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A Theory of Common Stock Returns Over Trading and Non-Trading Periods

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I. Introduction

CONVENTIONAL THEORY ASSUMES THAT the same return process operates over all trading and non-trading periods. There are reasons to assume that the return sequence when an organized market is formally open may differ from the return sequence during closed periods. For example, during a trading day, stock prices fluctuate as orders are executed. During nights, weekends, holidays, and holiday-weekends there are no transactions, but a share's value from close to open on the next trading day may still change to reflect revised expectations about a firm's productivity. In fact, capital changes and important news items are usually announced after the stock exchange closes.[1]

The purpose of this paper is to develop and test a general stochastic process for common stock returns that is applicable over any arbitrary time interval for which the return is computed. During a trading day we propose that the stochastic process of stock transaction returns is an autoregressive jump process. This process consists of a calendar time diffusion process and a jump process in which the magnitudes of the jumps may be autocorrelated. Over non-trading periods, it is assumed that the same diffusion process operates, but a separate jump process is specified for overnights, holidays, weekends, and holiday-weekends. The resultant multiple component jump process is used to investigate returns over days, weeks, and months. Several statistical analyses using actual transaction data over different time intervals lend support to the proposed theory.

The following analysis is divided into four parts. Section II specifies the general model for common stock returns over any arbitrary time interval. Section III describes the data, defines the return computations, and provides summary statistics for all returns. In Section IV, hypotheses consistent with the theory are set forth and empirical tests are presented. Section V contains a summary and concluding remarks.

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[1] An earlier empirical analysis of open and closed trading period returns by Granger and Morgenstern [4] suggests that an important and distinct aspect of a stock's return generating process through time goes on when the market is closed.

II. THE GENERALIZED RETURN PROCESS FOR ARBITRARY TIME INTERVALS

Suppose that transaction returns over trading and non-trading periods are generated according to a combination of a diffusion process and multiple jump processes that can be represented by the stochastic equation,

$$dP/P = \alpha dt + \beta dt\psi + z_d d\theta_d + z_o d\theta_o + z_h d\theta_h + z_w d\theta_w + z_{hw} d\theta_{hw} \qquad (1)$$

where P = share price;

α, β^2 = the instantaneous mean and variance return per unit time from the Wiener process $d\psi$;

$d\theta_d$ = a random jump process (when $d\theta_d = 1$, a transaction occurs, when $d\theta_d = 0$, no transaction occurs);

z_d = the random percent change in share price resulting from a jump, $Z_d = (1 + z_d)$ is the jump amplitude;

z_o = the random percent change in share price resulting from an overnight jump, $Z_o = (1 + z_o)$ is the overnight jump amplitude;

z_h = the random percent change in share price resulting from a holiday jump, $Z_h = (1 + z_h)$ is the holiday jump amplitudes;

z_w = the random percent change in share price resulting from a weekend jump, $Z_w = (1 + z_w)$ is the weekend jump amplitude;

z_{hw} = the random percent change in share price resulting from a holiday-weekend jump, $Z_{hw} = (1 + z_{hw})$ is the holiday-weekend jump amplitude;

$d\theta_o$, $d\theta_h$, $d\theta_w$, $d\theta_{hw}$ = non-random time indicators. If $d\theta_o = 1$, an overnight jump occurs; $d\theta_o = 0$ otherwise. When a holiday occurs $d\theta_h = 1$; otherwise $d\theta_h = 0$. $d\theta_w = 1$ over a weekend; $d\theta_w = 0$ otherwise. For holiday-weekends $d\theta_{hw} = 1$; otherwise $d\theta_{hw} = 0$.

Random variables z_d, z_o, z_h, z_w, z_{hw}, $d\theta_d$, and $d\psi$ are mutually independent by assumption. Transaction returns are assumed to be identically distributed and independent among days but they may be correlated within a trading day. Overnight returns are assumed to be independently and identically distributed as are holiday returns, weekend returns, and holiday-weekend returns. Thus the return generating process defined in equation (1) comprises five separate jump parts with each jump component operating at the appropriate time. In addition, a common diffusion component operates during all periods.

The solution for equation (1) for some arbitrary time interval s is

$$P(t + s) = P(t) \prod_{m=1}^{Nd} \prod_{i=1}^{N_m} Z_m(k) \cdot \prod_{j=o}^{hw} \prod_{k=1}^{N_j} Z_j(i)$$

$$\cdot \exp\{(\alpha - \beta^2/2)s + \beta\sqrt{s}\,\psi\}. \qquad (2)$$

Here $Z_m(i)$ denotes the i^{th} transaction increment in share price on the m^{th} trading day during s. $Z_m(0) = 1$ and $Z_m(i) > 0$ for all m. $Z_j(k)$ is the transaction increment in share price during the k^{th} closed period during s with $j = o, h, w, hw$. $Z_j(k) = 1$ if $d\theta_j = 0$. s is the elapsed time between observed prices $P(t + s)$ and $P(t)$. The number of jumps on any trading day m during the interval s is N_m with each N_m a random variable. Return over the interval s is measured by the natural logarithm of the price ratio of end price over start price

$$\ln[P(t+s)/P(t)] = (\alpha - \beta^2/2)s + \beta\sqrt{s}\,\psi + \sum_{m=1}^{Nd} \sum_{i=1}^{N_m} \ln Z_m(i)$$
$$+ \sum_{k=1}^{N_o} \ln Z_o(k) + \sum_{k=1}^{N_h} \ln Z_h(k) + \sum_{k=1}^{N_w} \ln Z_w(k) + \sum_{k=1}^{N_{hw}} \ln Z_{hw}(k). \quad (3)$$

The return equation (3) is written more compactly as

$$\ln[P(t+s)/P(t)] = (\alpha - \beta^2/2)s + \beta\sqrt{s}\,\psi$$
$$+ \sum_{m=1}^{Nd} \sum_{i=1}^{N_m} \ln Z_m(i) + \sum_{j=o}^{hw} \sum_{k=1}^{N_j} \ln Z_j(k). \quad (4)$$

In this condensed form, the return $\ln[P(t+s)/P(t)]$ during s is composed of several parts. The first two result from the continuous diffusion process which may operate during both trading and non-trading periods. The next five terms are explained by the multiple jump processes. The indices N_d, N_o, N_h, N_w, and N_{hw} in equation (4) represent the number of trading days, overnights, holidays, weekends, and holiday-weekends, respectively, during the interval s. For example, if $[t, t+s]$ is a weekly time interval from Friday's closing price $P(t)$ to the next Friday's closing price $P(t+s)$ and the market is open all days during the week, then $N_d = 5$, $N_o = 4$, $N_h = 0$, $N_w = 1$, and $N_{hw} = 0$. All these values are assumed to be known ex ante for any time interval (except for some unexpected exchange closings ignored here).

Assume $\ln Z(i)$, $\ln Z_o(k)$, $\ln Z_h(k)$, $\ln Z_w(k)$, and $\ln Z_{hw}(k)$ are each identically distributed with means μ, μ_{Z_o}, μ_{Z_h}, μ_{Z_w}, $\mu_{Z_{hw}}$ and finite variances σ^2, $\sigma^2_{Z_o}$, $\sigma^2_{Z_h}$, $\sigma^2_{Z_w}$, $\sigma^2_{Z_{hw}}$ respectively. Given independence among trading day processes and among Z_o, Z_h, Z_w, Z_{hw}, and $d\psi$, the unconditional density for returns $p\{\ln[P(t+s)/P(t)]\}$ over any time interval s is

$$p\{\ln[P(t+s)/P(t)]\} = [h_d\{\ln[P(t+v)/P(t)]\}]^{N_d} \cdot \prod_{j=o}^{hw} [h_{Z_j}(\Delta_{Z_j})]^{N_j} \quad (5)$$

where $h_*(\Delta_*) = \int_{-\infty}^{\infty} f_*\{[\ln(\Delta_*)] - (\alpha - \beta^2/2)v_* - \beta\sqrt{v_*}, \psi\}\,d\psi$ is the unconditional density for the log price ratio, denoted Δ_*, with elapsed time between prices of v_* over the $*$th closed trading period. For overnights, $* = Z_o$; for holidays, $* = Z_h$; for weekends, $* = Z_w$; and for holiday-weekends, $* = Z_{hw} \cdot h_d\{\ln[P(t+v)/P(t)]\}$ is the density for the jump component of the open trading period return.

