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"Stochastic Demand, Output and the Cost of Capital: A Clarification"

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MICHAEL LONG AND GEORGE RACETTE in an important article in this journal [1] were among the first to relate a firm's cost of capital to its production and output decisions using the CAPM of Sharpe, Lintner and Mossin. In particular they arrived at two important conclusions; first that the cost of capital was 'u' shaped in output and second that it was not necessarily minimized at that output level that maximized firm value. In their words "Though the cost of capital will be minimized where AC = MC, it will not in general be optimal to seek to minimize ρ alone. The optimality conditions for the firm are given by equation 9 (our 3) and there is no economic reason why AC will necessarily equal MC at this output." These conclusions have passed without criticism in the literature, possibly because of their 'reasonableness and familiarity'. However, this paper will show that their results come from a mis-specification in their model set up and a misunderstanding of their model's relevance.

Using the notation contained in Long and Racette's paper the cost of capital is,

$$\rho = \frac{iE(\pi) + \gamma\,\mathrm{COV}(\tilde{R}_M, \tilde{P}Q)}{E(\pi) - \gamma\,\mathrm{COV}(\tilde{R}_M, \tilde{P}Q)} \tag{1}$$

This equation is obtained by setting the valuation form of the CAPM equal to the definitional form $V = \dfrac{E(\pi)}{1+\rho}$ and solving for ρ, the cost of capital. In this model, only the product price P is stochastic. Long and Racette looked at short run changes in the cost of capital. Hence, differentiating (1) with respect to the control variable Q, output, we have,

$$\frac{d\rho}{dQ} = \frac{(1+i)\gamma\,\mathrm{COV}(\tilde{R}_M, \tilde{P}Q)}{(E(\pi) - \gamma\,\mathrm{COV}(\tilde{R}_M, \tilde{P}Q))^2}\,(\text{MC-AC}) \tag{2}$$

From (2), Long and Racette concluded that,

$$\frac{d\rho}{dQ} \gtreqless 0 \quad \text{as} \quad \text{MC} \gtreqless \text{AC}.$$

These results can be graphed in figure 1 along with the firm's value maximi-

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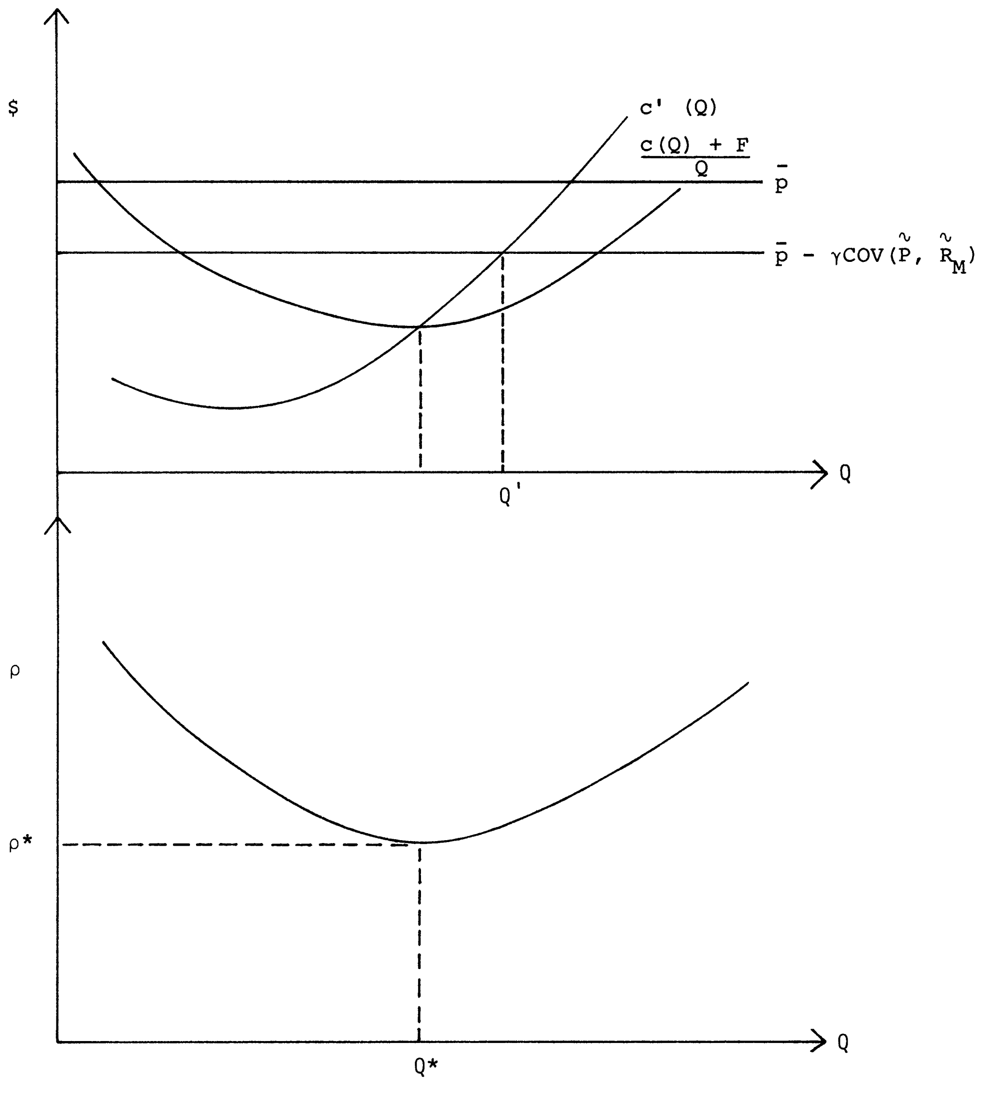

