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DISCUSSION

GORDON C. RAUSSER*: The principal results of Anderson and Danthine can be summarized as follows: (1) in general, cash and future positions should be determined simultaneously; (2) the optimal hedge should be viewed as a sum of risk-minimizing position and a speculative position; (3) basis risk is not only important for physical goods which are equivalent to goods defined by prespecified futures contracts but as well for those goods for which no futures market exists; and (4) the allocation signal to be used by hedger in selecting among alternative production levels is a linear combination of his spot market price expectations and the relevant futures prices. These results, aside from the clarifications needed to incorporate basis risk, follow immediately from conventional portfolio theory.

To be sure, the paper's distinguishing feature is the explicit recognition of basis risk in the context of conventional portfolio theory. This recognition provides one of the more interesting implications of the A-D analysis in terms of existing literature. Specifically, in the absence of basis risk, the risk premium disappears; and the relevant price signal to be used by the hedger does not include the spot price expectations component. This result has been noted recently by Holthausen and Feder *et al.*, as well as previous work of A-D. All risk-averse firms in the market key their production decisions to the futures price; thus, there is a separation of the real production decision and the futures market position decision. The incorporation of basis risk, however, eliminates this separation.[1]

* Department of Agricultural and Resource Economics, University of California, Berkeley.

[1] Note that, if the relevant covariance matrix among the multiple cash and futures prices is singular, we have an equivalence to the absence of basis risk; and, once again, we have a separation of physical and futures market position decisions.

Aside the results on basis risk, what is the ultimate value of the analysis advanced in this paper? Two possible purposes for the analysis can be distinguished. First, the framework could be advanced as a market-based theory or as a prescriptive paradigm of choice for various types of hedgers and speculators. In what follows, I shall focus on each of these two purposes.[2] A more fully developed version of these comments is in the Rausser working paper cited in the references.

Market-Based Theory

Although A-D do not advance their formulation in the context of a market equilibrium formulation, the spirit of their analysis could be used for this purpose. Unfortunately, such an extension of their analysis would provide a market-based theory which is inadequate. Available empirical evidence, knowledge of existing institutions, and advancements in theoretical formulation suggest that a framework for futures market behavior must admit (a) several groups of market participants, both rational and irrational and informed and uninformed; (b) risk aversion; (c) wealth limitations and imperfect capital markets; (d) indivisible capital and choice selections; and (e) transaction and information costs. Transaction and information costs are particularly important in a futures markets' attempt to form the relevant information set. Given this information base, the market must also perform a second function, namely, transmit this information into a futures price. The efficiency with which this transmission is accomplished depends upon (a) through (e). The A-D formulation explicitly admits only risk aversion and, thus, cannot capture those price transmission errors which emanate from the remaining influences.

Progress has been made in the context of futures market behavior on isolating the potential influences of irrational market participants by Stein, uninformed market participants by Grossman, and heterogeneous information expectations and the importance of wealth by Figlewski. However, no one has yet dealt squarely with the influence of imperfect capital markets and indivisible choice selections or, for that matter, related all of the various possible sources of errors in a single comprehensive model.

Prescriptive Paradigm-Based Theory

It is difficult to imagine a situation in which a decision-maker evaluating positions in futures markets would find the formulation advanced by A-D useful. At best, the A-D analysis represents only a first cut at an operational empirical formulation. The simplifying assumptions that are imposed in the development of their framework for most behavioral units are unacceptable. These simplifying assumptions include: *a two-period formulation*, in the context of portfolio theory, the A-D framework could be generalized along the lines of Samuelson, Merton,

[2] A technical problem exists in the analysis provided by A-D which should be noted. They implicitly assume that their optimal conditions provide an interior solution. This is not insured and, in fact, one would expect that, in the context of multiple cash goods and multiple futures, that corner solutions would arise. This problem can be easily corrected by restating the necessary conditions via the Kuhn-Tucker formulation and thus admitting noninterior solutions.

and Stein's previous work[3]; *the mean-variance framework*, the relevant statistical distributions for most agricultural commodities are not two-parameter distributions and are most certainly not Gaussian (see the empirical evidence summarized by Day) and the remaining condition of quadratic utility implies well-known absurdities (Arrow, Pratt); *omission of production uncertainty*, a general formulation which allows for skewed distributions and decreasing absolute and relative risk aversion would indeed modify the A-D results if production uncertainty was recognized; *omission of wealth constraints and perfect capital markets*, no hedger or speculator with whom I am aware is faced with such a perfect world or any set of circumstances even remotely resembling such a a perfect world; and *future and spot price relationships*, unfortunately, these price relationships are viewed as totally random and thus neglect the causal mechanisms that exist between futures and spot markets.[4]

Empirical Applications

Although A-D's investigation of two illustrative examples is commendable, for the above reasons, these treatments leave much to be desired. The estimated random price relationships, the neglect of the life remaining on futures contracts, the neglect of seasonality, and what appears to be the disregard for existing institutions in these applications is bothersome. For the grain storage example, A-D fail to recognize that a futures markets for barley exists in Winnipeg. The specified static cost function is ludicrous; and the static price expectations that are assumed make this example artificial. In the case of the debt portfolio interest rate futures example, we are informed that many agents have a satisfactory way of selecting positions in the cash markets; and, thus, a risk-minimizing formulation can be employed. Unfortunately, they provide no evidence to justify this observation. They also argue that high liquidity and low transaction costs of organized futures markets lead such agents to revise future positions more frequently than cash positions. As noted above, their framework has nothing whatsoever to say about such influences. Perhaps more importantly, they provide us with no information on the robustness of their results to the arbitrary assumptions imposed.[5]

[3] An important issue here is the degree approximation offered by myopic decision rules to their dynamic optimization counterparts. Tesfatsion shows that the degree of approximation is improved the smaller the uncertainty in a system and the higher the correlation among the gains of different periods. Moreover, he finds the bound for the approximation error which is expressed in terms of absolute risk aversion, variance of the stochastic element, and the marginal gain.

[4] They are certainly subject to change. Fortunately, econometric techniques are available for effectively capturing such changes (Mundlak and Rausser).

[5] Another technical problem arises in the A-D analysis. They use R^2 to confirm the intuitive notion that, for a 2-year bond, the T-bill futures are preferable if only one future is to be employed, whereas, for the 10-year bond, the T-bond futures are preferable. This application of R^2 completely neglects the literature on the simultaneous determination of futures and spot prices. For such simultaneous determination, the R^2 provides no evidence whatsoever and may, in fact, be misleading.

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