The unconditional mean and variance return over s when s contains N_d closed trading periods, N_o overnights, N_h holidays, N_w weekends, and N_{hw} holiday-weekends are

$$E\{\ln[P(t+s)/P(t)]\} = E\{\ln[P(t+v)/P(t)]\} + \sum_{j=o}^{hw} E\{\ln(\Delta_{Z_j})\}$$
$$= (\alpha - \beta^2/2)s + N_d[\mu E\{N\}] + \sum_{j=o}^{hw} N_j \mu_{Z_j} \quad (6)$$

$$\begin{aligned}\mathrm{Var}\{\ln[P(t+s)/P(t)]\} &= \mathrm{Var}\{\ln[P(t+v)/P(t)]\} \\ &\quad + \sum_{j=o}^{hw} \mathrm{Var}\{\ln(\Delta_{Z_j})\} \\ &= \beta^2 s + N_d[\sigma^2 E\{N\} + \mu^2 \mathrm{Var}\{\mathrm{N}\} \\ &\quad + 2\sigma^2 \sum_{N=2}^{\infty} \sum_{j=1}^{N-1} (N-j)\rho_j q(N)] \\ &\quad + \sum_{j=o}^{hw} N_j \sigma^2_{Z_j}. \end{aligned} \quad (7)$$

Of course, if returns among open and closed trading periods are not mutually independent, then (6) remains unchanged but (7) must be altered to include additional covariance terms.

The density defined in equation (5) incorporates the following unconditional density for transaction returns during a trading day, (Oldfield, Rogalski, and Jarrow [7]):

$$h_d\{\ln[P(t+v)/P(t)]\} = \left[1 - \frac{\gamma(r, \lambda v)}{\Gamma(r)}\right] c\{\ln[P(t+v)/P(t)] \mid N = 0\} + \textstyle\sum_{N=1}^{\infty} q(N) c\{\ln[P(t+v)/P(t)] \mid N\}. \quad (8)$$

The unconditional mean and variance for daily transaction returns are

$$E\{\ln[P(t+v)/P(t)]\} = (\alpha - \beta^2/2)v + \mu E\{N\} \quad (9)$$

$$\mathrm{Var}\{\ln[P(t+v)/P(t)]\} = \beta^2 v + \sigma^2 E\{N\} + \mu^2 \mathrm{Var}\{N\} + 2\sigma^2 \textstyle\sum_{N=2}^{\infty} \sum_{j=1}^{N-1} (N-j)\rho_j q(N) \quad (10)$$

where $q(N) = [\gamma(rN, \lambda v)/\Gamma(rN)] - [\gamma(rN + r, \lambda v)/\Gamma(rN + r)]$ is the density for N transactions in a trading day of length v with mean $E\{N\}$ and variance Var $\{N\}$. $q(N)$ is derived from the assumption that the time intervals between transactions are independent and identically distributed variables having a gamma density function with parameters r and λ.[2] The parameter λ is a scale factor while r indicates the height of the gamma density. The expression $c\{\ln[P(t + v)/P(t)] \mid N\}$ denotes the density of $\ln[P(t + v)/P(t)$ conditional on N. To derive equations (8), (9), and (10), the $\ln Z_m(i)$ are assumed to be identically distributed with mean μ and finite variance σ^2, where ρ_j is the autocorrelation between transactions at lag j. However, it is important to note that the derivations do not depend on a specific form for the distribution of the $\ln Z_m(i)$.

The mean and variance of $p\{\ln[P(t + s)/P(t)]\}$ are derived without specific functions assigned to the densities $h\{\ln[P(t + v)/P(t)]\}$ and $h_*(\Delta_*)$ for $* = Z_o$, Z_h, Z_w, and Z_{hw}. N_m transactions occur during a trading period m with the same $E\{N\}$ and $\mathrm{Var}\{N\}$ for any given trading day. All non-trading period returns are based on a single jump so during closed trading periods $E\{N\} = 1$, $\mathrm{Var}\{N\} = 0$, and no serial correlation is possible. In addition, if no diffusion process operates, equations (6) and (7) reduce to

$$E\{\ln[P(t+s)/P(t)]\} = N_d[\mu E\{N\}] + \textstyle\sum_{j=o}^{hw} N_j \mu_{Z_j} \quad (11)$$

$$\mathrm{Var}\{\ln[P(t+s)/P(t)]\} = N_d[\sigma^2 E\{N\} + \mu^2 \mathrm{Var}\{N\} + 2\sigma^2 \textstyle\sum_{N=2}^{\infty} \sum_{j=1}^{N-1} (N-j)\rho_j q(N)] + \sum_{j=o}^{hw} N_j \sigma_{Z_j}^2. \quad (12)$$

When no jump processes are present, equations (6) and (7) reduce to a simple

[2] For $r = 1$, a gamma density reduces to an exponential distribution. If $r = 1$, then the density $q(N)$ for the number of transactions in a trading day is Poisson. For a Poisson distribution the mean $E\{N\}$ and $\mathrm{Var}\{N\}$ are equal.

diffusion process. All overnight returns (i.e. Monday–Tuesdays, Tuesday–Wednesday, Wednesday–Thursday, Thursday–Friday) are assumed to be drawn from the same distribution. Also, all daily returns (i.e. Monday, Tuesday, Wednesday, Thursday, Friday) are assumed to be drawn from the same distribution. Finally, as the time interval s is made larger, both the unconditional mean and variance increase. However, returns over any trading or non-trading interval are not simply time scaled unless a diffusion process operates alone. For example, weekend returns are not constrained to average 3⅔ times overnight returns.

III. The Data and Return Computations

3.1 Nature of the Data

All data used were obtained from the Francis Emory Fitch Company and consist of individual transaction information for five New York Stock Exchange companies during the 39-month period October 1, 1974 to December 31, 1977. Names of the five companies selected appear in the first column of Table I. October 1, 1974 was chosen as the starting date because on that date the NYSE extended trading by one-half hour from 3:30 to 4:00. Since the return generating process described in the previous section is dependent on the number of transactions per day we sampled a period when the market was open for trading the same number of minutes each day. December 31, 1977 was the latest date available from Fitch when this study began.

3.2 Common Stock Returns

For each stock the price and time of day are recorded for every transaction during the sample period. Several returns are computed from the prices. Continuous compounding is always used in calculating returns.

Transaction returns are computed from the prices as $\ln [P(i, t)/P(i, t-1)]$ where $P(i, t)$ is the price of stock i at transaction t. Returns over a trading day are computed as the natural logarithm of the ratio of the last price of the day to the first price of that day. All closed trading period returns are based on the log of the ratio of the opening price of a trading day plus any dividend relative to the closing price of the last day the market was open. An overnight return refers to the return between adjacent week days such as Tuesday and Wednesday. A holiday return occurs if the market was closed for a holiday during the week. Then the opening price relative to the closing price on the trading day before the holiday is used. Weekend returns are measured using Monday's opening price relative to Friday's closing price. If a holiday occurs on Friday or Monday, we distinguish this longer period as a holiday-weekend return.

Weekly return relatives are based on Friday's closing price plus any dividend relative to the preceding Monday's opening price as well as Friday to Friday closing prices including dividends for weeks when the market was open all five days. Finally, monthly returns are simply the log of the last price each month plus any dividend minus the log of the previous month's last price.

