their portfolios. Hence, since they also assume that all investors hold the market portfolio, investors would be willing to pay a premium for assets which possess positive co-skewness with the market if

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[1] See Friend, Westerfield and Granito [4] and Friend and Westerfield [5]. These papers also raise serious questions about exclusive reliance on nondiversifiable risk in measuring the overall riskiness of assets.

[2] See Kraus and Litzenberger [6].

[3] Their basic approach is to expand a utility function beyond the second moment in a Tayler Series and to examine skewness effects. They do not consider higher order effects. Assuming separation all investors choose the market portfolio (in equilibrium). The market portfolio is not mean-variance efficient but is efficient with respect to the utility functions that lead to separation. Thus the Kraus-Litzenberger model is different from but can be regarded as an extension of the earlier forms of the CAPM.

that portfolio is characterized by positive skewness. If the market had negative skewness, investors would be averse to positive co-skewness with the market.[4]

The explicit empirical model developed by Kraus and Litzenberger is given by

$$\bar{r}_i = \gamma_0 + \gamma_1 \beta_i + \gamma_2 \delta_i + u_i \quad \text{for} \quad i = 1, 2, \cdots, N \quad \text{where} \quad \bar{r}_i = \frac{\overline{R_i - R_f}}{\bar{R}_f},$$

$$\delta_i = \frac{\sum_t (r_{mt} - \bar{r}_m)^2 (r_{it} - \bar{r}_i),}{\sum_t (r_{mt} - \bar{r}_m)^3}, \qquad \beta_i = \frac{\sum_t (r_{it} - \bar{r}_i)(r_{mt} - \bar{r}_m)}{\sum_t (r_{mt} - \bar{r}_m)^2},$$

$R_i$ is the dividend-adjusted relative for the i'th asset, $R_m$ is the market portfolio return relative, $R_f$ is the risk-free return relative, and $u_i$ is an independently distributed random variable. According to the Kraus-Litzenberger version of the CAPM, $\gamma_0 = 0$, $\gamma_1 + \gamma_2 = r_m$, $\gamma_1$ is positive, and the sign of $\gamma_2$ is opposite that of the skewness of the market portfolio. It should be noted that a zero intercept in their model ($\gamma_0 = 0$) is equivalent to a risk-free rate intercept ($R_f$) in the Sharpe-Lintner version of the CAPM.

In review of the importance of their finding if it is valid, this paper is devoted to a more comprehensive testing of the Kraus-Litzenberger thesis than is carried out in their original article. We incorporate bonds as well as stocks into the analysis, use a market portfolio including bonds and stocks as well as stocks alone, measure a market portfoilo of stocks by a value-weighted index as well as equal-weighted index, carry out tests for individual assets as well as groups of assets, use predictive as well as contemporaneous measures of risk, distinguish among different periods and types of markets and follow more satisfactory grouping procedures.[5] Our conclusion is that the Kraus-Litzenberger attempt to develop and substantiate a modified form of the Sharpe-Lintner CAPM is not successful though there is some but not conclusive evidence that investors may pay a premium for positive skewness in their portfolios.

Sections II-IV of this paper provide a wide variety of different tests of the Kraus-Litzenberger version of the CAPM, all based on a value-weighted index of stock and bond returns which is theoretically preferable to the equal-weighted index of stock returns they used. Unlike the Kraus-Litzenberger analysis, Section II also presents results for individual stocks as well as groups of stocks and compares stationary and non-stationary regressions; Section III covers bonds as well as stocks, both separately and combined; Section IV introduces predictive as contrasted with contemporaneous measures of risk.

Section V provides selected results comparable to those presented in the earlier sections but based both on a value-weighted and an equal-weighted index of stock returns alone to determine the sensitivity of the results to the market index used

[4] The Kraus-Litzenberger theory also holds if the market portfolio is symmetric in the returns distribution and individual assets exhibit co-skewness with the market.

[5] On the other hand, the January 1936-June 1970 period they cover is longer though not so current as our January 1952-December 1976 period. Unfortunately they do not provide results for sub-periods which would facilitate an assessment of the stability of their findings over time as well as a direct comparison with our results.

and to explain the difference between our findings and those obtained by Kraus-Litzenberger. The results do not seem to be very sensitive to the market index used but are more sensitive to the period covered and statistical procedures used. As the analysis in the earlier sections indicates, the results are especially sensitive to the relationship between the market rate of returns ($R_m$) and the risk-free rate ($R_f$) in the period examined. When $R_m > R_f$, the intercept in the Kraus-Litzenberger model is significantly positive; for $R_m < R_f$ the intercept is significantly negative. The Kraus-Litzenberger conclusion that the intercept is insignificantly different from zero is probably largely a reflection of the particular period they selected and the fact that they did not examine sub-periods.

## II. Tests of the CAPM Based on a Value-Weighted Index of Stock and Bond Returns

To construct an overall market return index which would be expected to be more satisfactory for testing the CAPM than the usual stock index, it was necessary to obtain appropriate market indexes for the major classes of marketable assets and then to apply the relevant market weights.[6] We used the Standard & Poor's 500 Composite Index to cover all common stocks from 1947 to 1964, and New York Stock Exchange (NYSE) Composite Index from 1964 through 1976;[7] the Salomon Brother's Total Performance Index to cover all corporate bonds from 1969 through 1976, and Moody's Composite Bond Index from 1947 to 1968;[8] and a U. S. Government bond index developed by John Bildersee[9] to cover all long-term marketable government issues from 1947 to 1973, and Salomon Brother's government bond yields from 1974 to 1976. The weights applied in estimating the overall market return ($R_m$) for 1973 from the three constituent returns were 60% for corporate equities, 30% for bonds other than U.S. Governments, and 10% for long-term marketable U.S. Government issues. These weights which varied from year to year were obtained from the annual Federal Reserve Board Flow of Funds data on the market value of stocks and bonds held by U.S. individuals and financial institutions. A potential limitation of the $R_m$ index as an estimate or return on all stocks and bonds is the absence of a satisfactory index for return on municipal bonds, which account for about 10% of the value of all stocks and bonds held by individuals and institutions. Municipal returns have been assumed to

[6] Correct weights on various classes of securities are difficult to determine because of problems associated with the treatment of government debt, financial intermediation and non-marketable assets. As a result several different sets of weights were tested. Of course, all of our tests hypothesize that our composite index of stocks and bonds is *ex ante* efficient and close to the true market portfolio.

