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Author(s): Robert H. Litzenberger and Krishna Ramaswamy
Source: *The Journal of Finance*, Vol. 35, No. 2, Papers and Proceedings Thirty-Eighth Annual Meeting American Finance Association, Atlanta, Georgia, December 28-30, 1979 (May, 1980), pp. 469-482
Published by: Blackwell Publishing for the American Finance Association
Stable URL: http://www.jstor.org/stable/2327407

THE JOURNAL OF FINANCE • VOL. XXXV, NO. 2 • MAY 1980

Dividends, Short Selling Restrictions, Tax-Induced Investor Clienteles and Market Equilibrium

ROBERT H. LITZENBERGER and KRISHNA RAMASWAMY*

I. Introduction

THE PURPOSE of this paper is to examine the effects of tax-induced investor clienteles on capital asset prices. In their seminal work on corporate dividend policy, Miller and Modigliani (1961) argued that in a world where dividend income is taxed at a higher rate than capital gains, and tax rates vary across investors, corporations would adjust their dividend policies until, in equilibrium, such policies coincided with the desires of shareholders. That is, tax-induced shareholder clienteles would form, with low (high) yielding stocks being held by investors in high (low) marginal tax brackets. Black and Scholes (1974) argue for a 'supply effect', such that in equilibrium, a marginal change in a corporation's dividend distribution would leave its stock price unaffected.

Brennan (1970) was the first to derive an After-Tax Capital Asset Pricing Model (CAPM) that accounted for the differential taxation of dividend and capital gains, and for a progressive tax scheme. The model was later extended by Litzenberger and Ramaswamy (1979) to account for restrictions on investors' borrowing. While in both models investors' tax brackets do influence their portfolio choices, it turns out that in equilibrium the expected return per unit of dividend yield is a constant across securities and independent of dividend yield. It is shown in the present study that this result obtains only in the absence of any restrictions on short sales.

While Brennan took corporate dividend policies as exogenous, his analysis under pure exchange indicates that an equilibrium where corporations are paying positive dividends is inconsistent with the required return per unit of dividend yield being zero. Put another way, the weighted average (using investors' global risk tolerances as weights) of investors' marginal tax brackets determines the expected return per unit of dividend yield. Since corporations[1] and tax-exempt investors would be indifferent between dividends and capital gains while other investors have a preference for capital gains, this weighted average tax rate would be positive. Litzenberger and Ramaswamy (1979) demonstrate that an equilibrium where corporations are paying positive dividends is consistent with the expected return per unit of yield being zero, if there are dividend related

* Stanford University and Columbia University, respectively. We are grateful for helpful discussions with George Constantinides and Stephen Schaefer.

[1] Dividend income and short term capital gains are eligible for the 85 percent dividend exclusion. Corporations have the option of realizing all gains short term; therefore, the common assertion that corporations' (as investors) preference for dividends cannot be based solely on the existence of the 85 percent dividend exclusion and the corporate tax rates of long-term capital gains.

constraints on either borrowing or on the tax deductibility of interest on margin borrowing.

Recently, Miller and Scholes (1979) have argued that investors can escape the penalty tax on dividend income by sufficiently levering their purchase of equities so as to offset taxable dividends with interest deductions. They argue that any unwanted risk in this levered position can be removed by the purchase of tax-deferred insurance. Their model makes the distinction between accumulators who are assumed to hold all risky assets and to employ the above strategy and non-accumulators (who are assumed not to hold risky assets). The thrust of their argument is that the marginal tax rate *applicable to dividend income* is zero for everyone by virtue of strategies the tax code permits, and hence that dividend policy is irrelevant. Clearly, unless the expected after tax return to non-accumulators from holding each equity is less than the after tax rate of interest they would hold some equities:[2] and even if they are in low tax brackets they should affect the market equilibrium. It is an empirical matter, then, whether these strategies are effectively employed by the two investor groups.

More recently, Ross (1977) and Bhattacharya (1979) have argued that dividend policy could be viewed as a signalling mechanism whereby firms with profitable projects are able and willing to pay higher dividends in order to segregate themselves from firms with less profitable projects. They provide a rationale for value maximizing firms paying positive dividends when the required return per unit of dividend yield is positive.