All returns are adjusted for any splits or capital changes. Transaction data also

Table I

Summary Statistics for Returns

Stock (1)	Sample Observations (2)	Mean (3)	Variance (4)	Skewness (4)	Kurtosis (6)	Studentized Range (7)
A. *Transactions (over all trading days)*						
Aetna	49053	.07434	.136319	−.0853*	5.0831*	19.728*
Caterpillar	44525	.04674	.034483	−.1057*	6.3837*	23.474*
Gulf Oil	134149	.09836	.124617	−.0148*	3.1921*	17.434*
Tandy	54138	.09768	.090032	−.1998*	11.0579*	21.942*
Westinghouse	152995	.02925	.298843	.0013	4.1767*	22.175*
Average	86972.0	.06927	.136859	−.0809	6.1227	20.951
B. *Trading Day (close to open)*						
Aetna	819	4.19448	3.708700	.3127*	7.2097*	10.594*
Caterpillar	818	2.43280	1.560561	−.2458*	5.3025*	8.552*
Gulf Oil	819	15.76791	1.427056	.1422	3.7526*	7.539
Tandy	817	6.23803	6.613056	−.2214*	6.6273*	10.877*
Westinghouse	820	5.50234	4.492731	.4086*	6.9488*	9.876*
Average	818.6	6.82711	3.560421	.0793	5.9682	9.488
C. *Weekly (Friday close to Monday open for complete weeks only)*						
Aetna	140	45.81266	22.847832	.6587*	4.3664*	5.819
Caterpillar	139	35.19563	10.837782	.5840*	6.6302*	8.072*
Gulf Oil	140	55.08693	8.684500	.7430*	4.8976*	6.592
Tandy	138	46.35830	34.644264	.1731	4.1504*	6.824*
Westinghouse	140	32.69400	21.117590	.0748	3.2829	5.805
Average	139.4	43.02950	19.626393	.4467	4.6655	6.622
D. *Weekly (Friday close to Friday close for complete weeks only)*						
Aetna	134	63.18886	22.544864	.6654*	4.1229*	6.076
Caterpillar	133	35.90269	11.181637	.5111	6.4092*	8.147*
Gulf Oil	134	30.39381	9.178694	.7938*	5.0060*	6.735*
Tandy	132	74.10603	36.964943	.2220	3.9782	6.606
Westinghouse	134	42.47328	22.890131	.2802	3.5055	6.070
Average	133.4	49.21293	20.552053	.4945	4.6044	6.727
E. *Monthly (close to close)*						
Aetna	38	157.20214	64.641624	−.1417	2.1761	3.939
Caterpillar	38	136.64610	35.051743	.3560	2.7975	4.333
Gulf Oil	38	159.69592	32.382966	1.1378*	4.5435	4.662
Tandy	38	416.58017	223.285170	.9140	4.3821	5.022
Westinghouse	38	224.11419	78.658681	.0412	2.8503	4.782
Average	38.0	218.84770	86.804037	.4615	3.3499	4.548
F. *Overnight (open to previous close)*						
Aetna	638	4.47948	1.231284	.4846*	10.4674*	12.778*
Caterpillar	637	5.24267	.466895	1.1391*	15.0302*	12.812*
Gulf Oil	638	−7.18029	.361372	.3158*	4.2182*	7.327
Tandy	635	10.79601	1.066032	−.3259*	11.5370*	13.622*
Westinghouse	639	.32401	1.050928	.0449	4.1235*	8.099*
Average	637.4	2.73238	.835296	.3317	9.0753	10.928

Means and variances are multiplied by 10^4. Skewness is measured by the third moment from the mean divided by the second moment to the $3/2$ power. Kurtosis is computed by dividing the fourth moment about the mean by the square of the second moment about the mean. The studentized range is the range of observations in the sample divided by the square root of the sample variance. Asterisks represent values which are significant at the 1% level.

Table I—*Continued*

Stock (1)	Sample Observations (2)	Mean (3)	Variance (4)	Skewness (4)	Kurtosis (6)	Studentized Range (7)
G. *Holiday (open to previous close)*						
Aetna	10	3.58727	.746857	−.5721	3.2228	3.888*
Caterpillar	10	17.85859	.685822	−.7007	3.7977	3.962*
Gulf Oil	10	−39.44870	2.247958	−2.2553*	6.8447	3.603
Tandy	11	106.73019	4.174524	1.3558	4.4115	3.946*
Westinghouse	11	23.98959	3.052494	−.7952	3.6029	3.903*
Average	10.4	22.54339	2.181551	−.5935	4.3759	3.860
H. *Weekend (open to previous close)*						
Aetna	152	21.76692	1.488006	.7708*	8.9743*	8.447*
Caterpillar	151	11.23036	.362434	1.8165*	11.2430*	7.541*
Gulf Oil	152	−2.60501	.412291	.3832	3.6087	5.700
Tandy	151	17.13379	2.818093	1.6825*	37.6913*	14.320*
Westinghouse	151	21.50125	1.898612	.6714*	4.7930*	6.448
Average	151.4	13.80546	1.395887	1.0649	13.2621	8.491
I. *Holiday-Weekend (open to previous close)*						
Aetna	18	−13.28424	1.057310	−1.1759	4.7346	4.456
Caterpillar	19	30.50964	.195795	−.2675	1.8761	3.151
Gulf Oil	18	−23.07053	.559897	−2.8710*	11.3826*	4.821*
Tandy	19	7.50979	1.970736	.8352	7.0997*	5.602*
Westinghouse	18	−5.18642	.912929	−1.3369	5.2504	4.493
Average	18.4	−.70435	.939333	−.9632	6.0687	4.505

have one curious anomaly termed a "sold sale." This is a transaction that appears out of sequence on the ticker tape but is picked up and coded as such by Fitch. We discard all sold sale transactions because of the difficulty in trying to determine the proper sequencing of such transactions. They account for less than one-half of one percent of the sampled transactions for any of the five companies.[3]

3.3 Statistical Properties of the Data

Summary statistics for all the described returns are presented in Table I. Column (1) names the stock; the number of sample observations is shown in column (2). Columns (3) to (6) give the mean, variance, skewness, and kurtosis in that order. The last column gives the value of the studentized range, an overall test of the normality of the return distribution.

An interesting aspect of Table I is the general decline in the kurtosis values as the time interval over which the return is computed gets longer. In particular, the weekly and monthly returns are on average less peaked in the central portion of their distributions than either the daily or transaction returns (see Panels A, B, C, D, E). Moreover, the studentized range values suggest that the weekly and monthly return distributions correspond more closely to the overall shape of a normal distribution than the daily or transaction returns. This implies that as the

[3] To be more specific, the number of transactions discarded for Aetna, Caterpillar, Gulf Oil, Tandy, and Westinghouse are 61, 79, 120, 71, and 116, respectively.

interval over which continuously compounded returns are calculated becomes larger, the return distributions seem to converge to normal distributions. This is consistent with Fama's [3] observation (after looking at monthly return distributions for the Dow Jones stocks) that as the summation of continuously compounded daily returns gets longer the observed distribution of returns tends to approach normality.

Returns for each period when the market is closed are given in Panels F to I of Table I. Generally, the summary statistics are quite distinct depending on the non-trading period. For example, weekend returns on average have a mean more than five times as large as the overnight returns with nearly double the variance. (See columns (3) and (4).) The weekend returns are also more skewed and more highly peaked relative to the overnight returns. (See columns (5) and (6).) Note that if returns during non-trading periods strictly follow a simple diffusion process then on average weekend returns should be about 3⅔ times the overnight returns instead of the observed difference of more than five times as large. Similar comparisons that are inconsistent with a diffusion model can be made between overnight and holiday returns, holiday and holiday-weekend returns, etc. This evidence implies that returns during non-trading periods may not be simply time scaled as if a diffusion process were operating.

In Table II, sample autocorrelations for the returns computed for lags one to five periods are presented. A review of the correlations in Panel A of Table II for transaction returns shows that 5 out of 5 at lag one, 3 out of 5 at lags two, three, and five, and 2 out of 5 at lag four are significantly different from zero at the 1% level. These correlations imply that some degree of dependence may exist among transaction returns, especially at lag one where the significant correlations are all negative. Recall that the multiple component jump process developed in Section II directly incorporates this observed autocorrelation for transaction returns. All other returns in Table II exhibit few significant correlations at any lag regardless of the time interval over which the return is computed. This lack of correlation between lagged returns suggests that daily, weekly, monthly, and all non-trading period return sequences are serially independent during the sample period.

When reviewing the estimated autocorrelations in Table II for various return sequences, keep in mind that the different number of observations in the estimates of serial correlation affects the estimate of its variance for testing hypotheses. Using standard statistical tests, one is more likely to conclude there is a difference in estimates where a large number of observations are available relative to where there are fewer, even though estimates may be from the same population. However, the estimated autocorrelations at lag one for transaction returns continue to be significantly different from zero at the 1% level for all five stocks with tests based on estimates of the variance of the lag one correlations for trading day returns which are computed with far fewer observations.