[7] Monthly dividend-adjusted investment relatives were computed from these indexes.

[8] Salomon Brother's Total Performance Index includes coupon income and reinvestment income as well as the capital gain. Monthly return relatives were estimated as $I_t/I_{t-1}$, where $I_t$ is the Index at time $t$. On the other hand, Moody's Index is basically the yield to maturity index and had to be converted to a price index before estimating the return relatives. The price $P$ was calculated using coupon rate $C$ and the Yield to maturity given at the end of the month with the assumption of 20-year maturity. The yield to maturity at the beginning of the month was used as the coupon rate $C$ on a new 20-year bond sold at par. Monthly price relatives were then estimated as $(P + C/12)/100$.

[9] See Bildersee [1].

move in the same fashion as corporate bonds and the weight on corporate bonds has been increased accordingly.

Following the general procedures used by Kraus and Litzenberger, Table I presents return-risk cross-section relationships in which the average monthly ratios ($\bar{r}_{it}$) of the differences $\overline{(R_i - R_f)}/R_f$ between returns on individual New York Stock Exchange (NYSE) common stock issues and on one-month Treasury bills (taken to be the risk-free asset) to the Treasury bill return are regressed on an estimate of the beta coefficient ($\beta_i$) and an estimate of co-skewness ($\delta_i$) for five

**Table I**

Return-Risk Regressions for Individual Common Stocks[a] Adjusted for Order Bias, January 1952–December 1976: Stationary (S) and Non-Stationary (NS) Estimates Using Value Weighted Common Stock and Bond Index

| | | Estimates of Regression Coefficients | | | | | |
|---|---|---|---|---|---|---|---|
| Period | | $\gamma_0$ | $\gamma_1$ | $\gamma_2$ | $\bar{R}^2$ | $\overline{R_m - R_f}$ | Skew$_m$(.0001) |
| 1952–56 | | 0.324 | 0.663 | −0.010 | | .0085 | .005 |
| | S | (3.8) | (9.8) | (−2.9) | .09 | | |
| | NS | (1.6) | (1.6) | (−0.7) | .06 | | |
| 1957–61 | | 1.627 | −.685 | 0.311 | | .0056 | −.049 |
| | S | (12.8) | (−7.3) | (8.0) | .07 | | |
| | NS | (4.8) | (−1.3) | (1.7) | .08 | | |
| 1962–66 | | −0.252 | 0.190 | 0.299 | | −.0021 | −.153 |
| | S | (−2.1) | (1.9) | (5.4) | .06 | | |
| | NS | (−0.6) | (0.3) | (1.2) | .08 | | |
| 1967–71 | | 0.137 | 0.297 | 0.015 | | .0021 | −.005 |
| | S | (1.3) | (4.2) | (4.4) | .04 | | |
| | NS | (0.3) | (0.5) | (1.0) | .09 | | |
| 1972–76 | | 0.879 | −0.356 | 0.032 | | .0005[b] | .377 |
| | S | (7.8) | (−4.6) | (1.1) | .02 | | |
| | NS | (1.5) | (−0.4) | (0.1) | .06 | | |
| 1952–76 | | .545 | .022 | .013 | | .0029 | .026 |
| | NS | (3.0) | (0.1) | (1.6) | .07 | | |

[a] These are cross-section regressions of the general form

$$\bar{r}_i = \gamma_0 + \gamma_i\beta_i + \gamma_2\delta_i \quad \text{where} \quad \delta_i = \textstyle\sum_t (r_{mt} - \bar{r}_m)^2(r_{it} - \bar{r}_i)/\sum_t (r_{mt} - \bar{r}_m)^3,$$

$$\beta_i = \textstyle\sum(r_{it} - \bar{r}_i)(r_{mt} - \bar{r}_m)/\sum(r_{mt} - r_m)^2, \qquad r_i = ((R_i - R_f)/R_f) \times 100$$

$$r_m = ((R_m - R_f)/R_f) \times 100 \quad \text{and} \quad \bar{R}_i, \bar{R}_m \quad \text{and} \quad \bar{R}_f$$

are the average monthly return relatives on an individual stock, on a composite value-weighted market index of stocks and bonds and on 30 day Treasury bills over a 60 month period. The t-test statistics are indicated by ( ). The number of observations is 942 for 1952–56. 924 for 1956–61, 966 for 1962–66, 961 for 1967–71, and 1144 for 1972–76. Skew$_m$ represents market skewness and equals $\frac{1}{T}\sum_t (R_{mt} - \bar{R}_m)^3$, where $T$ is the number of months.

[b] Not significantly different from zero at .05 probability level.

different 60 month periods from January 1952 through December 1976.[10] The returns for individual stocks were obtained from data of the Rodney L. White Center for Financial Research covering all NYSE issues, but for each 60 month period only those issues for which data were available for every month were included in the regressions.[11] The beta coefficients computed for these stocks have been adjusted for "order bias" using procedures originally suggested by Vasicek.[12]

The statistical analysis in Table I presents two sets of $t$-statistics shown in parentheses under the relevant regression coefficients. The first set (denoted S) assumes that the regression coefficients are constant or stationary over each 60 month period (though not from one period to the next). Clearly it is desirable to test, and where necessary allow, for possible non-stationarities in the values of these coefficients within a 60 month period, and this is done in Table I (for individual stocks) and Table II (for grouped stocks) by computing the cross-sectional regression coefficients in every month and deriving the appropriate standard errors from the time series 60 estimates of the regression coefficients. The regression coefficients presented are those associated with the stationary (S) regressions but also represent averages of the 60 monthly non-stationary regressions (NS). The coefficients in the non-stationary regressions are obviously characterized by much larger standard errors so that they lose much of any significance shown in the stationary regressions.[13]

The stationary regressions in Table I indicate that with the introduction of co-skewness as an additional explanatory variable in the regressions of stock returns on beta, the intercepts ($\gamma_0$) are in general significantly different from zero, which means that the expected returns on the zero-beta portfolio are significantly different from the risk-free rate. A similar result is obtained from the non-stationary regression for the period as a whole (1952-76). This is inconsistent with the hypothesis advanced by Kraus and Litzenberger if, as they assume in their

[10] Returns are dividend-adjusted investment relatives, i.e. $R_t = \dfrac{D_t + P_t}{P_{t-1}}$ where $D_t$ is the monthly dividend and $P_t$ the price at month $t$. The regression coefficients in the Kraus-Litzenberger model presented earlier $\gamma_0$, $\gamma_1$ and $\gamma_2$ are estimated using ordinary least squares.

