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Capital Asset Prices and the Temporal Resolution of Uncertainty*

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I. Introduction

THE PATTERN OF RETURNS that an investor expects over time partly depends upon the temporal resolution of uncertainty. Information pertaining to a firm may become available at any time and, in general, this information will affect price and thus the realisable capital gains in any particular period. While the amount and content of information is uncertain, investors may expect the characteristics of the flow of information to differ among securities and this should be reflected in prices.

Robichek and Myers [26] argue that early resolution of uncertainty might have a positive effect upon price, for it allows individuals to revise their consumption-investment decisions to compensate for unfavourable outcomes. If this is correct, then it implies that a firm can increase its market value by informing investors that it will release all information when and if it becomes available. However, it might be argued that the expectation of new information arriving in a particular period increases the risk of holding the particular stock. Investors will require a greater expected rate of return in order to hedge against adverse outcomes, thus causing a possible decrease in price.

The first objective of this paper is to examine how the temporal resolution of uncertainty affects market prices and consumption-investment decisions. We adopt the simplest model that makes possible the analysis of temporal resolution, namely a three period ($t = 1, 2, 3$) model. Individuals are assumed to act so as to maximize the expected utility of present and future consumption. It is assumed that individuals can trade in the shares of assets which provide uncertain returns and there exists a riskless asset which provides a riskless return over the three periods. Before the beginning of the second period new information arrives. As

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there is trading at date $t = 2$, investors will revise their portfolios and prices of the different assets will adjust to the new information.[1]

Information is produced by an experiment which the firm performs after undertaking an investment project. Given the production decision at date $t = 1$ the firm undertakes the experiment and informs investors of the results.[2] For concreteness consider the case of an automobile manufacturer who at date $t = 1$ announces that it will produce a fixed number of new sport cars for sale at date $t = 3$ and will undertake an experiment before date $t = 2$ that will provide investors with more information. The nature of the experiment could be of the form of a survey of the opinions of representatives of different car publications. Favourable opinions would indicate that the car might sell well at $t = 3$. Alternatively, the experiment could consist of a performance test on a prototype. For the individuals investing in the project, the firm in performing the experiment and informing them of the results, is providing the individuals with more information about the future cash flow at $t = 3$. For the firm, the experiment provides a monitoring service to investors and can also provide the firm with research information with which to improve its product, which is a description of the research and development function. The latter is the subject of separate research by the authors. In the analysis that follows we shall be concerned solely with the monitoring service provided by an experiment. Therefore, it will be assumed that no production decisions are taken at $t = 2$ after the experiment results become known.

The monitoring service provided by the experiment could also be interpreted as providing information to shareholders about the activities of management, if there is separation between ownership and control. Given separation, the owners of the firm face a moral hazard problem in that actions by management may not be in the best interests of the owners and will decrease the market value of their holdings. One means of minimizing this type of risk and associated costs is for the owners to insist upon the monitoring of managerial activities.[3]

The ordinal measure of the informativeness of an experiment that is adopted in this paper is described in Section II. It is based on the notion of sufficiency due to Blackwell [3] and Marschak and Miyasawa [21]. A theorem is derived proving the existence of an intuitive parameterization of the amount of information provided by an experiment.

The basic framework of the model, which is described in Section III, draws upon the recent work of Stapleton and Subrahmanyam [30]. The equilibrium asset prices are derived in a model where the temporal resolution of the uncertainty surrounding the cash flow at $t = 3$ is given for each asset but differs across assets. The derivation facilitates examination of how the temporal resolution of

[1] The assumption that risky assets may be traded at $t = 2$ is only a formal requirement, but the possibility of $t = 2$ trading in the safe asset is crucial to the analysis. See the discussion in Section III.1 and footnote 15.

[2] It is assumed that the firm communicates the information truthfully. This assumption avoids the moral hazard problem of the firm giving spurious information. (See also footnote 3.)

[3] The monitoring of managerial activities as a means of reducing agency costs is described in Jensen and Meckling [19].

uncertainty affects systematic risk and the structure of asset returns. This provides insight into the assertion of Robichek and Myers of an inverse relationship between risk and the timing of the resolution of uncertainty. The information provided by an experiment can be partitioned into two components: information that is specific to the cash flow of one firm and information about the market as a whole. The discussion distinguishes between these two cases due to their differing implications.

The second objective of the paper, presented in Section IV, is to analyse the amount of monitoring of investment projects that will be undertaken by firms that maximize market value. It is assumed that more informative experiments are more costly to undertake. Again it is useful to distinguish between the production of firm specific and market information as a free rider problem exists for the latter. We compare the amount of information produced by firms with the Pareto optimal amount. In general, market value maximizing firms produce a suboptimal (and in the firm specific case, excessive) amount of information. This result is related to the recent work on the inefficiency of the investment decision of market value maximizing firms.[4]

Some technical details are collected in two appendices. Appendix B contains only brief derivations of the basic equations used in sections III and IV. For more details the reader is asked to refer to the complete derivations contained in Stapleton and Subrahmanyam [30].

We should note that a number of recent papers have examined the process and effects of information generation. (See Hirshleifer [15], Fama and Laffer [9], Jaffe [17] and Ng [24].) This paper differs from these in several respects. First, to various degrees, the earlier papers (i) adopt the framework of Arrow-Debreu contingent markets and assume the existence of the complete set of markets; (ii) investigate the acquisition and social value of private information (available only to a single individual), and (iii) assume pure exchange models. In this paper we adopt a multiperiod CAPM framework and we consider only public information (available to everyone).[5] Also our model contains a form of productive activity, namely storage.