IV. Hypotheses About the Statistical Properties of Returns

Several propositions are developed and examined using the data described in the previous section to establish the accuracy of the multiple component jump process detailed in Section II.

Table II

Estimated Autocorrelation Function for Returns

Stock	Lag 1	2	3	4	5
A. *Transactions (over all trading days)*					
Aetna	−.220937*	−.040314*	−.007255	.003376	.004799
Caterpillar	−.147032*	−.007375	.030856*	.016867*	.018678*
Gulf Oil	−.396823*	−.046512*	−.013083*	−.005949	−.003021
Tandy	−.102649*	−.009817	.035753*	.029278*	.052265*
Westinghouse	−.430658*	−.033744*	−.006248	−.002140	.008129*
Average	−.259620	−.027552	.008005	.008286	.016170
B. *Trading Day (close to open)*					
Aetna	.032101	.006568	−.014918	.043876	.000393
Caterpillar	.105731*	.004644	.086144	−.068025	.114356*
Gulf Oil	.035400	.074743	.067968	−.037642	−.009174
Tandy	.061536	−.042815	−.006083	.018828	.054422
Westinghouse	−.023589	−.082520	−.021030	.026986	.070119
Average	.042236	−.007876	.022416	−.003195	.046023
C. *Weekly (Friday close to Monday open for complete weeks only)*					
Aetna	−.001882	−.002714	−.018376	.082779	.010099
Caterpillar	−.014022	−.138671	.210511	−.059150	−.064577
Gulf Oil	−.104000	.029620	−.149699	−.048715	−.037798
Tandy	.113621	.075115	.115991	−.001086	−.010037
Westinghouse	−.162781	.067500	−.133077	.151097	−.057603
Average	−.033813	.006170	.005070	.024985	−.031983
D. *Weekly (Friday close to Friday close for complete weeks only)*					
Aetna	−.049703	−.064015	.004248	.056045	−.023763
Caterpillar	.042830	−.084786	.179470	−.089912	−.130257
Gulf Oil	−.035446	.138606	−.157256	−.238818*	−.032821
Tandy	.168753	.052058	.164640	.047999	−.050958
Westinghouse	−.130703	.038365	−.072070	.178026	.013828
Average	−.000854	.016046	.023806	−.009332	−.044794
E. *Monthly (close to close)*					
Aetna	−.039623	−.204605	−.272284	.023797	.023195
Caterpillar	.010995	−.227840	.067356	−.069496	.006288
Gulf Oil	−.265164	−.199730	.151192	−.074352	.237847
Tandy	.302589	.007671	−.020278	−.005761	−.200903
Westinghouse	.110191	.056056	.117583	−.330703	−.104414
Average	.023798	−.113690	.008714	−.091303	−.007597
F. *Overnight (open to previous close)*					
Aetna	.044330	.077995	−.008430	.041665	.078125
Caterpillar	−.074076	−.071229	−.133546*	.077519	.126001*
Gulf Oil	.152656*	.078590	.069714	.030674	.113079*
Tandy	.036893	.015276	−.044820	.017159	.057653
Westinghouse	.029826	.010720	.059160	.014908	.068068
Average	.037926	.022270	−.011584	.036385	.088585
G. *Weekend (open to previous close)*					
Aetna	.055744	−.113127	−.004495	.006868	−.043371
Caterpillar	.167532	.198169	.008505	.174883	−.085204
Gulf Oil	.069843	−.057628	.034704	.105452	.090605
Tandy	.045654	.037685	−.029797	.029167	.005784
Westinghouse	.070095	−.008515	−.044355	.058552	.025586
Average	.081774	.011317	−.007088	.074984	−.001320

Values which are significantly different from zero at the 1% level are noted with asterisks.

4.1 Returns during Monday, Tuesday, Wednesday, Thursday, and Friday Trading Periods Are Identically Distributed

Market lore holds that different days of the week have different trading properties. This proposition is defined to locate such differences. A Kruskal-Wallis test is used to determine whether returns each trading day of the week are identically distributed, [8]. This test is based upon a statistic calculated from ranks established by pooling the returns from the five independent random samples taken for each day of the week. It is an all-purpose test that is "distribution-free" concerning the underlying distribution. The null hypothesis is that returns each day of the week are identically distributed.

Both transaction returns and trading day open to close returns (from Panels A and B of Table I) are segmented by day of the week and the Kruskal-Wallis test applied to their segmented rankings. None of the computed chi-square values for this test are significant at the 1% level so the null hypothesis cannot be rejected.[4] This means that the data are consistent with no significant differences in the distribution of returns by day of the week for any of the five stocks.

4.2 Overnight Returns Are Identically Distributed for Each Week Night

Proposition two maintains that overnight returns between Monday–Tuesday, Tuesday–Wednesday, Wednesday–Thursday, Thursday–Friday are drawn from the same population. A Kruskal-Wallis non-parametric test is again used to test the null hypothesis that the returns each night of the week are identically distributed. Small chi-square values imply that the population is the same for each random sample. None of the computed chi-square values are large enough to be significant at the 1% level.[5] Thus, the proposition that the overnight returns for each week night are identically distributed cannot be rejected with our data.

4.3 The Log Price Ratios Over Closed Trading Periods Are Independent of the Next Trading Period Returns

A measure of the degree of association between two variables is the coefficient of correlation, which is the square root of the coefficient of determination in regression analysis. Estimated correlation coefficients are computed between trading day returns and each type of non-trading period return. The only non-zero coefficients at the 1% level are between the daily and overnight returns of Aetna and Gulf Oil.

A rank correlation coefficient is a coefficient of correlation between two ranked variables. Tests of independence based on rank correlation coefficients do not assume that the underlying distribution is normal. The null hypothesis is that trading day returns are independent of non-trading period returns. Spearman

[4] Chi-square values from the Kruskal-Wallis test based on transaction returns each day are 6.08, 3.89, 3.78, 7.55, 3.86 for Aetna, Caterpillar, Gulf Oil, Tandy, and Westinghouse, respectively. Based on daily close to open each day the chi-square values are 10.98, 3.45, 2.36, 5.07, and 1.92. The 1% significance level for these values is 13.28.

[5] The chi-square values for the stocks listed in alphabetical order are .91, 8.33, 1.03, 1.71, and 2.49, respectively. The 1% significance level for these values is 11.34.

rank correlation coefficients are also computed between trading day returns and each type of non-trading period return. The only rank coefficients significantly different from zero at the 1% level are between daily and overnight returns for Aetna, Gulf Oil, and Westinghouse.[6]

In sum, both coefficients of correlation suggest that trading period returns are largely independent of the previous non-trading period returns.

4.4 The Mean and Variance Return for a Daily Trading Period Result from an Autoregressive Jump Process for Transactions

Three separate hypotheses are examined to establish this proposition. First, the density function for the time interval between transactions is gamma. Second, the mean and variance return observed between opening and closing prices correspond to those of the specified autoregressive jump process. Third, the daily transactions process does not contain a diffusion component.

Summary statistics are given for the time intervals between transactions in Table III. The time of day of a transaction is measured by the NYSE to the nearest preceding minute. On average about four minutes and twenty seconds elapse between transactions per stock. Evidence of a gamma shape is apparent in these statistics. The large kurtosis values depict peakedness and the strictly positive skewness measures imply a long-tailed but strictly positive distribution.

A gamma distribution is fit to the time intervals in Table III summed over transactions. It is necessary to sum over at least enough transactions to eliminate zero trading times. Sums of gamma distributions, however, are also gamma distributed. See Johnson and Katz [5]. Maximum likelihood estimates (MLE) of gamma parameters for the summed time intervals are given in columns (4) and (6) in Panel A of Table IV. These parameter estimates are used in the goodness of fit tests summarized in columns (7) to (9) in Panel A of Table IV.

Several goodness of fit statistics are computed to test whether theoretical

Table III

Summary Statistics for Time Intervals Between Transactions

Stock (1)	Sample Observations (2)	Mean (3)	Variance (4)	Skewness (5)	Kurtosis (6)	Studentized Range (7)
Aetna	49053	5.80902	58.979232	3.3457*	23.2269*	18.490*
Caterpillar	44525	6.38176	73.900401	3.1318*	19.6268*	15.471*
Gulf Oil	134149	2.17176	8.284742	3.0459*	24.9671*	28.836*
Tandy	54138	5.21473	94.913006	6.5295*	90.5236*	35.926*
Westinghouse	152995	1.89827	6.542894	3.2567*	31.0635*	32.448*
Average	86972	4.29511	48.524052	3.8619	37.8816	26.234

All statistics are computed as in Table I.