[11] This requirement can introduce some *ex post* selection bias in exaggerating the expected positive relationship between return and risk. However, an analysis carried out in Friend, Westerfield and Granito suggests that the qualitative results are not likely to be affected.

[12] See Vasicek [10]. Also, see Blume [2]. The exact procedure is found on pages 790–791 of Blume, except that sample means replace the population means and the error in measuring beta for each asset is not assumed to be the same.

[13] For each month in a five year period the cross-section or individual common stock returns are regressed on the estimated betas and co-skewness measures to obtain the regression coefficients in the relationship $\bar{r}_i = \gamma_0 + \hat{\gamma}_1 \hat{\beta}_i + \hat{\gamma}_2 \hat{\delta}_i + \hat{\epsilon}_i$. This procedure results in a time series of observations in the estimates ($\hat{\gamma}_0$, $\hat{\gamma}_1$ and $\hat{\gamma}_2$) and averaging these 60 cross-sectional estimates for each of the regression coefficients provides an estimate of the risk-return trade-off. Standard errors of these averages are taken from the time series of $\hat{\gamma}_0$, $\hat{\gamma}_1$, and $\hat{\gamma}_2$, thus incorporating the variability of the risk-return trade-off. $\bar{R}^2$ in these regressions are an average of the monthly values. An F-test of the stationarity of $\hat{\gamma}_1$ for each of the five 60 month periods, estimated by dividing the variance of the 60 values of $\hat{\gamma}_1$ by the average of the square of the standard errors of the 60 values of $\hat{\gamma}_1$, points to non-stationarity at the .95 level of significance for all five periods.

**Table II**

**Return-Risk Regressions for Common Stocks[a] Grouped to Minimize Measurement Errors, January 1952–December 1976: Stationary and Non-Stationary Estimates Using Value Weighted Common Stock and Bond Index**

| Period | | Estimates of Regression Coefficients | | | | | |
|---|---|---|---|---|---|---|---|
| | | $\gamma_0$ | $\gamma_1$ | $\gamma_2$ | $\bar{R}^2$ | $\overline{R_m - R_f}$ | $\text{Skew}_m(.0001)$ |
| 1952–56 | | 0.553 | 0.513 | 0.003 | | .0085 | .005 |
| | S | (4.3) | (5.1) | (0.2) | .49 | | |
| | NS | (2.5) | (1.3) | (0.1) | .30 | | |
| 1957–61 | | 2.447 | −1.258 | 0.399 | | .0056 | −.049 |
| | S | (9.2) | (−3.0) | (1.2) | .59 | | |
| | NS | (6.2) | (−2.5) | (1.2) | .34 | | |
| 1962–66 | | −0.500 | 0.898 | −0.267 | | −.0021 | −.153 |
| | S | (−2.5) | (2.9) | (−0.7) | .55 | | |
| | NS | (−1.1) | (1.6) | (−0.6) | .42 | | |
| 1967–71 | | 0.350 | 0.119 | 0.010 | | .0021 | −.005 |
| | S | (1.7) | (0.9) | (0.5) | .00 | | |
| | NS | (0.7) | (0.2) | (0.3) | .40 | | |
| 1972–76 | | 0.849 | −0.202 | −0.106 | | .0005[b] | .377 |
| | S | (3.7) | (−1.3) | (−0.6) | .10 | | |
| | NS | (1.3) | (−0.2) | (−0.2) | .38 | | |
| 1952–76 | | 0.740 | 0.014 | .008 | | .0029 | .026 |
| | NS | (3.5) | (0.1) | (0.4) | .37 | | |

[a] The regression forms are identical with Table I but now are estimated from 25 grouped observations.

[b] Not significantly different from zero at .05 probability level.

statistical analysis, realized returns are a satisfactory proxy for expected returns. On the other hand, most of the co-skewness co-efficients ($\gamma_2$) in the stationary regressions are statistically significant, and have the expected sign,[14] suggesting that investors may have a preference for positive skewness in their portfolios. In contrast, only one of the sub-period co-skewness coefficients in the nonstationary regressions is statistically significant with the expected sign (1957-61) and for the period as a whole the coefficient of co-skewness has the wrong sign. In view of the strange pattern of signs for the beta ($\gamma_1$) coefficients, the regression coefficients may reflect collinearity between the beta and co-skewness measures.[15]

The substantial difference between our results and those obtained by Kraus and Litzenberger so far as the intercept is concerned may reflect the different periods or the broader market index we used. However, there is one other

[14] This is true for the first four of the five periods covered. The expected sign according to Kraus and Litzenberger would be opposite the sign of market skewness.

[15] For example, the correlation coefficient of the estimates of $\beta_i$ and $\gamma_i$ over the 1962–66 period is .57 for individual securities and .92 for groups of securities. These values are representative and are similar to those reported by Kraus and Litzenberger.

important difference between their procedures and those followed in Table I. The results in that table rely on the Vasicek adjustment to take care of the substantial measurement errors in estimating $\beta_1$ from observations on individual stocks, whereas Kraus and Litzenberger relied exclusively on grouped data for this purpose. It is desirable to use both techniques to provide a check on the adequacy of the adjustments used for minimizing measurement errors, and tests of the Kraus-Litzenberger version of the CAPM based on grouped data will be presented in Table II (and subsequent tables) in this paper. Nonetheless, before proceeding to the tests based on grouped data, it should be noted that tests of capital asset pricing theory that rely only on grouped data, to the exclusion of tests based on individual assets, are not completely satisfactory, since it is the returns on individual assets which the theory is trying to explain, and some kinds of individual asset deviations from linearity may cancel out in the formation of portfolios.[16] Since Kraus and Litzenberger use grouped data and allow for non-stationarities in the regression coefficients, their results should be most comparable with those presented as NS in Table II.