In the first published empirical test of the effect of dividends on asset prices, Black and Scholes (1974) reported that their tests on the data did not find a significant effect. Subsequently, empirical evidence has been presented by Rosenberg and Marathé (1979), Litzenberger and Ramaswamy (1979), and Blume (1979) that supports the prediction of the After-Tax CAPM: that holding beta constant, before-tax expected returns are an increasing function of dividend yield. However, Litzenberger and Ramaswamy (1979) also presented preliminary evidence that this relationship is non-linear. They found that, holding beta constant, the incremental expected return per unit of dividend yield is a decreasing function of dividend yield. Using different procedures, Wells Fargo Investment Advisory Service (1979) documents a like finding of a non-linear relationship. This result is consistent with the Elton and Gruber (1970) evidence on the ex-dividend behavior of common stocks.

This finding is inconsistent with the Brennan and the Litzenberger and Ramaswamy versions of the After-Tax CAPM. It is shown in this paper that this finding is consistent with a capital market equilibrium under divergent personal tax rates and restrictions on short sales. A rigorous treatment of existing restrictions on short sales, and of the existing tax code is difficult. Indeed, the removal of the short sale assumption requires further assumptions on the tax scheme, investor preferences and/or return distributions for tractability. It makes the analysis considerably less elegant than the analyses in previous work. The

[2] See Arrow (1975) for a proof that a positive risk premium is a sufficient condition for any risk averse individual not to place all of his wealth in the riskless asset.

difficulty in modelling the effect of restrictions on short sales on capital market equilibrium does not, however, deny their importance.

Under restricted short sales it is shown that tax-induced clienteles form and the incremental expected return per unit of dividend yield is a decreasing function of dividend yield. A tax exempt bond is introduced and the dividend supply adjustment of corporations are analyzed. Empirical evidence is present that supports the "tax clientele CAPM".

II. Theory

In this section a "tax clientele CAPM" is derived under the assumption that no short sales are permitted. The following assumptions are made:

A1. There are no costs of transacting, no indivisibilities. Investors are price takers, and have a single period horizon.
A2. Dividends and interest income are taxed as ordinary income, capital gains are not taxed and personal interest payments are tax deductible; and
A3. Investors are not permitted to sell equities short.

A1 embodies competitive and frictionless market assumptions. In *A2* the assumption that there are no taxes on capital gains is made for tractability. *A3* is the central assumption of our analysis that gives different predictions from the Brennan (1970) and Litzenberger and Ramaswamy (1979) models.

Before introducing uncertainty, it is instructive to consider a simpler certainty model. This abstracts from the effects of covariances and concentrates on the role of dividends: it permits us to identify optimal portfolios for different tax clienteles in market equilibrium without knowledge of preferences. Under uncertainty it is not possible to identify the optimal portfolio (in equilibrium) of investors in specific tax brackets without specific knowledge of their preferences.

Certainty

In this section the following assumptions are added to the set *A1* to *A3*:
A4. Personal income taxes are progressive; and
A5. The return on equities is certain and corporations pursue an announced dividend policy. A (riskless) taxable bond exists and investors can lend at the bond interest rate,[3] and borrow at the bond interest rate subject to a margin constraint.

Let
W_k = the k-th investor's initial wealth;
t_k = the k-th investor's *average* tax rate, a function of his or her income;

[3] In this certainty framework the only distinction between a bond and a stock is that the bonds' total return is assumed to be interest and is taxed as ordinary income and the individual may borrow (i.e., short the bond) but is unable to short equities.

r_f = the rate of return on the taxable bond;
R_i = the rate of return on the i-th equity security, $i = 1, 2, \cdots, N$;
d_i = the dividend yield on the i-th security,
X_{ik} = the fraction of the k-th individual's wealth in the i-th security;
Y_k = the k-th individual's end of period pre-tax ordinary income.

From *A4*, the k-th individual's average tax rate is written as

$$t_k = g(Y_k), \tag{1}$$

a function of the individual's income. By a progressive scheme, it is meant that taxes paid are monotone increasing and convex in pre-tax income, and the tax scheme is not confiscatory:

$$0 \leq \frac{d(t_k Y_k)}{dY_k} = t_k + Y_k g'(Y_k) < 1 \quad \text{for} \quad Y_k \geq 0,$$

and $\dfrac{d^2(t_k Y_k)}{dY_k^2} > 0$ for $Y_k > 0$. Also, $g(0) = g'(0) = 0$. For use later, define the k-th individual's marginal tax rate as

$$T_k = t_k + Y_k g'(Y_k). \tag{2}$$

The investor's end of period taxable income is

$$Y_k = W_k\{(1 - \textstyle\sum_{i=1}^N X_{ik})r_f + \sum_{i=1}^N X_{ik}\, d_i\}. \tag{3}$$

where $(1 - \sum_{i=1}^N X_{ik})$ is the fraction of the individual's wealth in the taxable bond.