Second, and more significantly, none of the above papers address the issue of the temporal resolution of uncertainty. Hirshleifer and Ng distinguish between the two cases of, first, when information becomes available before trading and second, when individuals trade to reach their consumptive optimum positions prior to the release of the information with another round of trading permitted afterwards.[6] When all markets finally close at the initial point in time, individuals use the new information to derive their consumption optima. It is assumed that all uncertainty is resolved at the end of the first period and the possibility of

[4] See Jensen and Long [18], Fama [7], Stiglitz [31], Merton and Subrahmanyam [23], and Baron [2].

[5] Jaffe [17] also concentrates on public information. Note that information may be public not only because it is directly available to everyone, but also because market prices may reveal to "uninformed" traders all information available to "informed" traders. See Grossman [13, 14].

[6] An elaboration of these two cases is given by Ng [25].

subsequent information affecting current and future consumption decisions is not considered.

Finally, there is the difference in the ordinal measure of information that is used. Ng [24] introduces the notion of more accurate information. Information is defined to be more accurate than existing information if individuals are willing to revise their subjective homogeneous beliefs based upon the new information. In terms of the model of this paper, such a notion relates to a particular outcome of an experiment rather than to the overall nature of the experiment as defined by the entire spectrum of possible outcomes. Since the eventual result of the experiment is not known at $t = 1$, the Ng definition of more accurate information is not useful in addressing the issue of temporal resolution.

II. Some Preliminaries and Basic Assumptions

There exists a single commodity which may be consumed, saved or as described below, may be used as an input to produce information. A representative investor faces a three-period time horizon. The investor makes consumption-savings and portfolio decisions to maximize the expected utility of consumption where the temporal von Neumann-Morgenstern utility index is

$$u(c_1, c_2, c_3) = -\sum_{t=1}^{3} \delta^{1-t}\exp(-Ac_t).$$

A is the constant measure of absolute risk aversion with respect to uncertainty in consumption in any period and $\delta \geq 1$ is the subjective rate of discount.[7] Savings may be held in the form of any of three assets. There is a risk-free bond that may be bought or sold in unlimited amounts at $t = 1$ or $t = 2$ and yields the certain gross returns r_1 and r_2 respectively one period later. Markets for two risky assets are also open at $t = 1$ and $t = 2$. The assets are available, in fixed supplies, normalized to unity, in each trading period. They correspond to shares in firms having uncertain cash flows X and X' respectively at $t = 3$.[8]

The investor entertains a prior probability distribution at $t = 1$ regarding the eventual realization at $t = 3$ of the random variables X and X'. The investor is also aware that before the market at $t = 2$ opens, new information concerning X and X' will become available. The information will be forthcoming through the observation of a (vector) random variable Z, the outcome of an experiment, which is correlated with X and X'. After observing a particular realization z of Z the investor revises the probability distribution for X and X' according to Bayes' Rule. With the revised ex post probability beliefs, the investor adjusts the holdings of risky assets by trading on the market at $t = 2$. The random variables Z, X and X' are assumed to be jointly normally distributed.

[7] A general discussion of aggregation conditions is given in Rubinstein [27]. If there are H consumers each having negative exponential utility functions with different absolute risk aversion measures A_h, $h = 1, \cdots, H$, but with common expectations, then a representative individual exists with $A^{-1} = \sum_1^H A_h^{-1}/H$.

[8] For the most part upper case letters will denote random variables and lower case letters deterministic variables.

The investor's period 1 decisions are affected by the prospect of future information. The amount of information about X and X' provided by Z is a parameter in the model.

Our first objective is to compare the first period equilibrium prices for the risky assets in two situations that differ only with respect to the information provided by Z about the cash flows. Note that the prior uncertainty about X and X' is identical in the two situations. The latter differ only with respect to the amount of information that is expected to be forthcoming before the period 2 decisions are made. Thus it seems justifiable to describe the analysis below as an investigation of the effects on asset prices of the temporal resolution of uncertainty. We will say that the prior uncertainty about the cash flows is resolved earlier (later) the more (less) informative is Z concerning X and X'.[9]

The notion of greater information adopted in this paper is that due to Blackwell [3], and Marschak and Miyasawa [21]. Applications below will be confined to instances where a scalar random variable provides information about another scalar random variable. Thus let us for the purpose of the following brief digression assume that Z is a scalar that provides information about X alone. Let Z' correspond to another experiment that provides information about X alone. Denote by $h(z \mid x)$ and $h'(z' \mid x)$ the density functions of Z and Z' respectively, conditional on $X = x$. Then Z is said to provide more information about X than does Z' if there exists a conditional density function $m(z' \mid z)$ such that for all z' and x

$$h'(z' \mid x) = \int m(z' \mid z) h(z \mid x)\, dz, \tag{1}$$

where the integration is carried out over all possible realizations z of Z.

The critical feature of the conditional density $m(z' \mid z)$ is the fact that it is independent of x. Thus once Z is known to equal z, the random variable Z' is independent of x and observations of Z' convey no additional information about x. (Two extreme cases may be noted: Z' is said to provide no information at all about X if Z' and X are stochastically independent, while Z is said to provide perfect information if X conditional on Z is certain.) Blackwell and Marschak and Miyasawa prove that if Z is more informative than Z' then every user of information about x (such as the decision maker who solves problem (2) below), is at least as well off observing Z before making his decision as he would be if he based his decision on Z'.

When Z and X have a bivariate normal distribution there exists a convenient and intuitive parameterization of the amount of information provided by Z, as described by the following theorem:

THEOREM. *Let (Z, X) and (Z', X) have bivariate normal distributions with identical marginals. Then Z is more informative than Z' if $\rho_{zx}^2 > \rho_{z'x}^2$, where ρ_{zx} and $\rho_{z'x}$ denote the correlation coefficients in the respective probability distributions.*
See Appendix 1 for a proof.