The results in Table II based on grouped data for common stocks[17] are also inconsistent with the Kraus-Litzenberger findings and, so far as the intercepts are concerned, support the results in Table I based on individual stocks, with the intercepts generally significantly different from zero. However, in contrast to previous results, the co-skewness effects are also insignificantly different from zero. Moreover, in the non-stationary (group) regressions which are conceptually closest to those presented by Kraus and Litzenberger, the beta coefficients are never characterized both by significance and the sign conforming to expectations implicit in the market model (i.e., the same sign as $\overline{R_m - R_f}$). The grouping procedure followed in this analysis was to rank the individual stocks first by beta quintile on the basis of monthly data for the preceding 60 month period and then within each quintile by co-skewness into 5 sub-groups, resulting in a total of 25 groups. Each equation was then estimated using subsequent beta and co-skewness estimates for each group. As a result, only those common stocks for which returns data were available for 120 months were included in these regressions.[18]

The standard errors of the $\beta_i$ and $\delta_i$ coefficients in the nonstationary regressions, it should be noted, test the significance of the departures from zero of the mean values of these coefficients over the indicated period of time and hence reflect the variance of the regression coefficients over time as well as the variance of the estimation error terms. The results of the non-stationary regressions make no special assumption about the stochastic process generating returns and thus could tend to understate the significance of the $\beta_i$ and the $\delta_i$ coefficients within the time period covered (i.e., for individual months within a five year or longer

[16] See Lévy [7] and Roll [9].

[17] The values of $\beta_i$ and $\delta_i$ in the grouped regressions represent group means. They are simple averages of the values for the individual assets in the group.

[18] Portfolios are formed on the basis of estimates of beta and co-skewness for individual securities in an initial five year period and then estimates of portfolio beta and co-skewness are computed by re-estimating beta and co-skewness for individual securities in a subsequent five year period. This process is repeated five times.

period of time). The stationary regressions, in contrast, probably tend to overstate significance.

## III. Stocks and Bonds

It is possible for a more limited period to provide additional evidence on the relative explanatory power of beta and co-skewness in determining returns on risky assets by using the available quarterly data on two samples of individual corporate bonds as well as common stocks. The first sample of 891 individual bonds as obtained from a data tape compiled by the Rodney L. White Center containing quarterly rates of return from the fourth quarter of 1968 through the third quarter of 1973 for every corporate bond listed on the NYSE, after removing a small number of bonds for which satisfactory price and interest data were not available for as many as 10 quarters. The same data tape was used to obtain for this period an equally-weighted quarterly index of market return on bonds (the RLW index) based on all issues covered by this tape. To compare the risk-return relationships for bonds with those for stocks, the quarterly rates of return for the same period were obtained for 867 NYSE common stocks from a second Center tape.[19] The second sample of 86 individual bonds covering the period from the first quarter of 1964 through the third quarter of 1968 consisted of a 10% sample of 891 bonds covered in the subsequent period, except that not more than one bond was included from a single corporation. The S&P Composite AAA Bond Price Index was used to obtain for this period a quarterly index of market return on bonds. Again, to compare the risk-return relationships for bonds with those for stocks, the quarterly rates of return over this period were obtained from the relevant RLW data tape for 802 NYSE stocks having continuous return data from the 2nd quarter 1959 to the 3rd quarter of 1968. The appropriate overall quarterly market return indexes, both for 1964-68 and 1968-73, were then constructed following the same general procedures as for the 1952-76 monthly returns described earlier. The grouping procedure used for stocks was the same as that described earlier (see footnote 17), but for bonds an instrumental variable technique was used.[20]

Table III presents both for individual assets and groups of assets risk-return relationships separately for stocks, for bonds, and for stocks and bonds combined, assuming stationarity of the regression coefficients, for one of these two five year periods, 1968-73.[21] The results, especially those for groups of assets, are again

[19] The common stocks were required to have 19 quarters of return data in each period from the 1st quarter 1964 to the 2nd quarter 1968 and from the 3rd quarter of 1968 to the 2nd quarter of 1973. Again this requirement can introduce some *ex post* selection bias which is not likely to affect the qualitative results.

[20] See Friend, Westerfield and Granito [4], p. 912.

[21] $\overline{R_m - R_f} < 0$ in 1968-73 and Skew$_m$, the skewness of the return distribution of the market portfolio, is also less than zero in that period. $\overline{R_m - R_f} > 0$ and Skew$_m > 0$ in 1964-68. Skew$_m = \frac{1}{T} \sum_t (R_{mt} - \bar{R}_m)^3$, where $T$ is the number of quarters. Similar results are obtained for the earlier period; see Friend, Westerfield and Granito [4].

**Table III**

Stationary Return-Risk Regressions for Individual and Groups of Assets[b] 4th Quarter 1968–3rd Quarter 1973 Using Value Weighted Common Stock and Bond Index

| Individual Assets[b] | Estimates of Regression Coefficients $\gamma_0$ | $\gamma_1$ | $\gamma_2$ | $\bar{R}^2$ | $\overline{R_m - R_f}$ | $\text{Skew}_m(.001)$ |
|---|---|---|---|---|---|---|
| Stocks | .033 | −1.119 | .143 | .09 | −.0038 | −.042 |
| | (0.2) | (−8.3) | (3.9) | | | |
| Corp. Bonds | .089 | .601 | −.042 | .02 | −.0038 | −.042 |
| | (1.6) | (4.8) | (−1.5) | | | |
| Stocks and Corp. Bonds | .596 | −1.327 | .101 | .20 | .0038 | −.042 |
| | (7.3) | (−20.7) | (4.1) | | | |
| Groups of Assets[c] | | | | | | |
| Stocks | .976 | −1.770 | .232 | .35 | −.0038 | −.042 |
| | (1.8) | (−5.3) | (1.4) | | | |
| Corp. Bonds | .478 | −.683 | .373 | .02 | −.0038 | −.042 |
| | (2.3) | (−1.2) | (2.6) | | | |
| Stocks and Corp. Bonds | .903 | −1.689 | .174 | .73 | −.0038 | −.042 |
| | (8.1) | (−14.2) | (1.9) | | | |

[a] The regression forms are identical with Table I.