The investor's problem is to maximize the after-tax rate of return on his or her portfolio, subject to non-negativity conditions on X_{ik}, $i = 1, 2, \cdots, N$, and a margin constraint on borrowing $\sum_{i=1}^N X_{ik} \leq \alpha \leq 1$.

$$\text{Max}_{\{X_{ik},\ \forall_i\}}\ L$$
$$= r_f(1 - t_k) + \textstyle\sum_{i=1}^N X_{ik}[R_i - r_f - t_k(d_i - r_f)] - a_k\left[\sum_i X_{ik} - \alpha\right] + \sum_{i=1}^N \lambda_{ik} X_{ik} \tag{4}$$

At the optimum, for any asset i held long in the k-individual portfolio,

$$R_i - r_f = a_k + T_k(d_i - r_f) \quad \text{if} \quad X_{ik} > 0, \tag{5}$$

and for assets not held in his portfolio,

$$R_i - r_f \leq a_k + T_k(d_i - r_f) \quad \text{if} \quad X_{ik} = 0. \tag{6}$$

If the strict inequality held, the k-th individual would desire to short the security, but is unable to do so.

Note that a_k, the shadow price on the margin constraint, is equal to $R_k - r_f - T_k[d_k - r_f]$, the after-tax excess rate of return on the k-th individual's optimal equity portfolio. This shadow price would be identical for all individuals in the same marginal tax bracket. Equities would be held by the tax clientele willing to bid the highest price or accept the lowest before tax return. It follows from relation (6) that if in market equilibrium individuals who are not in the lowest tax bracket clientele hold equities with positive excess dividend yields, the a_k's would be inversely related to the T_k's. It also follows from relation (6) that if the market equilibrium individuals who are not in the highest tax bracket clientele hold equities with negative excess dividend yields the a_k's would be directly related to the T_k's.

Assume that in market equilibrium individuals are in G different marginal tax brackets and that each of these G tax clienteles has the same amount of wealth invested in equities. The maximum price for the Gth of the market value of all stocks with the highest dividend yields would be paid by the investors in the lowest tax bracket. The maximum price for the Gth of the market value of stocks with the lowest yields would be paid by the investors in the highest brackets, and so on. The resulting equilibrium would exhibit a linear relationship between returns and yields within each group:

$$R_i - r_f = \sum_{g=1}^{G} \delta_{ig} a_g + \sum_{g=1}^{G} T_g \delta_{ig}(d_i - r_f), \tag{8}$$

where $\delta_{ig} = 1$ if stock is in group g and 0 otherwise. It is straightforward in this simple case to identify the group a stock belongs to in market equilibrium. If group 1 (group G) corresponds to the portfolio of equities which has the lowest (highest) dividend yield, and these portfolios have equal market value, then a ranking of all equities by yield provides the group that contains a given stock. Under this (piecewise linear) tax scheme, the hypothesis is that $1 > T_1 > T_2 \cdots > T_G \geq 0$.

Uncertainty

Under uncertainty, it is no longer possible to isolate the optimal portfolio for an investor tax clientele in market equilibrium by knowledge of yield alone. It is necessary now to consider investor preferences for risk and after-tax returns. In doing so, a version of the mean-variance after-tax Capital Asset Pricing Model that accounts for restrictions on short selling is derived. Under uncertainty, the demand for assets is influenced by their diversification potential (covariance with optimal portfolio) as well as their dividend yields.

In addition to assumptions *A1*, *A2* and *A3*, the following assumptions are made:

A6. Security rates of return have a multivariate normal distribution;
A7. Investors have von Neumann-Morgenstern utility functions that are monotone increasing and strictly concave functions of end of period after-tax wealth.
A8. The dividends are paid at the end of the period and are known with certain *ex ante*; and
A9. Individuals have homogeneous expectations.