[9] This terminology was introduced by Epstein [5].

Below we will compare experiments for which the corresponding random variables have identical marginal distributions. Applying the theorem we will use the square of the correlation coefficient as a measure of the amount of information. Note that Z provides no information if (and only if) $\rho_{zx} = 0$.

Before we formulate our model more precisely, some comments are in order regarding the assumptions we have made. The analysis could be extended to include uncertain cash flows at $t = 2$. The uncertain period 2 cash flows might provide information about period 3 cash flows. Our results, suitably modified, remain valid however if we consider the informativeness of Z about X and X' conditional on second period cash flows. The framework of two risky assets is sufficient to achieve the objectives of this paper; with some added mathematical complexity, any finite number of assets could be considered.

Apart from the usual set of assumptions used in deriving the capital asset pricing model—all assets are infinitely divisible, taxes and transaction costs are zero, short selling is permitted, investors act as price takers, the price of the aggregate consumption good is fixed and normalized to equal unity, the probability distribution of all cash flow is multivariate normal, there is no limited liability, there is no non-investment income, and the investor's initial wealth w_1 consists of the value of the endowment of the riskless bond and risky assets—three additional assumptions are necessary in order that asset prices can be determined in a general equilibrium multi-period framework: (a) the expected future risk free rates of interest are non-stochastic; (b) negative exponential utility; and (c) the probability distribution of all cash flows over time is multivariate normal.[10, 11] Finally, expectations of $t = 2$ asset prices are assumed to be rational in the sense that they are generated by a future general equilibrium of prices. It should be emphasized that markets are not assumed to be complete.[12]

III. The Basic Valuation Model

The investor solves the following problem:

$$\max_{s_1,\alpha_1,\alpha_1',b_1} u(w_1 - s_1) + E_Z \max_{s_2,\alpha_2,\alpha_2',b_2}$$

$$\left\{\frac{1}{\delta} u(w_2 - s_2) + E_{X,X'|z} \frac{1}{\delta^2} u(b_2 r_2 + \alpha_2 X + \alpha_2' X')\right\}$$

$$\begin{aligned} \text{subject to}\quad & p_1\alpha_1 + p_1'\alpha_1' + b_1 = s_1 \\ & w_2 = r_1 b_1 + P_2\alpha_1 + P_2'\alpha_1' \\ & P_2\alpha_2 + P_2\alpha_2' + b_2 = s_2. \end{aligned} \tag{2}$$

The following notation is adopted:

$u(c) = -\exp(-Ac)$;

[10] See Fama [8], or Stapleton nd Subrahmanyam [30].

[11] However, the analysis remains valid if utility is a multiplicatively separable exponential function or is an exponential function of terminal wealth alone. See Stapleton and Subrahmanyam (p. 1087–9).

[12] For a definition of a complete market see Arrow [1].

p_1 and p'_1 are the respective first period prices of the risky assets;

P_2 and P'_2 are the first period expectations of the $t = 2$ asset prices; by the assumption of rational expectations the realizations of P_2 and P'_2 are uniquely determined by the realization of Z, i.e., we may write $P_2 = P_2(z)$ and $P'_2 = P'_2(z)$;

α_t, α'_t and b_t denote holdings of risky assets and the bond respectively, $t = 1, 2$;

w_t is the investor's wealth at the beginning of period t; by assumption, $w_1 = p_1 + p'_1 + \bar{b}$, where $\bar{b}$ is the initial endowment of bonds;

s_t denotes t^{th} period savings out of beginning of period wealth w_t;

E_Z denotes the expected value with respect to Z (and hence also with respect to P_2 and P'_2) and $E_{X,X'|z}$ denotes the expected value with respect to X and X' conditional on $Z = z$.

Note that s_2, α_2, α'_2 and b_2 are chosen after observing Z. Also, the return to purchasing the risky assets in period 1 is no longer the cash flows X and X' but rather the capital gains due to the (uncertain) change in asset prices between periods 1 and 2.

Problem (1) may be solved by the standard dynamic programming argument. (Since the solution of a more general problem is described by Stapleton and Subrahmanyam, we relegate mathematical details to Appendix 2.) First, for a given realization z of Z, solve the period 2 decision problem, namely the problem inside the expectation operator E_Z in (1). The portfolio decision is made within the standard single period, mean-variance framework. Therefore asset demand functions are readily derived. Equating demand and supply in the market for each risky asset we derive[13]

$$P_2(z) = \frac{1}{r_2}\{E[X \mid z] - A[\operatorname{var}(X \mid z) + \operatorname{cov}(X, X' \mid z)]\},$$

$$P'_2(z) = \frac{1}{r_2}\{E[X' \mid z] - A[\operatorname{var}(X' \mid z) + \operatorname{cov}(X, X' \mid z)]\}. \tag{3}$$

From the elementary properties of the normal distribution described in Appendix 1, it follows that $P_2(z)$ and $P'_2(z)$ are jointly normally distributed.

The complete solution of the period 2 decision problem implies a derived utility function for wealth w_2 which is negative exponential. Therefore, the period 1 decisions may be viewed in a standard two period framework (see Fama [6]) and are readily determined. Equilibrium in the period 1 markets implies

$$p_1 = \frac{1}{r_1}\left\{EP_2 - \frac{r_2 A}{1 + r_2}[\operatorname{var} P_2 + \operatorname{cov}(P_2, P'_2)]\right\},$$

and

$$p'_1 = \frac{1}{r_1}\left\{EP'_2 - \frac{r_2 A}{1 + r_2}[\operatorname{var} P'_2 + \operatorname{cov}(P_2, P'_2)]\right\}. \tag{4}$$

[13] Equations for X and X' are obviously symmetric. Therefore, we will frequently write only one of them. $E[X \mid z]$ is the expected value of X conditional on $Z = z$. Similarly for the conditional variance and covariance.