[b] The number of observations is 867 for stocks, 891 for bonds, and 1758 for the combined regression.

[c] The number of groups is 50 for each of stocks and bonds and 100 for stocks and bonds combined.

generally inconsistent with the modified form of the Sharpe-Lintner CAPM proposed by Kraus and Litzenberger. However, there is evidence in these stationary regressions that investors may pay a premium for positive skewness in their portfolios as predicted by the Kraus-Litzenberger theory. Similar results were obtained for the 1964-68 and 1968-73 periods when different corporate bond indexes were used (such as Moody's Composite bond index or Salomon Bros. Total Performance Bond Index), when bonds were reduced in weight to 20% (vs. 80% for equities) or increased to 50% (vs. 50% for equities), or when the logarithms of return relatives were substituted for the returns themselves.

## IV. Predictive Measures of Risk

So far in our analysis, we have followed customary practice and used contemporaneous rather than predictive values of betas and co-skewness in testing CAPM theory; i.e., we have used current rather than prior period risk measures to explain current period returns. While not altogether clear, this seems also to have been the procedure followed by Kraus and Litzenberger. However, it is possible on several grounds to justify the use of predictive values, and at least two previous studies have done so.[22] The information actually available to investors is probably intermediate between that generally implicitly assumed in the use of contemporaneous and predictive values. Thus, we have introduced predictive values of $\beta_i$ and $\delta_i$ in the return-risk relationships presented in Table IV and the following tables,[23] though the significance of the regression coefficients as a test of positive CAPM theory probably tends to be understated in these analyses.[24]

Tables IV and V present, for individual common stocks and groups of stocks respectively, return-risk relationships which both allow for nonstationarities in the regression coefficients and incorporate predictive rather than contemporaneous risk measures. With these changes in the analysis, which may understate the significance of the relevant regression coefficients, the intercepts in both the individual stock and group regressions remain significantly different from zero for the 1952-76 period as a whole and for the first two sub-periods. On the other hand, co-skewness does seem to affect returns significantly in the expected direction over the period as a whole for individual common stocks and for groups of stocks. For the subperiods, however, the co-skewness coefficients have the wrong sign at least as often as they have the right sign.

In view of the fact that virtually all tests of the CAPM as well as those in this paper can be considered joint tests of the CAPM theory (using an empirical surrogate for the market portfolio) and of a return generating process, we have in Table 6 divided the predictive risk results for 1952-76 as a whole both for individual stocks and groups of stocks into two periods—one including all months in which the market rate of return was higher than the risk-free ($R_m > R_f$) and the other including all months in which the reverse was true ($R_m < R_f$).[25]

[22] See Fama and MacBeth [3] and Pettit and Westerfield [8].

[23] The procedures followed for estimating the predictive values of the risk measures are similar to those described in Fama and MacBeth. *Groups*: (a) Portfolios are formed on the basis of estimates of betas and co-skewness for individual securities in an initial five year period, and then estimates of portfolio beta and co-skewness are computed by reestimating beta and co-skewness for individual securities in a subsequent five year period. The portfolio betas and co-skewness values are equal-weighted averages taken from the individual securities and are formed by ranking securities on estimated beta (from high to low) and dividing them into five groups and within each group ranking the securities on estimated co-skewness (from high to low) and dividing the group into five subgroups. Thus 25 groups of securities are obtained. (b) Monthly returns are calculated for these groups for the twelve months subsequent to those used in estimating the betas and co-skewness value. (c) In each month these returns are regressed on the group betas and co-skewness. The process is repeated in each year. The average of the monthly OLS estimates $\gamma_1$ ($\beta_i$) and $\gamma_2$ ($\delta_i$) from the cross-sectional regression are the estimates of the risk-return trade-off. Note 132 months of return data are required for a set of regression estimations. *Individual securities*: The procedures for the cross-sectional estimates involving predictive measures of risk for individual securities closely resemble those just described, except (a) Estimates of beta and co-skewness are computed by OLS for all (NYSE) securities with continuous data in an initial five year period, (b) A 'Vasicek' weighting procedure is applied to each estimated beta, (c) Monthly returns are calculated for the next twelve month period for each security and the cross-sectional regression model is estimated. The process is repeated in each year. Now 72 months of return data are required for a set of regression estimates.

[24] This is true if the measurement error in $\hat{\beta}_{i\,(t-1)}$ contains random estimation error and "true" nonstationarity error. See Blume [2].

[25] The standard errors are computed from the resulting time series of coefficients.

**Table IV**

Non-Stationary Return-Risk Regressions for Individual Common Stocks[a] Adjusted for Order Bias and Incorporating Predictive Risk Measures January 1952–December 1976 Using Value Weighted Common Stock and Bond Index

| Period | Estimates of Regression Coefficients | | | $\bar{R}^2$ | $\overline{R_m - R_f}$ | Skew$_m$(.0001) |
|---|---|---|---|---|---|---|
| | $\gamma_0$ | $\gamma_1$ | $\gamma_2$ | | | |
| 1952–56 | 0.573 | 0.332 | 0.046 | .03 | .0085 | .005 |
| | (3.0) | (1.5) | (3.1) | | | |
| 1957–61 | 1.759 | −0.543 | −0.057 | .03 | .0056 | −.049 |
| | (−1.8) | (−1.8) | (−1.6) | | | |
| 1962–66 | −0.123 | 0.615 | −0.238 | .03 | −.0021 | −.153 |
| | (−0.4) | (1.6) | (−1.8) | | | |
| 1967–71 | 0.446 | 0.062 | 0.071 | .03 | .0021 | −.005 |
| | (1.3) | (0.1) | (1.7) | | | |
| 1972–76 | 0.462 | 0.066 | −0.186 | .03 | .0005 | .377 |
| | (0.9) | (0.1) | (−1.9) | | | |
| 1952–76 | 0.624 | 0.107 | −0.073 | .03 | .0029 | .026 |
| | (3.9) | (0.6) | (−2.0) | | | |

[a] The regression forms are identical with Table I. The number of observations averages 900 for 1952–56, 916 for 1957–61, 912 for 1962–66, 890 for 1967–71, 1037 for 1972–76, and 931 for 1952–76.