These assumptions permit the characterization of the investor k's portfolio problem as the maximization of a function $f_k(W_k\mu_k, W_k^2\sigma_k^2)$ defined over the mean $(W_k\mu_k)$ and variance $(W_k^2\sigma_k^2)$ of after-tax end of period wealth. By assumption *A7* the partial derivatives of f_k with respect to the first (second) argument is positive (negative). By assumption *A8* the variance of pre-tax end of period wealth is identical to the variance of post-tax end of period wealth.

Some additional notation is necessary.

Let

$$\tilde{R}_i = \text{the total return on stock } i \text{ over the period}$$
$$E(\tilde{R}_i) = \text{the expectation of } \tilde{R}_i,$$
$$\text{and } \sigma_{ij} = \text{Cov}(\tilde{R}_i, \tilde{R}_j).$$

Then the portfolio problem for investor k in average tax bracket t_k is

$$\text{Max}_{\{X_{ik} \geqq 0\}} f_k(W_k\mu_k, W_k^2\sigma_k^2) \tag{9}$$

where

$$\mu_k = r_f(1 - t_k) + \sum_i X_{ik}[E(\tilde{R}_i) - r_f - t_k(d_i - r_f)] \tag{10}$$

and

$$\sigma_k^2 = \sum_i \sum_j X_{ik}X_{jk}\sigma_{ij}. \tag{11}$$

The Kuhn-Tucker conditions for this problem are

$$E(\tilde{R}_i) - r_f - T_k(d_i - r_f) - \left[\frac{-2f_{2k}W_k}{f_{1k}}\right] \sum_j X_{jk}\sigma_{ij} \leqq 0 \qquad \text{for} \quad i = 1, 2, \cdots, N. \tag{12}$$

Note that $[-2f_{2k}/f_{1k}] = W_kE(-U'')/E(U')$ and is denoted as the individual's global relative risk aversion (see Gonzalez-Gaverra (1973) and Rubinstein (1973)).

It follows directly from the first order conditions that in market equilibrium a sufficient condition for all individuals in the same marginal tax bracket (T_g) to hold identical risk asset portfolios is either (A) each individual's tax bracket is not a function of his income, or (B) the individuals in the same tax bracket have identical initial wealths. Assume that either (A) or (B) is satisfied for individuals in marginal tax bracket T_g: the first order conditions for assets held long in the optimal risk asset portfolio are

$$E(\tilde{R}_i) - r_f - T_g(d_i - r_f) = \left[\frac{-2f_{2k}}{f_{1k}}\right] W_k(\textstyle\sum_j X_{jk})\sigma_{ig} \tag{13}$$

for all i held in the optimal portfolio, where $\sigma_{ig} = (\sum_j X_{jk})^{-1} \sum_j X_{jk}\sigma_{ij}$.

The wealth of investors in group g invest in risky assets is

$$\sum_k W_k(\textstyle\sum_j X_{jk}) = W_g, \tag{14}$$

The conservation relation in the market for risky assets is

$$\sum_g W_g\tilde{R}_g = W_m\tilde{R}_m, \tag{15}$$

where $\tilde{R}_g = (\sum_j X_{jk})^{-1} \sum_j X_{jk}\tilde{R}_j$, for any k in group g, and $\tilde{R}_m$ = the return on the market portfolio risky assets. Using the relation (14) and aggregating (13) over all individuals k in marginal tax bracket T_g, gives relation (16)

$$E(\tilde{R}_i) - r_f = b_g \beta_{ig} + T_g(d_i - r_f), \tag{16}$$

for all assets i held in group g's optimal portfolio, where

$$b_g \equiv \left[\sum_{k \in g} \left(\frac{-2f_{2k}}{f_{1k}} \right)^{-1} \right]^{-1} W_g \operatorname{Var}(\tilde{R}_g), \tag{17}$$

and

$$\beta_{ig} \equiv \operatorname{Cov}(\tilde{R}_i, \tilde{R}_g)/\operatorname{Var}(\tilde{R}_g). \tag{18}$$

If the assumption is made that the covariances of *all* securities are attributable to their common covariances with the market portfolio return $\tilde{R}_m$, then

$$\beta_{ig} = \beta_{im}\beta_{mg}, \quad \text{where} \quad \beta_{mg} \equiv \operatorname{Cov}(\tilde{R}_m, \tilde{R}_g)/\operatorname{Var}(\tilde{R}_g) \tag{19}$$

and relation (16) reduces to

$$E(\tilde{R}_i) - r_f = b'_g \beta_{im} + T_g(d_i - r_f) \tag{20}$$

where $b'_g \equiv b_g \beta_{mg}$. This coefficient b'_g on the "market" beta will be constant across all groups if (C) the harmonic mean of global relative risk aversion for group g, $\left[\sum_{k \in g} \left(\frac{-2f_{2k}}{f_{1k}} \right)^{-1} \right]^{-1}$ is the same across all groups, the groups all have identical amounts invested in risky assets, and the covariance $\operatorname{Cov}(\tilde{R}_g, \tilde{R}_m)$ is identical across all groups.