The valuation equations in the two periods, (3) and (4), are identical in form. The only significant difference between them is that the price of period 3 risk is A and the price of period 2 risk is $r_2A/(1 + r_2)$.

In analysing (3) and (4), it is instructive to adopt two alternative specifications regarding Z corresponding to the natural conceptual decomposition of information into a component that is related to the market as a whole and another component that is purely asset specific. Sharpe's [29] simplified model can be used to illustrate the discussion of asset specific information. Suppose that

$$X = a\Phi + \theta, \qquad X' = a'\Phi + \theta', \qquad \operatorname{cov}(\theta, \theta') = 0, \tag{5}$$

where a and a' are constants; and θ, θ', and Φ are random variables. The terms $a\Phi$ and $a'\Phi$ represent the parts of X and X' respectively that depends on the general level of economic activity, while θ and θ' represent the unique asset specific aspects of each cash flow. A random variable $Y(Y')$ which is correlated with $\theta(\theta')$ and is independent of Φ, can be interpreted as providing information that is purely asset specific. A random variable that provides information about Φ alone, to the exclusion of θ and θ', can be thought of as providing market information.

Asset Specific Information

Let $Z = (Y, Y')$ where Y and Y' are scalar random variables which provide information about X and X' respectively. To capture the specific nature of the information provided by each random variable, we assume that $\operatorname{cov}(X, Y') = \operatorname{cov}(X', Y) = \operatorname{cov}(Y, Y') = 0$.

From Appendix 2, we see that at time $t = 1$,

$$EP_2 = \frac{1}{r_2}\{EX - A[\operatorname{var} X\cdot(1 - \rho_{xy}^2) + \operatorname{cov}(X, X')]\}, \tag{6}$$

where $\operatorname{var} P_2 = \operatorname{var} X\cdot\rho_{xy}^2/r_2^2$, and $\operatorname{cov}(P_2, P_2') = 0$.

Thus the more informative is Y for X, as measured by ρ_{xy}^2, the larger are both the mean and variance of P_2. However, the increase in the mean dominates in the sense that p_1 varies positively with ρ_{xy}^2. It can be shown—see Appendix 2—that

$$p_1 = \frac{1}{r_1r_2}\left\{EX - A\left[\operatorname{var} X\cdot\left(1 - \frac{r_2}{1 + r_2}\rho_{xy}^2\right) + \operatorname{cov}(X, X')\right]\right\}. \tag{7}$$

Clearly, if the experiment Y provides no information about $X(\rho_{xy}^2 = 0)$ then $p_1 = EP_2/r_1$, the expected value of the asset at $t = 2$ is discounted at the risk free rate of interest to determine its present value. The systematic risk for the return over the first period is zero.[14]

The value of the experiment Y, as measured by its effect on the market value of the cash flow X, is equal to $A\cdot\operatorname{var} X\cdot\rho_{xy}^2/r_1(1 + r_2)$. Thus we have the intuitive result that the firm's valuation of an experiment varies inversely with the risk

[14] This is equivalent to the case discussed by Fama [8, p. 9].

free rates of interest, and directly with the investor's aversion to risk, the variance of X, and the informativeness of the experiment. The overall market risk associated with X is unambiguously reduced by the asset specific information. The standard valuation equation (7) is in terms of the smaller "effective" variance of X, $\operatorname{var} X \cdot (1 - r_2\rho_{xy}^2/(1 + r_2))$, and asset specific information does not affect $\operatorname{cov}(X, X')$. (Note that the factor $r_2/1 + r_2$ is necessary because of the different prices of risk, or measures of absolute risk aversion, for uncertainty in periods 2 and 3.)

One might be tempted to at least partially explain the positive market value of an experiment by suggesting that investors speculate on the outcome of the experiment. Feiger [10] has called speculators "agents who trade to a bundle of goods from which they hope to trade away profitably after some price movements". Adopting the implied definition of speculation we see that there is speculative activity in our model only if the investor behaves differently than described above if the market for risky assets is not reopened at $t = 2$, while the market for the safe asset remains open at $t = 2$.[15] But in fact the equilibrium prices of risky assets at $t = 1$ are unchanged form (7) in such a case, as can be demonstrated by adapting the arguments of Appendix 2 to the new model specification. That there can be no speculative holding of risky assets when trading in these assets is possible at $t = 2$ is perfectly clear when the model is formulated as above in terms of a single representative investor, for if markets are in equilibrium he must hold the endowment vector of risky assets in each period. Thus he will not wish to revise his holding of risky assets at $t = 2$ even if the market is open. If there are several investors with different initial endowments and attitudes towards risk, none will wish to revise their portfolios of risky assets at $t = 2$ if they have common expectations.[16] Thus in the framework of this paper, as in Hirshleifer [16], only differences in expectations can motivate speculation on emerging information. We may conclude, given our assumption of homogenous expectations, that the positive market value of an experiment is completely due to the improved consumption-investment decisions which the emerging information allows. The information is used at $t = 2$ to alter the investment in the riskless asset and current consumption. A similar conclusion is valid with respect to the value of an experiment that provides market information, as analysed below.