**Table V**

Non-Stationary Return-Risk Regressions for Individual Common Stocks Grouped to Minimize Measurement Errors and Incorporating Predictive Risk Measures January 1952–December 1976 Using Value Weighted Common Stock and Bond Index

| Period | Estimates of Regression Coefficients | | | $\bar{R}^2$ | $\overline{R_m - R_f}$ | Skew$_m$(.0001) |
|---|---|---|---|---|---|---|
| | $\gamma_0$ | $\gamma_1$ | $\gamma_2$ | | | |
| 1952–56 | 0.861 | 0.153 | 0.059 | .21 | .0085 | .005 |
| | (3.6) | (0.8) | (1.0) | | | |
| 1957–61 | 2.007 | −0.718 | −0.139 | .31 | .0056 | −.049 |
| | (6.7) | (−1.8) | (−0.9) | | | |
| 1962–66 | −0.585 | 1.450 | −0.928 | .34 | −.0021 | −.153 |
| | (−1.2) | (2.7) | (−2.3) | | | |
| 1967–71 | 0.402 | 0.242 | −0.138 | .25 | .0021 | −.005 |
| | (0.8) | (0.5) | (−0.8) | | | |
| 1972–76 | −0.173 | 0.363 | −0.091 | .26 | .0005 | .377 |
| | (−0.3) | (0.7) | (−0.5) | | | |
| 1952–76 | 0.502 | 0.298 | −0.248 | .27 | .0029 | .026 |
| | (2.5) | (1.5) | (−2.4) | | | |

[a] The regression forms are identical with Table I but are now estimated from 25 grouped observations.

Presumably in the first of these periods (all months where $R_m > R_f$) there would be a closer coincidence between *ex ante* and *ex post* returns.[26]

The results of this breakdown of the entire period 1952-76 into two sets of months, depending on the relationship between $R_m$ and $R_f$, are quite striking. Unlike the insignificant or marginally significant results for the period as a whole, in the months when $R_m > R_f$, $\beta_i$ makes a highly significant contribution to the explanation of returns both in the individual and group regressions, and the signs of its regression coefficients are consistent with CAPM theory. In the months when $R_m < R_f$, and it is necessary to incorporate some plausible return generating function to make the transition from an *ex ante* or expected relationship between returns and risk to an *ex post* or observed relationship, $\beta_i$ is significantly and—as would be expected from the market model customarily used—negatively related to returns in both sets of regressions. It is not clear what return generating function would best reflect the incorporation of co-skewness in the explanation of the difference between observed and anticipated returns, but there is no obvious reason to expect the sign of the co-skewness coefficient to depend on the relationship between $R_m$ and $R_f$. However, over the months when $R_m > R_f$ in the 1952-76 period the market skewness is positive and when $R_m < R_f$ market skewness is negative, and it is only in the former months that co-skewness has a significant effect. This effect is in the direction predicted by Kraus and Litzenberger. On the other hand, the estimated intercepts are significantly different from zero and strongly inconsistent with their thesis. It should be noted that when contemporaneous risk measures are substituted for predictive risk in Table VI, even co-skewness no longer has the predicted effect. Thus, the significance of co-skewness is quite sensitive to the particular statistical methodology used.

## V. Tests of the CAPM Based on Indexes of Stock Returns

Obviously, the remaining question to be answered in explaining this difference in findings between the Kraus-Litzenberger and our studies is whether it would disappear if the market proxy in our analysis is confined to stocks alone rather than to the more theoretically satisfactory stocks and bonds. Tables VII and VIII, therefore, duplicate the return-risk estimations using predictive measures of beta and co-skewness for individual and groups of common stocks corresponding to those presented in the preceding analysis and Tables IV, V and VI, except that now the market proxies are a value-weighted stock index (the Standard and Poor's Composite Index) and an equal-weighted stock index (the Fisher Arithmetic Index) rather than the value-weighted index of stocks and bonds combined described earlier.

The use of a value-weighted stock index instead of the value-weighted stock and bond index does not seem to change significantly the results of the analysis.

[26] While it is obvious that *ex post* returns are below *ex ante* returns when $R_m < R_f$, the relationship between the two rate of returns is not so clear when $R_m > R_f$. However, over very long periods omitting the sub-periods with low market returns might be expected to yield *ex post* returns in excess of *ex ante* returns. This procedure truncates the sample and has the risk of introducing sample bias into the estimates.

**Table VI**

Non-Stationary Return-Risk Regressions for Individual and Groups of Common Stocks[a] Incorporating Predictive Risk Measures and Segregating Periods with Positive and Negative Risk Differentials Using Value Weighted Common Stock and Bond Index January 1952–December 1976

| Period | Type of Asset | Estimates of Regression Coefficients $\gamma_0$ | $\gamma_1$ | $\gamma_2$ | $\bar{R}^2$ | $Skew_m(.0001)$ |
|---|---|---|---|---|---|---|
| | Individual Stocks[b] | | | | | |
| $R_m > R_f$[d] | | 1.326 | 1.554 | −0.103 | .02 | .115 |
| | | (6.6) | (7.5) | (−1.9) | | |
| $R_m < R_f$[e] | | −0.259 | −1.711 | −0.035 | .04 | −.061 |
| | | (−1.1) | (−7.3) | (−0.9) | | |
| Period as whole | | 0.624 | 0.107 | −0.073 | .03 | .026 |
| | | (3.9) | (0.6) | (−2.0) | | |
| | Groups of Stocks[c] | | | | | |
| $R_m > R_f$ | | 1.320 | 1.485 | −0.514 | .27 | .115 |
| | | (5.2) | (6.0) | (−3.4) | | |
| $R_m < R_f$ | | −0.548 | −1.193 | 0.091 | .28 | −.061 |
| | | (−1.8) | (−4.1) | (0.8) | | |
| Period as whole | | 0.502 | 0.298 | −.248 | .27 | .026 |
| | | (2.5) | (1.5) | (−2.4) | | |

[a] The regression forms are identical with Table I. The number of observations corresponds to those reported in Table IV in the individual stock regressions and is 25 in the grouped regressions.

[b] Adjusted for order bias.

[c] Grouped to minimize measurement errors.

[d] Regression computed for months in which rate of return on market is higher than risk-free rate.

[e] Regression computed for months in which rate of return on market is less than risk-free rate.