[6] We considered this correlation might be due to the "overlap" of the two return intervals as defined in Section III. By recomputing both coefficients so the daily return is the log ratio of the next to last price to the second price each day, the significant correlations disappear and none of the other coefficients are significantly different from zero at the 1% level. All tables are available on request.

Table IV

Gamma Parameters which Maximize the Likelihood of the Samples

Stock	SS	Sample Observa-tions	r	$r \div$ SS	λ	Goodness of Fit Comparisons KS	CVM	AD	Log-Likelihood Ratio
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
A. *All Months*									
Aetna	14	3125	3.66993	.26214	.04592	.02497	.54911	3.26708	79480.9
Caterpillar	10	4093	3.38389	.33839	.05360	.02288	.32085	2.10811	67802.4
Gulf Oil	36	3334	4.60275	.12785	.05984	.02058	.20016	11.9348	293005.0
Tandy	12	4142	1.49364	.13280	.02788	.05623*	3.68562*	21.12520*	72104.2
Westinghouse	40	3412	3.52561	.08814	.04730	.02058	.26329	1.85986	319240.0
B. *Even Month Sample*									
Aetna	14	1616	3.62342	.25882	.04564	.02101	.11913	.82580	41149.4
Caterpillar	10	2051	3.54776	.35478	.05492	.02504	.14013	.93916	34251.7
Gulf Oil	36	1705	4.72239	.13118	.06134	.02793	.22952	1.35190	151239.0
Tandy	12	2193	1.46541	.12212	.02666	.07256*	3.16073*	17.51630*	37470.1
Westinghouse	40	1717	3.58131	.08953	.04715	.02793	.15700	1.02260	161852.0
C. *Odd Month Sample*									
Aetna	14	1509	3.72240	.26589	.04625	.04257*	.57172	3.18317	38337.8
Caterpillar	10	2042	3.24467	.32447	.05262	.02834	.27072	1.82847	33590.6
Gulf Oil	36	1629	4.48397	.12455	.05834	.01684	.06431	.40960	141802.0
Tandy	12	1949	1.77969	.14831	.02984	.04796*	1.04965*	6.19026*	34816.3
Westinghouse	40	1695	3.47917	.08698	.04759	.02713	.15858	1.16851	157429.0

SS is the number of transactions summed. r and λ are maximum likelihood estimates of the parameters of a gamma distribution. The goodness of fit tests use the maximum likelihood estimates in columns (4) and (6) to represent theoretical distributions. KS, CVM, and AD are the Kolmogorov-Smirnov, Cramér-Von Mises, and Anderson-Darling statistics, respectively. Asterisks indicate those KS, CVM, and AD values significant at the 1% level. The log-likelihood ratio is log [likelihood of gamma distribution $\div$ likelihood of exponential distribution] where the likelihood is computed using maximum likelihood estimates of each density for time intervals between individual transactions.

gamma distributions correspond to empirical distributions. Specifically, the Kolmogorov-Smirnov (KS), Cramér-Von Mises (CVM), and Anderson-Darling (AD) statistics are calculated.[7] All of these statistics are based on ranks. As seen in Panel A of Table IV only Tandy has goodness of fit statistics significant at the 1% level. Taken together these statistics indicate that the theoretical gamma density fits the summed data quite well.

To recover the gamma parameters for the non-summed transactions, the estimated r is divided by the sum size in column (5) of Table IV while λ remains unchanged because it is a scale parameter. See Johnson and Katz [5]. To see if the distribution of time intervals between individual transactions is better represented by a gamma density than an alternative density such as the exponential distribution a likelihood ratio test is used. A positive log-likelihood ratio supports the gamma hypothesis.

The last column in Panel A of Table IV contains the log-likelihood ratio values computed using the time interval data in Table III. These are based on MLE's for the gamma density (given in columns (5) and (6) of Table IV and MLE's for the exponential density. The positive values allow us to conclude that a gamma distribution provides a better fit than an exponential to the sample time intervals between individual transactions.

To further investigate whether the transaction process is autoregressive a comparison is made between trading day sample moments and the theoretical moments given by expressions (11) and (12).[8] Let $N_d = 1$ and $N_o = N_h = N_w = N_{hw} = 0$ in (11) and (12). If α and β^2 are approximately zero then the theoretical and observed moments should be nearly identical.

It is important to note that to compute the theoretical mean and variance using equations (11) and (12) with the sample data requires the use of an estimate of the transaction mean and variance for a trading day. If trading day mean and variance are taken to be the sample mean and variance of close to open daily returns, then, clearly, the observed mean and variance include the transaction mean and variance per trading day which are used to estimate the theoretical mean and variance. In essence, the theoretical mean and variance computed over the entire sample period are not independent drawings from the same (or even an alternative) population as the observed mean and variance calculated over the same period.

To circumvent this problem the original sample of 39 months is split by months into two distinct sub-samples. One sub-sample consists of the twenty "even" months (i.e. October, December, . . .) and the other of the nineteen "odd" months (i.e. November, January, . . .). For each sub-sample the observed mean and variance plus the theoretical mean and variance are calculated for each stock.

[7] A brief description of the KS, CVM, and AD statistics is given in footnote ten of Oldfield, Rogalski, and Jarrow [7, p. 412]. For more details see Anderson and Darling [1].

[8] The autoregressive jump process given in expressions (8), (9), and (10) is actually a subset of the multiple component jump process specified by equations (5), (6), and (7) with $N_d = 1$, $N_o = 0$, $N_h = 0$, $N_w = 0$, and $N_{hw} = 0$. Therefore, testing whether daily returns follow an autoregressive jump process is identical to testing whether daily returns correspond to a multiple component jump process.

Then observed means and variances computed for even months are compared with theoretical means and variances obtained for odd months, and vice versa.

For completeness a gamma distribution is again fit separately to the summed time intervals in the even month sample and the odd month sample. (See Panels B and C of Table IV.) Goodness of fit tests continue to support the theoretical gamma density for the summed data in each sub-sample. The positive log-likelihood ratio values in column (10) reveal that a gamma density function for the time interval between transactions in each segment of the split sample is relatively better than an exponential distribution.

Estimated parameters imputed to equations (11) and (12) for the even month and odd month samples are documented in Panels A and B of Table V. Columns (3) and (4) are sample moments for transaction returns. The mean and variance of the distribution $q(N)$ for the number of transactions during a trading day are presented in columns (5) and (6). For each stock the distribution $q(N)$ is specified by the estimated parameters r and λ from columns (5) and (6) in Panels B and C of Table IV. In the last column of Table V the first summation is truncated at 2000 minutes. It is also assumed for the second sum that any true autocorrelations beyond lag five are zero.[9] Autocorrelations for lags one to five are estimated from transaction returns in each sub-sample.

Table VI presents the observed and theoretical means and variances for daily returns. Observed means and variances in columns (3) and (6) are sample statistics. Theoretical means and variances in columns (4) and (7) of Panel A are computed from (11) and (12) with $N_d = 1$ and $N_o = N_h = N_w = N_{hw} = 0$ using the estimated parameters in Panel B of Table V. Theoretical means and variances in columns (4) and (7) of Panel B are computed from (11) and (12) with $N_d = 1$ and $N_o = N_h = N_w = N_{hw} = 0$ using the estimated parameters in Panel A of Table V.

The τ-values in column (5) of Table VI allow us to test whether the theoretical means are equal to the observed means. These are computed as

$$\tau = \frac{\text{observed mean} - \text{theoretical mean}}{\sqrt{\text{observed variance/number of sample observations}}}$$

and are approximately normally distributed for samples of the size given in Table VI. Using a two-tail test, if the τ-value is in the interval ± 2.5758 we cannot reject the hypothesis that both means are the same at a 1% significance level. As seen in Table VI all of the τ-values lie within this range and it is concluded that the null hypothesis of identical observed and theoretical trading day means cannot be rejected.