Before considering this case, it is of interest to note that with one minor qualification these results confirm the assertions of Robichek and Myers. The minor qualification concerns the period by period structure of asset returns. If there is no information provided by the experiment ($\rho_{xy}^2 = 0$) then the required rate of return for the first period from $t = 1$ to $t = 2$ is the risk free rate of interest, r_1, implying that the first period systematic risk is zero. If there is temporal resolution of uncertainty ($\rho_{xy}^2 > 0$), then the expected rate of return for the first period is unambiguously greater than the risk free rate of interest, implying that

[15] If the market for the safe asset is also closed at $t = 2$, then all decisions would be made before the outcomes of the experiments are observed and information would have no private or social value. First period equilibrium prices would be given by (7) with ρ_{xy}^2 (and $\rho_{x'y'}^2$) set equal to zero.

[16] Each investor will hold A/A_h units of each risky asset in each period, where $A^{-1} = \sum_{h=1}^{H} A_h^{-1}$, and A_h is the measure of absolute risk aversion of investor h.

systematic risk is now positive.[17] However, the increase in the expected market value of the asset at $t = 2$ more than off-sets the increase in the required rate of return.

Market Information

Let Z be a scalar random variable. From Appendix 2 we have that

$$p_1 = \frac{1}{r_1 r_2}\left\{EX - A\left[\operatorname{var} X \cdot \left(1 - \frac{r_2}{1+r_2}\rho_{xz}^2\right) + \operatorname{cov}(X, X') - \frac{r_2}{1+r_2}(\operatorname{var} X \cdot \operatorname{var} X')^{1/2}\rho_{xz}\rho_{x'z}\right]\right\}. \quad (8)$$

where the ρ's denote the appropriate correlation coefficients.

The expression inside the square bracket may be viewed as the "generalized" market risk associated with X. The effect of the experiment represented by Z is to reduce the variance of X by the factor $(1 - r_2\rho_{xz}^2/(1 + r_2))$ and to change the covariance of X and X' in a direction depending on the sign of $\rho_{xz} \cdot \rho_{x'z}$.[18] (The presence of the factor $r_2/1 + r_2$ may be explained precisely as in the preceding section.) Because of the latter effect, the qualitative impact on price of the information provided by Z is ambiguous. Both p_1 and p_1' will rise, however, if $\rho_{xz} \cdot \rho_{x'z} > 0$. In general, the market information will induce an increase in $p_1 - p_1'$ if and only if $\operatorname{cov}^2(X, Z) - \operatorname{cov}^2(X', Z) > 0$. Finally, from (8) and the corresponding expression for p_1', it follows that $p_1 + p_1'$ is always larger (or at least never smaller) given Z than in the case of no market information. The explanation is that the information reduces total market variance by the non-negative amount

$$\frac{r_2}{1+r_2}[\rho_{xz}(\operatorname{var} X)^{1/2} + \rho_{x'z}(\operatorname{var} X')^{1/2}]^2,$$

inducing a shift in demand from the bond to the risky assets.

In this case it is seen that only with qualifications is it possible to accept the assertions of Robichek and Myers. Not surprisingly, the effects upon the systematic risk for the first period return is also ambiguous, depending upon the sign of $\rho_{xz} \cdot \rho_{x'z}$.

The interpretation of Z as supplying market information is clearer in the context of the Sharpe model defined by (7). If we assume that $\operatorname{cov}(Z, \theta) = \operatorname{cov}(Z, \theta') = 0$ and that $\operatorname{cov}(Z, \Phi) \neq 0$, then Z provides information only about the market component Φ of the cash flows. Equation (8) becomes

$$p_1 = \frac{1}{r_1 r_2}\left\{EX - A\left[\operatorname{var} X + \operatorname{cov}(X, X') - \frac{r_2}{1+r_2}a(a + a')\operatorname{var}\Phi \cdot \rho_{z\Phi}^2\right]\right\}. \quad (9)$$

Thus greater market information, as measured by $\rho_{z\Phi}^2$, increases p_1 if and only if $a(a + a') > 0$, and increases $p_1 - p_1'$ if and only if $a^2 > a'^2$. (By redefining Φ if

[17] From Appendix 2 it can be shown that the systematic risk or the beta coefficient is given by $\beta_x = (\rho_{xz}^2 \cdot \operatorname{var} X/p_1)/[(\rho_{xz}^2 \cdot \operatorname{var} X + \rho_{x'z}\operatorname{var} X')/V_M]$ where $V_M = p_1 + p_1'$.

[18] Note that $\operatorname{cov}(X, X' \mid z) < (>) \operatorname{cov}(X, X')$ according as $\rho_{xz} \cdot \rho_{x'z} > (<) 0$. (See Appendix 1.)

necessary we may assume that $a + a' \geq 0$. If $a + a' > 0$, $a(a + a') > 0$ is equivalent to $a > 0$, which we would expect to be true for most assets.) The impact on $p_1 + p_1'$ is always non-negative and vanishes only if $a = -a'$.[19]

IV. Competitive and Optimal Production of Information

In this section we use the price equations derived in Section III to describe the monitoring decisions of a firm that maximizes its market value. Then we compare these decisions with those that would maximize the welfare of the representative investor.

Asset Specific Information

Consider first the framework of III.1 and the production of firm specific information. At $t = 1$, a typical firm has two decisions to make. It must choose its level of investment, (i.e., its cash flow X or X') and the nature of the experiment Y(or Y') that it will perform at $t = 2$ in order to provide specific information about the progress of the investment project and hence about X. Suppose that X has been chosen and focus on the choice of an experiment. From (7) and the following remark, the gross value of an experiment to the firm increases with ρ^2_{xy}. The cost, in aggregate consumption units, of conducting an experiment is represented by $C(\rho^2_{xy})$. Presumably more informative experiments are more costly to perform so that C is an increasing function. We assume that it also exhibits increasing marginal costs. ($C(0)$ may be viewed as the investment cost alone with no monitoring being undertaken.)