The intercepts are still inconsistent with the Kraus-Litzenberger modification of Sharpe-Lintner theory both in the grouped but especially in the individual stock regressions. Co-skewness has the predicted effect at the .95 level of statistical significance in the individual stock regressions for the period as a whole, but only in one of the five 5 year sub-periods (1972-76), and not in any of the grouped regressions.

Another difference between the Kraus-Litzenberger and our procedures even where both are based on a market index of stock return, is our use of a value-weighted index while they used an equal-weighted stock index constructed from the sample of common stocks included in their tests. To further examine the sensitivity of the regression estimates and standard errors to alternative specifications of the market portfolio, we have computed the non-stationary regression estimates for individual and groups of stocks on predictive values of $\beta_i$ and $\delta_i$ using an equally weighted common stock index (i.e. The Fisher's Arithmetic

**Table VII**

Non-Stationary Return-Risk Regressions for Individual Common Stocks[a] Adjusted for Order Bias and Incorporating Predictive Risk Measures With Risk Measures Based on Value-Weighted Stock Index[b]

January 1952–December 1976

| Period | Estimates of Regression Coefficients $\gamma_0$ | $\gamma_1$ | $\gamma_2$ | $\bar{R}^2$ | $\overline{R_m - R_f}$ | $\text{Skew}_m(.0001)$ |
|---|---|---|---|---|---|---|
| 1952–56 | 0.738 | 0.330 | 0.130 | .02 | .0145 | .02 |
| | (3.6) | (1.1) | (4.0) | | | |
| 1957–61 | 1.596 | −0.677 | −0.085 | .03 | .0083 | −.22 |
| | (5.3) | (−1.8) | (−1.2) | | | |
| 1962–66 | 0.087 | 0.508 | −0.221 | .03 | .0024 | −.16 |
| | (0.3) | (1.4) | (−2.2) | | | |
| 1967–71 | 0.584 | 0.193 | 0.013 | .04 | .0035 | −.17 |
| | (1.6) | (0.0) | (0.3) | | | |
| 1972–76 | 0.539 | −0.023 | −0.121 | .03 | −.0001 | .70 |
| | (1.0) | (−0.0) | (−1.7) | | | |
| 1952–76 | 0.709 | 0.032 | −0.057 | .03 | .0057 | .00 |
| | (4.4) | (0.2) | (−1.8) | | | |
| Segregating Periods with Positive and Negative Risk Differentials | | | | | | |
| $R_f > R_m$ | 1.998 | 1.468 | −0.039 | .02 | .0324 | .28 |
| | (10.5) | (7.1) | (−0.9) | | | |
| $R_f < R_m$ | −0.910 | −1.772 | −0.079 | .04 | −.0316 | −.18 |
| | (−4.5) | (−7.4) | (−1.9) | | | |

[a] The regression forms are identical with Table I. The number of observations is 900 for 1952–56, 916 for 1957–61, 9.2 for 1962–66, 890 for 1967–71, 1087 for 1972–76, and 931 for 1952–76. The sampling procedures are the same used in preparing Table IV.

[b] The Standard and Poor's 500 Composite Index from 1/1947 to 12/1963 and the New York Stock Exchange Composite from 1/1964 to 12/1976.

Index) for the months of the period January 1952 through December 1974, the months for which the Fisher Index was readily available. The results are presented in Tables IX and X. Again the estimated intercepts are significantly different from zero the value predicted by Kraus-Litzenberger, both for the period as a whole and for two of the four sub-periods, but now co-skewness is not significant for the period as a whole in either the individual or group regressions. For the non-stationary group regression, which is closer conceptually to the statistical relationship estimated by Kraus-Litzenberger than is the individual regression, the new equation for the 1952-74 period is

$$\begin{array}{llll} \bar{r}_i = 0.404 + 0.135\beta_i - 0.035\delta_i & \overline{R}^2 = .30 & \overline{(R_m - R_f)} = & .0033 \\ \quad\ (2.1) \qquad (0.6) \qquad (-0.6) & & \text{Skew}_m(.0001) = & -.21 \end{array} \tag{1}$$

where the variables are as defined earlier and parentheses indicate $t$-values. For the individual stock regression over the same period, the intercept has a $t$-value of 3.65 and the other regression coefficients are again insignificant.

**Table VIII**

Non-Stationary Return-Risk Regressions for Common Stocks[a] Grouped to Minimize Measurement Errors and Incorporating Predictive Risk Measures With Risk Measures Based on Value-Weighted Stock Index[b]
January 1952–December 1976

| | Estimates of Regression Coefficients | | | | | |
|---|---|---|---|---|---|---|
| Period | $\gamma_0$ | $\gamma_1$ | $\gamma_2$ | $\bar{R}^2$ | $\overline{R_m - R_f}$ | $Skew_m(.0001)$ |
| 1952–56 | 0.751 | 0.369 | 0.186 | 0.22 | .0145 | .02 |
| | (3.2) | (0.9) | (1.4) | | | |
| 1957–61 | 2.001 | −1.232 | −0.199 | 0.33 | .0083 | −.22 |
| | (7.2) | (−1.9) | (−1.1) | | | |
| 1962–66 | −0.685 | 1.374 | −0.399 | 0.35 | .0024 | −.16 |
| | (−1.3) | (2.2) | (−1.3) | | | |
| 1967–71 | 0.163 | 0.191 | 0.187 | 0.22 | .0035 | −.17 |
| | (0.3) | (0.4) | (1.2) | | | |
| 1972–76 | −0.199 | 0.311 | 0.202 | 0.22 | −.0001 | .70 |
| | (−0.31) | (0.5) | (1.0) | | | |
| 1952–76 | 0.406 | 0.202 | −0.005 | 0.27 | .0057 | .00 |
| | (2.0) | (0.8) | (−0.0) | | | |
| | Segregating Periods with Positive and Negative Risk Differentials | | | | | |
| $R_m > R_f$ | 1.551 | 1.959 | −0.184 | 0.25 | .0324 | .28 |
| | (6.0) | (6.2) | (−1.5) | | | |
| $R_m < R_f$ | −1.032 | −2.003 | 0.219 | 0.28 | −.0316 | −.18 |
| | (−3.6) | (−6.0) | (1.6) | | | |

[a] The regression forms are identical with Table I but are now estimated from 25 grouped observations using sampling procedures embodied in the tabulations of Table V.
See Table VII.