A chi-square test is used to test the equality of variances. The χ^2-values in column (8) of Table VI are calculated as the ratio of the observed variance to the theoretical variance times the number of sample observations less one. These values are chi-square distributed with degrees of freedom equal to the sample observations given in column (2) minus one. A two-tail chi-square test is con-

[9] To examine whether the assumptions that $\rho_j = 0$ for $j > 5$ is valid all the tests to follow in the body of the paper are also performed using estimated autocorrelations of transaction returns from lags one to twenty-five. As expected, the results are unaffected. The reason is that almost all of the estimated autocorrelations beyond lag five are insignificantly different from zero at a 1% level.

Table V

Estimated Parameters of the Trading Day Theoretical Means and Variance

Stock (1)	Sample Observations (2)	$\hat{\mu}$ (3)	$\hat{\sigma}^2$ (4)	$E\{N\}$ (5)	$\text{Var}\{N\}$ (6)	$\sum_{N=2}^{2000}\sum_{j=1}^{N-1}(N-j)\hat{\rho}_j q(N)$ (7)
A. *Even Month Sample*						
Aetna	25404	.06129	.140871	64.91457	244.12080	−18.11605
Caterpillar	22353	.11754	.037070	56.63952	156.50680	−5.76911
Gulf Oil	68950	.08799	.126824	171.65750	1278.58600	−79.88372
Tandy	28613	.03852	.088826	82.17451	637.95460	−.33808
Westinghouse	77365	.01847	.315936	194.64960	2106.95800	−90.24602
B. *Odd Month Sample*						
Aetna	23649	.08836	.131430	64.00123	234.42210	−17.58174
Caterpillar	22172	−.02463	.031873	59.41993	179.21440	−5.68079
Gulf Oil	65199	.10932	.122281	172.14580	1348.58700	−80.02145
Tandy	25525	.16401	.091383	75.31261	484.74080	−1.21407
Westinghouse	75630	.04028	.281360	202.23170	2253.78300	−94.03136

$\hat{\mu}$, $\hat{\sigma}^2$ are sample means and variances for transaction returns. $\hat{\rho}_j$ are estimated autocorrelations of transactions returns for lags $j = 1, \cdots, 5$. All autocorrelations greater than five are assumed to be zero. $q(N)$ is the distribution for the number of transactions during a trading day as specified by the parameters r and λ in columns (5) and (6) of Table 4. $E\{N\}$, $\text{Var}\{N\}$ are the mean and variance of the distribution $q(N)$ for a daily trading period.

Table VI

Mean and Variance for Daily Returns: Theoretical vs. Observed

		Mean			Variance		
Stock (1)	Sample Observations (2)	Observed Mean (3)	Theoretical Mean (4)	τ-value (5)	Observed Variance (6)	Theoretical Variance (7)	χ^2-value (8)
A. *Even Month Sample*							
Aetna	422	3.50479	5.65515	−.2223	3.949549	3.790329	438.6849
Caterpillar	422	5.79342	−1.46351	1.1397	1.711073	1.531774	470.2794
Gulf Oil	422	13.72752	18.81898	−.8952	1.365127	1.481566	387.9128
Tandy	421	2.23601	12.35202	−.7756	7.161127	6.661706	451.4870
Westinghouse	423	3.29824	8.14589	−.4872	4.187789	3.986951	443.2578
B. *Odd Month Sample*							
Aetna	397	4.92760	3.97861	.1018	3.452582	4.040620	338.3695
Caterpillar	396	−1.14846	6.65741	−1.3139	1.397681	1.672121	330.1699
Gulf Oil	397	17.93680	15.10414	.4621	1.491970	1.508935	391.5478
Tandy	396	10.49270	3.16536	.5939	6.026873	7.239266	328.8475
Westinghouse	397	7.85079	3.59518	.3863	4.816577	4.472954	426.4217

All means and variances are multiplied by 10^4. Observed means and variances are sample statistics. Theoretical means and variances in Panel A are based on equations (11) and (12) with $N_d = 1$, $N_0 = 0$, $N_h = 0$, $N_w = 0$, $N_{hw} = 0$ using the estimated parameters in Panel B of Table V. Theoretical means and variances in Panel B are based on equations (11) and (12) with $N_d = 1$, $N_0 = 0$, $N_h = 0$, $N_w = 0$, $N_{hw} = 0$ using the estimated parameters in Panel A of Table V. The τ-value tests the null hypothesis that the observed mean equals the theoretical mean. Chi-square values test the null hypothesis of the equality of the observed and theoretical variances. None of the τ- or χ^2-values are significant at the 1% level.

structed for the appropriate degrees of freedom, but none of the chi-square values in Table VI is significant using a 1% level. (See column (8) of Table VI.) On the basis of this evidence it is concluded that the theoretical variances postulated by the autoregressive jump process for daily returns are not significantly different from the observed variances based on the sample data.

Remember that in the above tests we assume $\alpha \equiv 0$ and $\beta^2 \equiv 0$. Accepting the null hypotheses above only implies that there is no diffusion component in the return process. If there is a diffusion variance term the theoretical variances in column (7) should be systematically less than the observed variances in column (6). Five of the ten theoretical variances are greater than the observed variances and five are less, which suggests $\beta^2 \equiv 0$. Even if $\beta^2 \equiv 0$, however, the α could still be positive or negative so that we are unable to establish whether a drift term exists.[10]

In sum, it appears that a gamma distribution describes time intervals between transactions quite well. The evidence based on mean and variance return does not allow rejection of the proposition that common stock returns over a trading day follow an autoregressive jump process. The drift term on the diffusion component may or may not be zero, but $\beta^2 \equiv 0$ is consistent with our data indicating there is no diffusion variance.

4.5 The Mean and Variance Return over any Arbitrary Time Interval s Result from a Multiple Component Jump Process

To empirically examine proposition five, three hypotheses are defined. Each of these is tested by using expressions (11) and (12) which are valid for any arbitrary distributions assigned to the unconditional return density in (5). This assumes $\alpha \equiv 0$, $\beta^2 \equiv 0$ which means there is no diffusion process. All tests to follow utilize the split sample procedure adopted above.

Hypothesis one is that the mean and variance return of Monday's open price to Friday's close price of the same week when the market was open all five days is the sum of the mean and variance return for five daily periods and four overnight periods. Let $N_d = 5$, $N_o = 4$, $N_h = 0$, $N_w = 0$, and $N_{hw} = 0$ in equations (11) and (12). Then the theoretical means and variances are computed by multiplying the means and variances in columns (4) and (7) of Table VI by five and adding them to four times the sample means and variances for overnight returns. The resulting theoretical means and variances are reported in columns (4) and (8) of Table VII along with the observed means and variances of the weekly returns in columns (3) and (7). As seen in Table VII, the theoretical means and variances correspond very closely to the observed means and variances.

To test the equality of the means and variances both τ-values and chi-square values are again computed and subjected to a two-tail test. In column (5) of Table VII are presented τ-values to test the null hypothesis that the observed and

[10] To further complicate inferences about the diffusion drift term we test the null hypothesis that the true daily mean is zero. Only Gulf Oil for both the even and odd samples has a significant τ-value at the 1% level under this zero mean hypothesis. This implies that daily returns may have a zero mean which in turn implies a zero diffusion drift term.