The firm solves

$$\max_{\rho^2_{xy}} p_1 - C(\rho^2_{xy}), \quad p_1 \text{ given by (7).} \tag{10}$$

Thus the experiment chosen is described by the condition that the marginal cost equals the marginal change in market value:

$$dC(\rho^2_{xy})/d\rho^2_{xy} = A \cdot \operatorname{var} X/r_1(1 + r_2). \tag{11}$$

To determine the socially optimal level of firm specific monitoring we must find the derived utility of the investor as a function of the parameters that are exogenous to him; most importantly the distribution of $X(X')$ and $\rho^2_{xy}(\rho^2_{x'y'})$. This we do in Appendix 2. We obtain the following expression for the maximum value of the objective function in (2):[20]

$$W(\rho^2_{xy}, \rho^2_{x'y'}) \sim$$

$$-\exp\left[-\frac{A}{1 + r_2 + r_1 r_2}\left\{-r_1 r_2 C(\rho^2_{xy}) - r_1 r_2 C(\rho^2_{x'y'}) + E(X + X')\right.\right.$$

[19]

$$p_1 + p_1' = \frac{1}{r_1 r_2}\left\{E(X + X') - A\left[\operatorname{var}(X + X') - \frac{r_2}{1 + r_2}(a + a')^2 \operatorname{var} \Phi \cdot \rho^2_{xz}\right]\right\}$$

[20] Throughout the paper $\sim$ denotes equality apart from multiplicative constants which are independent of the important distributional parameters.

$$- \frac{1}{2} A \left[\operatorname{var}(X + X') - \frac{r_2}{1 + r_2} (\rho^2_{xy} \operatorname{var} X + \rho^2_{x'y'} \operatorname{var} X') \right] \Bigg\} \Bigg]. \quad (12)$$

Therefore, the value of ρ^2_{xy} which maximizes welfare is given by solving

$$\max_{\rho^2_{xy}} \frac{1}{2} \frac{A}{1 + r_2} \operatorname{var} X \cdot \rho^2_{xy} - r_1 C(\rho^2_{xy}). \quad (13)$$

The first order condition is

$$dC(\rho^2_{xy})/d\rho^2_{xy} = \tfrac{1}{2} A \operatorname{var} X / r_1 (1 + r_2). \quad (14)$$

Since C is convex, (13) and (14) imply that the market value maximizing firm will choose to produce an excessive amount of firm specific information.

That market value maximizing firms will not necessarily make Pareto optimal investment decisions has been observed and discussed in several papers.[21] Our observation concerning the inefficient choice of experiments is, of course, not unrelated. Consider (7) and (12). The source of the divergence is the difference in the trade-off between the expected value of X (or $C(\rho^2_{xy})$) and the generalized market risk of X in the price and welfare equations. The risk premium in (7) is twice what would be required by the consumer to accept additional variance. (See also Stiglitz [31, p. 44].) Thus the firm that maximizes market value will produce too little risk. This suboptimal production of risk takes two forms. First, too little investment will be undertaken.[22] Second, too much information will be produced concerning the chosen investment project.[23]

The source of the inefficiency has recently been elucidated by Mayshar [22] "... while correlations between firms are considerable, each firm's risky profits still include a major component of residual risk ... which is particular to the firm and independent of the risks of other firms.... It can thus be regarded as selling in the stock market a wholly differentiated 'product' and thus as possessing monopoly power.... Increasing the number of such incorporated independent companies will not eliminate that kind of inefficiency ...". The monopolistic power in the production of its risk induces each firm to produce too little risk and thus, as shown above, too much firm specific monitoring.[24]

Note that there is an additional monopolistic element in our model in that monitoring of the risks specific to a firm may be carried out only by that firm. For public information, even if entry to such monitoring were free no one else could find it profitable. More generally, whether the information is public or

[21] For a survey of this literature and a discussion on the use and implications of market value maximization as an objective of the firm, see Baron [2].

[22] See Mayshar [22, p. 191–3] and Jensen and Long [18, p. 159–60]. They assume that the pattern of returns across states of the world is independent of the level of investment.

[23] The inefficient investment decisions of market value maximizing firms imply that a first best optimum is impossible regardless of the monitoring decisions made. In this section we are pointing out that the monitoring decisions are generally inconsistent with a second-best optimum, i.e., for given investment decisions.

[24] Fama [7] and Merton and Subrahmanyam [23] have objected to the earlier findings of Stiglitz and Jensen and Long on the grounds of their "noncompetitive" assumptions regarding the capital market. As noted by Mayshar, however, it is not clear whether their "competitive" assumptions are of more than theoretical interest.

private, it seems reasonable to expect that a firm would control access to the monitoring of its specific risks.[25]

Market Information

For greater clarity we analyse the production of market information in the context of the Sharpe model.