For a complete characterization of the market equilibrium conservation, relation (15) must hold in addition to relation (16). In general, the composition of each group's optimal portfolio cannot be identified in market equilibrium without specific knowledge of preference and distributions. However, under the special case when condition (C) holds, the equilibrium may be characterized as

$$E(\tilde{R}_i) - r_f = b\beta_{im} + \sum_g \delta_{ig} T_g(d_i - r_f), \tag{21}$$

where $\delta_{ig} = 1$ if security i is in group g and 0 otherwise. In this special case δ_{ig} can be identified *ex ante*: in the equilibrium the highest tax bracket group would hold $\frac{1}{G}$ fraction of the market value of all risky stocks with the lowest yields, the next tax bracket group would hold $\frac{1}{G}$ fraction with the next lowest yields, and so on. In this special case under uncertainty as in the certainty case, we can empirically identify the optimal portfolios for each clientele in market equilibrium.

Existence of a tax exempt riskless bond

This section considers the implications of the existence of a tax-exempt municipal bond. Assume that investors are prohibited from selling municipal bonds short, and that they may not deduct interest on margin borrowing when they hold municipal bonds. If the kth investor's marginal tax bracket exceeds the tax bracket implicit in the municipal bond rate, the investor would prefer holding municipals to holding taxable riskless bonds. That is, if

$$T_k > 1 - \frac{r_{\text{mun}}}{r_f} \equiv T_{\text{mun}}, \tag{22}$$

(where r_{mun} is the yield and T_{mun} is the implicit marginal tax bracket on the municipal bond) the after-tax marginal rate of return on the municipal bond would exceed the after-tax marginal return on the taxable bond. The first order conditions for individuals who do not hold municipals is identical to condition (12). The first order condition for an individual who holds municipal bonds is

$$E(\tilde{R}_i) - r_f - T_k d_i + T_{\text{mun}} r_f - \left[\frac{-2 f_{2k} W_k}{f_{1k}}\right] \sigma_{ik} \leqq 0 \ \forall_i. \tag{23}$$

A sufficient condition for low income individuals ($T_k \leqq T_{\text{mun}}$) in the same marginal bracket to hold identical portfolios of risky assets is either (A) or (B), as discussed above. These conditions are also sufficient for all high income individuals ($T_k > T_{\text{mun}}$), who are lenders, to hold the same optimal risk asset portfolio. The subsequent analysis assumes that all high tax bracket individuals hold municipals. Aggregating over all individuals in tax brackets T_g, $g = 1, 2, \cdots, h, \cdots, G$.

$$E(\tilde{R}_i) - r_f$$
$$= \sum_{g=1}^{h-1} \delta_{ig}(T_g d_i - T_{\text{mun}} r_f) + \sum_{g=h}^{G} \delta_{ig} T_g (d_i - r_f) + \sum_{g=1}^{G} \delta_{ig} b_g \beta_{ig}, \tag{24}$$

where h is the group index with $T_h = T_{\text{mun}}$.

Under the assumption of the single model and condition (C), $b_g = b$, a constant across all groups. Then the equilibrium reduces to

$$E(\tilde{R}_i) - r_f = \sum_{g=1}^{h-1} \delta_{ig}(T_g d_i - T_{\text{mun}} r_f) + \sum_{g=h}^{G} \delta_{ig} T_g (d_i - r_f) + b\beta_{im}, \ \forall_i \tag{25}$$

Note that for stocks held in the optimal portfolios of all high income groups ($g = 1, 2, \cdots, h-1$), the same coefficient occurs on the riskless rate. The coefficient on dividend yield decreases as the group income decreases ($T_1 > T_2 > \cdots > T_G$).

Supply Effects

This section considers the supply adjustments that value maximizing firms would make, given the security market pure exchange equilibrium discussed in the previous section.