Table VII

Mean and Variance for Weekly Returns: Theoretical vs. Observed (Friday close to Monday open for complete weeks only)

		Mean				Variance			
Stock (1)	Sample Observations (2)	Observed Mean (3)	Theoretical Mean (4)	τ-value (5)	Diffusion τ-value (6)	Observed Variance (7)	Theoretical Variance (8)	χ^2-value (9)	Diffusion χ^2-value (10)
A. *Even Month Sample*									
Aetna	64	73.7335	31.8868	.7162	−.1718	21.8461	23.0343	59.0579	23.4489*
Caterpillar	64	56.7956	12.0490	1.0429	1.7787	11.7823	9.3465	79.4188	31.2402*
Gulf Oil	64	54.4281	60.8357	−.1873	−7.3241*	7.4865	8.7670	53.7980	18.5955*
Tandy	63	26.4652	101.5716	−.9311	−1.8833	40.9893	37.6401	67.5167	24.8040*
Westinghouse	64	50.2715	23.1091	.5005	−1.5328	18.8526	24.4051	48.6667	14.5052*
B. *Old Month Sample*									
Aetna	59	23.8034	52.6580	−.4648	−.5764	22.7333	25.7502	51.2046	19.6378*
Caterpillar	58	6.9845	57.8654	−1.2547	−2.2564	9.5382	10.3939	52.3074	18.6907*
Gulf Oil	59	60.7270	47.8476	.3022	−4.0509*	10.7160	8.9695	69.2934	26.7818*
Tandy	58	86.8567	64.2163	.3010	.6494	32.8071	40.3672	46.3249	15.3607*
Westinghouse	59	20.5739	42.2459	−.3526	−.5774	22.2940	26.2723	49.2174	18.1628*

All means and variances are multiplied by 10^4. Observed means and variances are sample statistics. Theoretical means and variances in Panel A are based on equations (11) and (12) with $N_d = 5$, $N_0 = 4$, $N_h = 0$, $N_w = 0$, $N_{hw} = 0$ using the estimated parameters in Panel B of Table 5. Theoretical means and variances in Panel B are based on equations (11) and (12) with $N_d = 5$, $N_0 = 4$, $N_h = 0$, $N_w = 0$, $N_{hw} = 0$ using the estimated parameters in Panel A of Table V. The τ-values in column (5) test the null hypothesis that the observed weekly mean equals the theoretical mean. Diffusion τ-values in column (6) test the null hypothesis that the observed weekly mean equals the theoretical mean for a simple diffusion process. Chi-square values in column (9) test the null hypothesis of the equality of the observed and theoretical weekly variances. Diffusion chi-square values in column (10) test the null hypothesis of the equality of the observed variance and the theoretical variance for a simple diffusion process. Asterisks indicate τ- or χ^2-values significant at the 1% level.

theoretical means from Monday open to Friday close for complete weeks are identical. Values close to zero support the null hypothesis. None of the τ-values is significant at the 1% level which means that the null hypothesis concerning the equality of means cannot be rejected.

A chi-square test is used to determine whether the theoretical variances equal the observed ones. The resulting χ^2-values are reported in column (9) of Table VII. The insignificance of these values for all five stocks at the 1% level leads to the conclusion that the theoretical and observed variances for returns computed as the log ratio of Friday close to Monday open for complete weeks are not significantly different.

A reasonable alternative hypothesis is that the mean and variance return of Monday's open price to Friday's close price of the same week when the market was open all five days correspond to a simple diffusion process which operates over all open and closed periods. To calculate the theoretical mean and variance of a simple diffusion process the sample mean and variance of trading day returns is compounded up by the number of 360 minute time intervals per week. τ-values in column (6) of Table VII test the null hypothesis that the observed weekly mean equals the theoretical mean for a simple diffusion process. Chi-square values in column (10) of Table VII test the null hypothesis of the equality of the observed variance and the theoretical variance for a simple diffusion process. Only the τ-values for Gulf Oil are significant at the 1% level. However, the magnitude of the χ^2-values casts doubt on the variances of a simple diffusion process being identical to the observed variances. On a comparative basis the multiple component jump process provides a better description of weekly mean and variance return than a simple diffusion process.

Hypothesis two is that the mean and variance return from Friday's closing price to Friday's closing price one week hence when the market was open Friday and all days the next week equals the sum of five daily moments, four overnight moments, and one weekend moment. In (11) and (12), let $N_d = 5$, $N_o = 4$, $N_h = 0$, $N_w = 1$, and $N_{hw} = 0$. The theoretical means and variances are calculated by taking the previous ones in columns (4) and (8) of Table VII and adding to them the sample weekend means and variances. These theoretical means and variances are presented in Table VIII. (See columns (4) and (8).) Based on the τ- and χ^2-values, we cannot reject the hypothesis that the observed weekly means and variances from Friday close to Friday close for complete weeks equal the theoretical means and variances which result from a multiple component jump process.

Two reasonable alternative hypotheses can be tested. One is that the mean and variance return of Friday's close to Friday's closing price one week hence for complete weeks correspond to a simple diffusion process. τ- and χ^2-values to test the equality of the observed and simple diffusion means and variances are listed in columns (6) and (10) of Table 8. Comparing the τ- and χ^2-values in columns (5) and (9) with those in columns (6) and (10) allow us to conclude that the observed weekly means and variances in Table VIII more closely correspond to the theoretical means and variances of the multiple component jump process than the means and variances of a simple diffusion process.

Another alternative hypothesis is that a single jump occurs each trading day of a Friday to Friday complete week while during all closed periods the same diffusion process operates. To test this hypothesis a comparison is made of the sample means and variances of overnight and weekend returns. If returns over these non-trading intervals are time scaled by the same diffusion process then weekend means and variances should be about 3⅔ times overnight means and variances.

Sample means and variances of overnight returns are multiplied by 3⅔ so that the null hypothesis to be considered is that the overnight and weekend means and variances are equal. To test the equality of means an analysis of variance is performed using an F-test. A Bartlett test using chi-square values is used to determine whether overnight and weekend variances are equal.

Using the entire sample of 39 months, the F-values calculated for all five stocks exceed the 1% significance level which implies that overnight means after time scaling are not equal to weekend means. Chi-square values based on all 39 months of sample data for Aetna, Caterpillar, Gulf Oil, Tandy, and Westinghouse are 59.0401, 104.3740, 64.5119, 6.0398, 26.0617, respectively. Except for Tandy, all of these values are greater than the 1% significance level of 6.6349 which implies that the overnight variances after time scaling are not equal to the weekend variances.

These tests are also conducted on the even month and odd month samples with generally similar results. In particular, all F-values are greater than the 1% significance level. With the exception of Tandy all of the chi-square values for the even month sample are significant at the 1% level. All five of the chi-square values for the odd month sample exceed 6.6349. On the basis of this evidence, it is concluded that observed Friday to Friday weekly mean and variance return are better represented by a multiple component jump process than a proposition of a single jump over trading days combined with the same diffusion over closed trading periods.

It is worthwhile noting in Tables VII and VIII that more than half of the observed weekly variances are less than the theoretical weekly variances, i.e. seven out of ten in Table VII and six out of ten in Table VIII. In computing the theoretical variances it is assumed that $\beta^2 \equiv 0$. If a diffusion variance term exists then another term $\beta^2 s$ must be added to the theoretical variances in column (8) of Table VII and the difference between the observed and theoretical variances would be even greater. In other words, a zero diffusion variance term for weekly returns is consistent with our results. Unfortunately, no definite conclusion can be drawn concerning the existence of a drift term for weekly returns.[11] Recall

[11] Similar to the daily return means we test the null hypothesis that the true weekly mean is zero. The resultant τ-values under a zero mean hypothesis for Panel A of Table VII are 5.8987, 4.5436, 4.3542, 2.1006, 4.0217 and for the lower panel of Table VII are 1.8284, .5319, 4.6645, 6.6148, 1.5803 for Aetna, Caterpillar, Gulf Oil, Tandy, and Westinghouse, respectively. Corresponding τ-values under a zero mean hypothesis for the even sample of Table 8 are 4.3272, 4.2071, 1.5482, 5.9337, 1.0465 and for the odd month sample are 1.5072, .8824, 6.1984, 4.2504, 2.0264.

τ-values that exceed ± 2.5758 are significant at the 1% level. Comparing these τ-values with those reported in column (5) of Tables VII and 8 lead us to the inference that observed weekly means correspond more closely to the theoretical means of the multiple component jump process than a zero mean proposition.