Consider first the attitude of the representative investor towards market information. In Appendix 2, the following analogue of equation (12) is derived:

$$W(\rho^2_{z\Phi}) \sim -\exp\left[-\frac{A}{1+r_2+r_1r_2}\left\{E(X+X') - r_1r_2C(\rho^2_{z\Phi}) - \frac{1}{2}A\left[\operatorname{var}(X+X') - \frac{r_2}{1+r_2}\operatorname{var}\Phi\cdot(a+a')^2\rho^2_{z\Phi}\right]\right\}\right]. \quad (15)$$

The social value of information varies directly with A, var Φ and $(a + a')^2$. The latter term provides a measure of the significance of Φ in determining aggregate cash flows. The value of information is small when a and a' have opposite signs essentially because then $\operatorname{cov}(X, X' \mid z)$ exceeds $\operatorname{cov}(X, X')$. (See footnote 18)

Turn to the production of market information by market value maximizing firms. Compare (8) and (15). As in the case of specific information, the social and private valuations of systematic risk differ by a factor of 2. This is a reflection of the monopolistic element in a situation where cash flows are differentiated, i.e. not perfectly correlated. Since information acts via its effect on market risk, there is a corresponding divergence between the social and private valuations of market information. Thus, as in the case of firm specific monitoring, there is an incentive for firms to produce too much market information.

However, there is clearly an offsetting incentive in the case of market information. Information can be viewed as an intermediate good used as an input by firms which "produce" cash flows with means and generalized market risks. Market information is available equally to all firms and is thus public in the sense of Samuelson [28]. Therefore, the well-known free riding problem exists. There exists an incentive for firms to let others produce the market information. Too little and in the extreme case, no market information would be produced. Alternatively, firms may internalize the externality and cooperate in conducting market experiments. It can be shown that the incentives provided by the monopolistic and public good aspects of the problem implies that firms will generally produce a sub-optimal amount of market information.[26]

[25] Besides information there are, of course, other tools available to the firm, such as its debt-equity ratio for example, that might under particular circumstances provide the firm with a degree of monopolistic power.

[26] The only complication in the formal analysis is the need to specify the degree to which different market experiments overlap one another and therefore, the total amount of market information provided by such experiments. Two polar cases where the monitoring activities of different firms exhibit, by technological necessity, perfect or no duplication, can be readily formulated and analysed. The analysis of optimal government intervention is also straightforward.

V. Summary and Conclusions

A theorem is proved establishing the existence of an intuitive measure of the informativeness of an experiment. The theorem is then applied to a formal analysis of the effects on asset prices of the temporal resolution of uncertainty. For asset specific information, the temporal resolution of uncertainty will unambiguously increase the market price of an asset even though it will cause the required rate of return over the initial period to increase. For market specific information the result is ambiguous, depending upon how information affects the covariance of different assets. Assuming that more informative experiments are more costly to perform, the welfare implications for the production of information are examined. For asset specific information, value maximizing firms will tend to produce more than the socially optimal level of information. For market information the existence of a free riding problem affects the level of production and in the extreme case zero information will be produced.

Appendix 1

This appendix contains some facts about the multivariate normal distribution which are critical for the above analysis (see Feller [11] or Goldberger [12]). A proof of the theorem of section II is also provided.

Let Z and X be vector random variables jointly normally distributed with mean vector $\begin{bmatrix} EZ \\ EX \end{bmatrix}$ and variance-covariance matrix $\sum$. Adopt the obvious notation and partition $\sum$ in the form

$$\sum = \begin{bmatrix} \sum_{zz} & \sum_{zx} \\ \sum_{xz} & \sum_{xx} \end{bmatrix}.$$

Then X conditional on $Z = z$ is also normally distributed with the distribution $N[EX + \sum_{xz}\sum_{zz}^{-1}(z - EZ), \sum_{xx} - \sum_{xz}\sum_{zz}^{-1}\sum_{zx}]$. Consider some special cases. If Z and X are scalars, then $E[X/z] = EX + \rho_{zx}(\text{var } X/\text{var } Z)^{1/2}(z - EZ)$, $\text{var}[X \mid z] = \text{var } X \cdot (1 - \rho_{zx}^2)$. Apply the result to the vector random variable (X, X', Z) where each component is a scalar. Then $\text{var}[X \mid z] = \text{var } X \cdot (1 - \rho_{xz}^2)$, $= \text{cov}(X, X' \mid z) = \text{cov}(X, X') - \rho_{xz}\rho_{x'z}(\text{var } X \cdot \text{var } X')^{1/2}$. Finally consider$(X, X', Y, Y')$ where each component is a scalar and $\text{cov}(Y, Y') = \text{cov}(X, Y') = \text{cov}(X', Y) = 0$. Define $Z = (Y, Y')$. It follows that $\text{var}[X \mid z] = \text{var } X \cdot (1 - \rho_{xy}^2)$, $\text{var } [X' \mid z] = \text{var } X' \cdot (1 - \rho_{x'y}^2)$, $\text{cov}(X, X' \mid z) = \text{cov}(X, X')$.

Let us now prove the theorem. Recall the notation in the statement of the theorem. For simplicity suppose that $EZ = EZ' = 0$. Define $Y \equiv \rho_{z'x}Z/\rho_{zx}$. Then $Y \mid X = x$ is $N(E[Z' \mid x], \rho_{z'x}^2 (1 - \rho_{zx}^2) \text{ var } Z/\rho_{zx}^2)$. Let θ be an independently distributed random variable with the distribution $N(0, \text{var } Z \cdot (1 - \rho_{z'x}^2/\rho_{zx}^2))$. Then $Z'/X = x$ has the same distribution as $\theta + Y \mid X = x$ or $\theta + ((\rho_{z'x}//\rho_{zx})Z) \mid X = x$. For the function m required in (1) simply take the density function corresponding to $N(z\rho_{z'x}/\rho_{zx}, \text{var } Z \cdot (1 - (\rho_{z'x}^2/\rho_{zx}^2))$. The appropriate version of (1) is readily verified.

Appendix 2

In this appendix we derive the solution of problem (2) and the equilibrium prices which the solution implies.