It is assumed that a change in a firm's dividend policy is sufficiently large so as to shift the stock into the portfolio of an adjacent group of investors, and that firms are price takers with respect to all T_g's and with respect to b. Consider a discrete change in dividend yield Δd_i, which moves the ith firm from the midpoint in the range of yield of one high income group ($T_g > T_h$) to the mid-point of an adjacent high income group. The change in the expected return is

$$\Delta(T_g d_i) = T_g \Delta d_i + d_i \Delta T_g + \Delta T_g \Delta d_i. \tag{26}$$

If Δd_i is greater (less) than zero then ΔT_g is less (greater) than zero. For the change in required return to be zero, the change in the tax rate ΔT_g must be

$$\Delta T_g^* = -T_g \Delta d_i/(d_i + \Delta d_i) \quad \text{for} \quad T_g > T_h. \tag{27}$$

Value maximizing firms would make supply adjustments until the difference in the tax brackets between adjacent groups of high tax bracket investors was equal to ΔT_g^*. The corresponding difference between the tax brackets of adjacent low income groups is

$$\Delta T_g^* = -T_g \Delta d_i/(d_i - r_f + \Delta d_i) \quad \text{for} \quad T_g \leqq T_h. \tag{28}$$

It is assumed that the denominator is positive for the low tax bracket group. This seems plausible since this clientele would hold high dividend yield equities.

Finally, consider the change in required return for change in yield between the mid-point of the group $h - 1$ (lowest high income group) to the mid-point of the group h (highest low income group).

$$\Delta(T_{h-1} d_i) = T_{h-1} \Delta d_i + d_i \Delta T_{h-1} + d_i \Delta T_{h-1} - r_f(T_{h-1} - T_{\text{mun}} + \Delta T_{h-1})$$

Solving for a zero change in expected return,

$$\Delta T_{h-1}^* = -(T_{h-1} \Delta d_i - r_f T_{h-1} + r_f T_{\text{mun}})/(d_i + \Delta d_i - r_f) \tag{29}$$

This analysis suggests that in equilibrium discrete changes in dividends between the mid-points of dividend yield groups may not affect the expected return. The focus of our analysis on discrete changes is arbitrary: indeed the possibility of intra-group dividend yield changes suggests it is not an equilibrium. The role of dividend increases in signalling future profitability was ignored. The existence of short selling restrictions tend to mitigate the tax effects of dividend changes: and suggests the cost of signalling through dividend increases is less than that implied by previous versions of the After-Tax CAPM.

Empirical Tests

The empirical tests presented here assume that individuals fall into five tax clienteles and that each clientele holds one-fifth of the market value of all New York Stock Exchange (NYSE) stocks. The portfolios of the clienteles are assumed to correspond to the optimum portfolios in market equilibrium under certainty: that is, having ranked available stocks in a given year by their (past) annual dividend yield, five portfolios are formed by proceeding down this ranking until a fifth of the market value of stocks is reached, and then until two-fifths is reached, and so on. The first (group) portfolio is then a value weighted portfolio of the lowest dividend yield stocks, comprising a fifth of the market value of all stocks. The next portfolio (a fifth of the market value) contains the next lowest dividend yield stocks, and so on.

The most general model of the tax-clientele CAPM in the absence of tax exempt bonds, is

$$E(\tilde{R}_i) - r_f = b_g \beta_{ig} + T_g(d_i - r_f) \; \forall \; i \in g, \qquad g = 1, 2, \cdots, 5. \tag{30}$$

In testing the above model using cross-sectional data, the econometric model used, in a given month t, is

$$\tilde{R}_{it} - r_{ft} = \gamma_{0g} + \gamma_{1g}\beta_{1gt} + \gamma_{2g}(d_{it} - r_{ft}) + \tilde{\epsilon}_{igt}, \tag{31}$$

where: $\tilde{R}_{it}$ = the total return on stock i in month t,

$\tilde{\epsilon}_{igt} = \tilde{R}_{it} - E(\tilde{R}_{it})$,

r_{ft} = the return on the riskless asset in month t,

$\beta_{igt} = \text{Cov}(\tilde{R}_{it}, \tilde{R}_{gt})/\text{Var}(\tilde{R}_{gt})$,

$$d_{it} = \begin{cases} 0 & \text{if } t \text{ was not an ex-dividend month for } i \text{ or the dividend was non recurring and not announced prior to } t; \\ \dfrac{D_{it}}{P_{it-1}} & \text{if } t \text{ was an ex-dividend month for } i \text{ and the dollar dividend } D_{it} \text{ was announced earlier;} \\ \dfrac{\hat{D}_{it-1}}{P_{it-1}} & \text{if } t \text{ was a recurring ex-dividend month for } i \text{ and the dividend per share was not announced earlier,} \end{cases}$$

$\hat{D}_{it}$ = previous (going back at most 12 months) recurring taxable dividend per share adjusted for any change in number of shares in the interim,

P_{it-1} = Price per share of security i at the end of month $t - 1$.