Table VIII

Mean and Variance for Weekly Returns: Theoretical vs. Observed (Friday close to Friday close for complete weeks only)

Stock (1)	Sample Observations (2)	Mean: Observed Mean (3)	Mean: Theoretical Mean (4)	Mean: τ-value (5)	Mean: Diffusion τ-value (6)	Variance: Observed Variance (7)	Variance: Theoretical Variance (8)	Variance: χ^2-value (9)	Variance: Diffusion χ^2-value (10)
A. *Even Month Sample*									
Aetna	53	59.4385	60.2264	−.0129	−1.2825	19.8723	25.2240	40.9673	10.6893*
Caterpillar	53	57.7889	23.1234	.7057	1.8310	12.7897	9.7552	68.1754	16.9941*
Gulf Oil	53	21.2659	56.4859	−.9066	−12.3801*	7.9993	9.1988	45.2191	9.9572*
Tandy	52	82.2856	126.6230	−.4875	−2.3258	43.0045	38.6102	56.8044	12.9967*
Westinghouse	53	14.3747	44.4340	−.4854	−3.3174*	20.3277	26.2665	40.2428	7.8378*
B. *Odd Month Sample*									
Aetna	52	20.9006	62.4729	−.6236	−1.1585	23.1118	27.0642	43.5519	10.6585*
Caterpillar	51	12.3556	69.9919	−1.2283	−3.1938*	11.2285	10.7536	52.2081	11.7183*
Gulf Oil	52	85.9569	43.2473	.8967	−6.2652*	11.7968	9.3733	64.1862	15.7400*
Tandy	51	59.5177	76.9034	−.2040	−.0363	37.0397	45.2628	40.9162	9.2363*
Westinghouse	52	28.1018	57.2077	−.3990	−.8808	27.6703	28.0610	50.2900	12.0349*

All means and variances are multiplied by 10^4. Observed means and variances are sample statistics. Theoretical means and variances in Panel A are based on equations (11) and (12) with $N_d = 5$, $N_0 = 4$, $N_h = 0$, $N_w = 1$, $N_{hw} = 0$ using the estimated parameters in Panel B of Table 5. Theoretical means and variances in Panel B are based on equations (11) and (12) with $N_d = 5$, $N_0 = 4$, $N_h = 0$, $N_w = 1$, $N_{hw} = 0$ using the estimated parameters in Panel A of Table V. The τ-values in column (5) test the null hypothesis that the observed weekly mean equals the theoretical mean. Diffusion τ-values in column (6) test the null hypothesis that the observed weekly mean equals the theoretical mean for a simple diffusion process. Chi-square values in column (9) test the null hypothesis of the equality of the observed and theoretical weekly variances. Diffusion chi-square values in column (10) test the null hypothesis of the equality of the observed variance and the theoretical variance for a simple diffusion process. Asterisks indicate τ- or χ^2-values significant at the 1% level.

that we were also unable to establish if a diffusion drift term existed for daily returns.

The last hypothesis is that the observed return each month is drawn from a distribution from which the mean and variance return are determined each month by expressions (11) and (12). The multiple component jump process implies that each month the observed return may be drawn from a different distribution with distinct mean and variance. This is because each month the N_d, N_o, N_h, N_w, and N_{hw} may change. Months do not contain the same number of calendar days. More importantly, each month may contain a different number of trading days, overnights, weekends, and holiday-weekends. On the one hand, if the number of trading and non-trading periods in any two months is the same, then the two monthly returns should be drawn from a distribution with the same mean and variance return. On the other hand, returns for "dissimilar" months should be drawn from distributions with different moments.

To test this last hypothesis, monthly returns based on the log ratio of the closing price each month plus any dividends relative to the previous month's closing price are computed for each month sampled. In addition, each month the number of trading days, overnights, holidays, weekends, and holiday-weekends are counted. For example, the month of December, 1974 had 20 trading days, 14 overnights, 4 weekends, 1 holiday and 1 holiday-weekend. That is, with $N_d = 20$, $N_o = 14$, $N_h = 1$, $N_w = 4$, and $N_{hw} = 1$ in expressions (11) and (12) the mean and variance return of stocks for December, 1974 can be obtained. Since December is an "even" month, the theoretical mean and variance are calculated using estimated parameters from the odd month sample. If it is assumed that the distribution of monthly returns with these moments is nearly normal (see section 4.6) then an approximate confidence interval can be constructed. The observed return for December, 1974 is computed and a determination made of whether the return falls within the confidence interval. This procedure is duplicated for each month sampled and a count made of the number of times that the observed return each month is within the approximate confidence interval. For a given percentage confidence interval, roughly that percentage of the monthly returns should fall within the constructed intervals.

Table IX presents the results of the confidence interval test for monthly returns. There are 19 monthly close-to-close returns for each stock during each sub-sample. Columns (3) and (4) of Table IX contain the cumulative number of observed returns over the 19 months that are outside the specified confidence intervals. Approximate intervals are constructed using the mean and variance returns in expressions (11) and (12) for the appropriate N_d, N_o, N_h, N_w, and N_{hw} each month under an assumption of normality. For the 99% confidence intervals it is expected by chance alone that about $19(.01) = .19$ of the returns on average should fall outside the intervals. This compares favorably with the averages of .20 and .20 per stock which actually exceed the bands. Turning to the 95% confidence interval in column (4), on average 1.20 of the observed monthly returns in the even month sample are outside the 95% interval which is greater than the approximate number expected by chance alone, i.e. $19(.05) = .95$. For

Table IX

Results of Confidence Interval Tests for Monthly Returns

Stock (1)	Sample Observations (2)	Number of Sample Returns Outside the Approximate 99% Confidence Interval (3)	Number of Sample Returns Outside the Approximate 95% Confidence Interval (4)
A. *Even Month Sample*			
Aetna	19	0	0
Caterpillar	19	0	1
Gulf Oil	19	1	2
Tandy	19	0	2
Westinghouse	19	0	1
Average	19	.20	1.20
Number of sample returns expected outside the interval by chance alone		.19	.95
B. *Odd Month Sample*			
Aetna	19	0	0
Caterpillar	19	0	0
Gulf Oil	19	0	0
Tandy	19	1	2
Westinghouse	19	0	0
Average	19	.20	.40
Number of sample returns expected outside the interval by chance alone		.19	.95

Sample returns are actual month by month returns computed as the log ratio of the last price each month plus any dividends to the last price of the previous month. (See Panel E of Table I.) Each month approximate confidence intervals are constructed using mean and variance returns computed with expressions (11) and (12) based on values of N_d, N_0, N_h, N_w, and N_{hw} each month and assuming that the density for returns each month is nearly normal. Theoretical means and variances underlying Panel A use the estimated parameters in Panel B of Table V. Theoretical means and variances underlying panel B use the estimated parameters in Panel A of Table V.

the odd month sample on average .40 of the observed monthly returns are outside the 95% interval which is less than the expected .95.

In sum, the evidence does not allow us to reject the proposition that weekly and monthly common stock returns follow a multiple component jump process.

4.6 *The Return over Weeks and Months, Measured as the Log Price Ratio, Is Normal*

A close examination of Panels C, D, and E of Table I indicates that three of the stocks have positively skewed weekly returns whereas monthly returns appear symmetric as required by a normal density. The weekly returns also have more area than a normal density in the central region. Studentized range statistics give a test for departure from normality in the tails. None of the five studentized range values for monthly returns is significant at the 1% level but the weekly returns of two stocks exceed the upper .99 percentage points.

In sum, no definite conclusions concerning normality of the weekly returns can be made on the basis of our data. Alternatively, we find no evidence against the proposition that the monthly returns of these five stocks are approximately normally distributed over the period sampled.

V. SUMMARY AND CONCLUDING REMARKS

This paper extends earlier work on an autoregressive jump process. A generalized stochastic process, termed a multiple component jump process (MCJP), is developed for common stock returns over any arbitrary time interval s, where s can be days, weeks, months, etc. The MCJP is a mixture of a diffusion process and several separate jump processes. The diffusion process operates over the entire interval s, while a different jump component is specified for the transaction process during a trading day and for overnight, holiday, weekend, and holiday-weekend transactions. Transaction returns over a trading day are assumed to follow an autoregressive jump process. In this way the MCJP still relies on the distribution of random time intervals between transactions. The MCJP also incorporates empirically observed negative dependence in transaction returns.

Several propositions consistent with the new model are subjected to statistical analysis. Three major conclusions are drawn from the results. First, for open trading periods, trading day returns are identically distributed each day of the week. In addition, regardless of the night of the week, overnight returns are also identically distributed. This implies that different days or nights of the week have the same return properties. Second, based on sample mean and variance return, stock returns for a trading day period seem to follow an autoregressive jump process. Implicit in this conclusion is that the probability density for time intervals between transactions is gamma. There also appears to be no diffusion variance component, although a drift term may exist. Last, returns computed over weekly and monthly intervals seem to follow a multiple component jump process based on sample mean and variance return. Again, there appears to be no variance on the diffusion component over these longer intervals. It is important to note that none of these empirical results for the MCJP depends on identifying the densities of trading or non-trading period returns.

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