Consider the second period problem, (the maximization inside the expected value operator E_Z in (2)), for given w_2 and realization z of Z. Both savings (s_2) and portfolio (α_2, α_2', b_2) decisions must be made. We determine the latter conditional on s_2 and then determine the optimal s_2. The portfolio problem is

$$\max_{\alpha_2,\alpha_2'} - E_{X,X'|z} \exp[-A\{\alpha_2(X - r_2P_2(z)) + \alpha_2'(X' - r_2P_2'(z))\}] \qquad (16)$$

where the expected value is conditional on $Z = z$. Since X, X' and Z have a multivariate normal distribution, X and X' have a bivariate normal distribution conditional on $Z = z$. (We assume that neither X nor X' is certain given a realization of Z.) Therefore the solution of (16) is well-known. The asset prices implied by equilibrium in the market for each asset are described in (3). (Note that the random variables $P_2(Z)$ and $P_2'(Z)$ have a joint normal distribution.) Of course in equilibrium $\alpha_2 = \alpha_2' = 1$.

Return to the savings decision. It is determined as the solution of the problem

$$\max_{s_2} -\frac{1}{\delta}\exp[-Aw_2 + As_2] - \frac{1}{\delta^2}\exp\left[-Ar_2s_2 - \frac{1}{2}A^2\{\text{var}(X + X' \mid z\}\right] \qquad (17)$$

The solution of (17) is straightforward. The maximum value of the objective function is found to be $J(w_2, z)$ where

$$J(w_2, z) = -\frac{1}{\delta}\exp\left[\frac{-r_2}{1 + r_2}Aw_2\right] \cdot B,$$

$$B = \left(\frac{r_2}{\delta}\right)^{1/1+r_2}\left(\frac{1 + r_2}{r_2}\right)\exp\left[-\frac{1}{2}\frac{A^2}{1 + r_2}\text{var}(X + X' \mid z)\right] \qquad (18)$$

Note that B is nonstochastic by the properties of the normal distribution.

Now analyse the first period decisions in problem (2). For given s_1, the portfolio decisions are determined as solutions of the problem

$$\max_{\alpha_1,\alpha_1'} -\frac{1}{\delta}BE_ZJ[r_1s_1 + \alpha_1(P_2(z) - r_1p_1) + \alpha_1'(P_2'(z) - r_1p_1'), z], \qquad (19)$$

where the expected value is taken with respect to realizations z of Z. But J is a negative exponential function with absolute risk aversion measure $r_2A/(1 + r_2)$. Therefore (19) has a structure similar to (16). Equilibrium in the market for each asset implies the equilibrium asset prices in (4). In equilibrium $\alpha_1 = \alpha_1' = 1$.

First period savings solve the problem

$$\max_{s_1} - \exp[-Aw_1 + As_1] - \frac{1}{\delta}BC\exp\left[-\frac{r_2}{1 + r_2}Ar_1s_1\right],$$

$$\text{where} \quad C = \exp\left[-\frac{1}{2}\frac{A^2r_2^2}{(1 + r_2)^2}\text{var}(P_2 + P_2')\right]. \qquad (20)$$

The maximum value of the objective function is found to be

$$V(w_1) = -\exp\left[-\frac{Ar_1r_2}{1+r_2+r_1r_2}w_1\right]\left[\frac{BCr_1r_2}{\delta(1+r_2)}\right]^{\gamma}\left[\frac{1+r_2+r_1r_2}{r_1r_2}\right]. \tag{21}$$

where $\gamma = (1+r_2)/(1+r_2+r_1r_2)$.

Consider the case of asset specific information where $Z = (Y, Y')$, $\text{cov}(X, Y') = \text{cov}(X', Y) = \text{cov}(Y, Y') = 0$, and $\rho_{xy}^2 < 1, \rho_{x'y'}^2 < 1$. From (3) and the normality we obtain

$$P_2(z) = \frac{1}{r_2}\left\{EX + \rho_{yx}\left(\frac{\text{var}\,X}{\text{var}\,Y}\right)^{1/2}(y - EY) - A[\text{var}\,X\cdot(1-\rho_{xy}^2) + \text{cov}(X, X')]\right\} \tag{22}$$

and a corresponding expression for $P_2'(z)$. (6) follows immediately. (7) is implied by (4) and (6). Finally (12) follows from (21) and (6).

Turn to the case of market information where Z is a scalar. Suppose $\rho_{xz}^2, \rho_{x'z}^2 < 1$ so that X and X' have positive variances conditional on Z. Now

$$P_2(z) = \frac{1}{r_2}\left\{EX + \rho_{xz}\left(\frac{\text{var}\,X}{\text{var}\,Z}\right)^{1/2}(z - EZ) - A[\text{var}\,X\cdot(1-\rho_{xz}^2) + \text{cov}(X, X') - \text{cov}(X, Z)\,\text{cov}(X', Z)/\text{var}\,Z]\right\},$$

$$EP_2 = \frac{1}{r_2}\left\{EX - A[\text{var}\,X\cdot(1-\rho_{xz}^2) + \text{cov}(X, X') - \rho_{xz}\rho_{x'z}(\text{var}\,X\cdot\text{var}\,X')^{1/2}]\right\},$$

$$\text{var}\,P_2 = \frac{1}{r_2^2}\rho_{xz}^2\,\text{var}\,X, \quad \text{cov}(P_2, P_2') = \frac{1}{r_2^2}\rho_{xz}\rho_{x'z}(\text{var}\,X\cdot\text{var}\,X')^{1/2}. \tag{23}$$

(8) follows from (22) and (4). (9) is implied by the special structure of the Sharpe model. Similarly, (15) is implied by (21).

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