Turning finally to the period 1952-1970 (June) and using contemporaneous rather than predictive estimates of beta and co-skewness (see footnote 13) to make our sample period and statistical procedures as close as possible to those used by Kraus-Litzenberger, the results are

$$\bar{r}_i = .400 + .214\beta_i + .105\delta_i \quad \bar{R}^2 = .39 \quad \overline{(R_m - R_f)} = .0059 \qquad (2)$$
$$(2.1) \qquad (.6) \qquad (1.2) \qquad \text{Skew}_m(.0001) = -.19$$

The estimated regression for individual stocks over this period is virtually identical with the group regression. The most important reason for the difference between Equation 2 above and the corresponding Kraus-Litzenberger result appears to be the difference in the sample period covered. It should be noted in comparing Equations (1) and (2) that a small difference in the period covered and the use of contemporaneous estimates of beta and co-skewness result in a fairly substantial shift in the co-skewness coefficient. Thus, the difference between their findings and ours taken in their entirety does not appear to depend

**Table IX**

Non-Stationary Return-Risk Regressions on Equal-Weighted Stock Index[a] for Individual Common Stocks Adjusted for Order Bias and Incorporating Predictive Risk

| Period | Estimates of Regression Coefficients | | | | | |
|---|---|---|---|---|---|---|
| | $\gamma_0$ | $\gamma_1$ | $\gamma_2$ | $\bar{R}^2$ | $\overline{R_m - R_f}$ | Skew$_m$(.0001) |
| 1952–56 | 0.771 | 0.267 | 0.129 | 0.02 | 0.0111 | .08 |
| | (3.65) | (0.87) | (2.72) | | | |
| 1957–61 | 1.571 | −0.581 | 0.004 | 0.04 | 0.0091 | .11 |
| | (6.09) | (−1.71) | (0.11) | | | |
| 1962–66 | −0.026 | 0.507 | −0.023 | 0.04 | 0.0031 | −.25 |
| | (−0.08) | (1.21) | (−0.57) | | | |
| 1967–71 | 0.612 | −0.016 | −0.000 | 0.04 | 0.0045 | −.16 |
| | (1.69) | (−0.03) | (−0.00) | | | |
| 1952–74 | 0.592 | −0.123 | 0.023 | 0.04 | 0.0033 | −.21 |
| | (3.65) | (−0.60) | (1.29) | | | |
| $R_m > R_f$ | 1.657 | 1.804 | 0.035 | 0.03 | 0.0343 | .29 |
| | (8.68) | (8.13) | (1.51) | | | |
| $R_m < R_f$ | −0.834 | −2.703 | 0.007 | 0.05 | −0.0382 | −.40 |
| | (−3.78) | (−12.66) | (0.26) | | | |

[a] The stock market index used is the Fisher Arithmetic Index. The regression forms and number of observations are identical with Table VII.

**Table X**

Non-Stationary Return-Risk Regressions on Equal-Weighted Stock Index[a] for Grouped Common Stocks[a] Incorporating Predictive Risk Measures
January 1952–December 1976

| Period | Estimates of Regression Coefficients | | | | | |
|---|---|---|---|---|---|---|
| | $\gamma_0$ | $\gamma_1$ | $\gamma_2$ | $\bar{R}^2$ | $\overline{R_m - R_f}$ | Skew$_m$(.0001) |
| 1952–56 | 0.928 | 0.595 | −0.330 | .20 | 0.0111 | .08 |
| | (3.75) | (1.64) | (−1.99) | | | |
| 1957–61 | 1.909 | −0.947 | 0.047 | .34 | 0.0091 | −.11 |
| | (6.49) | (−1.79) | (0.43) | | | |
| 1962–66 | −0.592 | 1.001 | 0.039 | .40 | 0.0031 | −.25 |
| | (−1.24) | (1.66) | (0.45) | | | |
| 1967–71 | 0.088 | 0.367 | 0.099 | .28 | 0.0045 | −.16 |
| | (0.20) | (0.63) | (0.98) | | | |
| 1952–74 | 0.404 | 0.135 | −0.035 | .30 | 0.0033 | −.21 |
| | (2.07) | (0.55) | (−0.63) | | | |
| $R_m > R_f$ | 1.460 | 2.090 | −0.073 | .28 | 0.0343 | .29 |
| | (6.21) | (7.22) | (−0.91) | | | |
| $R_m < R_f$ | −1.012 | −2.484 | 0.016 | .32 | −0.0382 | −.40 |
| | (−3.59) | (−8.83) | (0.23) | | | |

[a] The regression forms and number of observations are identical with Table VIII. The stock market index used is the Fisher Arithmetic Index.

importantly on the particular index used, and their results seem to apply only to the specific period and statistical procedures which they used.

## VI. Conclusion

In an earlier paper Kraus and Litzenberger presented a three parameter capital asset pricing model which extended the Sharpe-Lintner capital asset model to incorporate the effects of skewness on asset prices. They concluded that (1) co-skewness in addition to co-variance is required to explain the returns on individual risky assets and that (2) the implied riskless market rate of return is not significantly different from the actual risk-free rate of return. Our analysis provides some but not conclusive evidence in support of the first of these propositions, suggesting that investors may be willing to pay a premium for positive skewness in their portfolios. However, our tests provide no support for the second of their conclusions, so that we conclude that the Kraus-Litzenberger attempt to develop and substantiate a modified form of the Sharpe-Lintner CAPM is not successful.

Although our tests are not conclusive, we do investigate the sensitivity of our statistical inferences to alternative specifications of the market portfolio, and to other aspects of test construction. We find that the inference that co-skewness in addition to co-variance is required to explain individual asset prices is significantly affected by the different market indexes used and other testing and estimation procedures followed. In contrast, our empirical evidence uniformly suggests that the estimated risk-free rate of return is significantly higher than the actual risk-free rate of return. The latter finding is insensitive to the range of market indexes used in this study and all other aspects of our test construction. Thus, we conclude that the contrary finding of Kraus and Litzenberger is most likely a reflection of the particular time period and estimation procedures they used.

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