Figure 1.

zation decision. In this short run model the firm value maximizes when (L & R's 7′),

$$E(P) - \gamma \frac{d\,\mathrm{COV}(\tilde{R}_M, \tilde{P}Q)}{dQ} = c'(Q) \tag{3}$$

i.e. when the certainty equivalent product price is set equal to the marginal cost, this occurs in figure 1 at Q'. However, unless the firm is in long run equilibrium (which with only price uncertainty is when MC = AC), this output will not be at the output that minimizes the cost of capital. Hence, 'logically' at $Q \neq Q^*$, $\rho > \rho^*$ and there are competitive pressures towards equilibrium.

We disagree with the above results. If we substitute the cost of capital minimizing conditions into the cost of capital by substituting for $E(P)$, we get the minimum cost of capital as,

$$\rho = \frac{iE(\pi) + \gamma \operatorname{COV}(\tilde{R}_M, \tilde{P}Q)}{(\text{MC-AC})Q} \tag{4}$$

i.e. the cost of capital is minimized at some undefined value.

The resolution of this "paradox" is simple, since Long and Racette's problem was in not recognizing that the CAPM is a one period model. At the end of the second period, the firm's return is the uncertain profit and return of initial capital. Subtracting k from the valuation equation we find

$$NPV = \frac{E(\pi) - ik - \gamma \operatorname{COV}(\tilde{R}_M, \tilde{P}Q)}{1 + i} \tag{5}$$

Equating this to the NPV form of the definitional equation we get

$$\rho = \frac{i(E(\pi) + k) + \gamma \operatorname{COV}(\tilde{R}_M, \tilde{P}Q)}{(E(\pi) + k - \gamma \operatorname{COV}(\tilde{R}_M, \tilde{P}Q))} \tag{6}$$

and as output is varied we find

$$\frac{d\rho}{dQ} = \frac{(1 + i)\gamma \operatorname{COV}(\tilde{R}_M, \tilde{P}Q)}{(E(\pi) + k - \gamma \operatorname{COV}(\tilde{R}_M, \tilde{P}Q))^2} \left[k/Q + c'(Q) - \frac{c(Q) + f}{Q} \right] \tag{7}$$

This formulation gives very different conclusions from Long and Racette's. First even when the firm sets AC = MC, $d\rho/dQ \neq 0$ since $k/Q > 0$. Second at the value maximizing output level we find,

$$\frac{d\rho}{dQ} = \frac{(1 + i)\, dE(\pi)/dQ}{(E(\pi) + k - \gamma \operatorname{COV}(\tilde{R}_M, \tilde{P}Q))} \tag{8}$$

which is positive if the marginal risk premium on output is positive. Finally, in long run equilibrium under uncertainty a firm's $NPV = 0$, just as under certainty its profit equals zero, using this condition we find,

$$\frac{d\rho}{dQ} = \frac{dE(\pi)/dQ}{k} \tag{9}$$

which again can be positive or negative. Hence, the 'conflict' between value maximization and cost of capital minimization hinted at in figure 1 is evident, since the cost of capital always increases with output.

This situation can be explained by differentiating the definitional form of the cost of capital with respect to output and using the equilibrium condition $V = k$. Equation (9) results again. The result is in fact entirely caused by Long and Racette's assumption of only product price uncertainty. Since changing output changes uncertain revenue, uncertainty is linear in output. Thus, we would expect the cost of capital to be a positive function of output. If, we consider the case of uncertain factor prices [2] then uncertainty is no longer linear in output. In this case the cost of capital will depend on the interrelationships between the uncertain product price and the uncertain factor costs. However, there is still absolutely no reason for the cost of capital to be minimized at the value maximizing output. There is in fact no specific relationship between the two in the general case.

A final comment is in order. Long and Racette's model is short run, since capital is fixed and output is varied. Fama and Laffer [3] have concluded that a corollary of the perfectly competitive firm being a price taker is that it exhibit constant returns to scale. In the long run, (the usual scenario for discussing the cost of capital) if capital and output are varied, it can be easily shown that the firm's cost of capital is constant under constant returns to scale. Hence, the importance of the link between output and the cost of capital is obscure.

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