Note that β_{igt} is the 'group' beta of stock i with respect to its group portfolio return $\tilde{R}_{gt}\cdot\gamma_{1g}$ and γ_{2g} corresponds to b_g and T_g, respectively. The testable hypotheses are that $\gamma_{1g} > 0$, from the assumption that individuals are risk averse, and $1 > \gamma_{21} > \gamma_{22} > \cdots, > \gamma_{25} \geqq 0$. The theory predicts that in absence of margin restrictions, $\gamma_{0g} = 0$.

The covariance matrix of the vector disturbances in each cross sectional regression is specified from the Sharpe single index model, where the index is the group portfolio return $\tilde{R}_{gt}$.

$$\tilde{R}_{it} - r_{ft} = \alpha_{it} + \beta_{igt}(\tilde{R}_{gt} - r_{ft}) + \tilde{e}_{it} \tag{32}$$

where $\text{Cov}(\tilde{e}_{it}, \tilde{e}_{jt}) = \begin{cases} 0 & i \neq j, \\ S_j^2 & i = j. \end{cases}$

The econometric procedures are identical to those detailed in Litzenberger and Ramaswamy (1979). It is shown there that under the assumption of the Sharpe single index model described above the Generalized Least Squares estimator is obtained by deflating the variables in (31) by S_i and using an Ordinary Least Squares procedure. The resulting estimators for γ_{0g}, γ_{1g} and γ_{2g} have interpretations as returns on specific portfolios.

The true group betas, β_{igt}, are unobservables. An estimate of the beta of stock i with respect to its group portfolio, using data prior to month t can be obtained. Assuming that the groups are correctly identified, the variance of the measurement error in the estimate of the group beta can be estimated. If the observed betas were used in the above GLS procedure, the estimates of γ_{0g}, γ_{1g} and γ_{2g} would be inconsistent. The existence of measurement error in the beta estimates results in their cross-section variance being an upward biased estimate of the cross-sectional variance in the true betas. Assuming that the variance of the measurement error in beta is known, the cross sectional variance in the true betas is replaced by the difference in the variation of the observed betas and the variance of the measurement error. It is shown in Litzenberger and Ramaswamy

(1979) that this estimator is consistent, and under certain conditions, is the maximum likelihood estimator.

The cross sectional regression in each month results in a series of estimates $\{\hat{\gamma}_{0gt}, \hat{\gamma}_{1gt}, \hat{\gamma}_{2gt}, t = 1, 2, \cdots, T\}$ for each group g. Under the assumption that these monthly coefficients are serially uncorrelated simple time series averages are the most efficient overall estimators.

$$\hat{\gamma}_{rg} = \sum_t \frac{\hat{\gamma}_{rgt}}{T}, \tag{33}$$

Similar econometric procedures are employed to test the hypothesis with 'market' betas:

$$\tilde{R}_{it} - r_{ft} = \gamma'_{0g} + \gamma'_{1g}\beta_{imt} + \gamma'_{2g}(d_{it} - r_{ft}) + \tilde{\epsilon}_{it}. \tag{34}$$

The following model, which constrains the coefficient on market betas to be identical across all groups, is also estimated

$$\tilde{R}_{it} - r_{ft} = \gamma''_0 + \gamma''_1\beta_{imt} + \sum_g \delta_{ig}\gamma''_{2g}(d_{it} - r_{ft}) + \tilde{\epsilon}_{it} \tag{35}$$

Blume (1979) reports that zero dividend yield stocks tend to have higher rates of return than those predicted by After-Tax CAPM. To test this hypothesis, a dummy varianble (δ_{i0}) is introduced into the above model. δ_{i0} is 1 if stock i had a zero yield last calendar year, and 0 otherwise.

$$\tilde{R}_{it} - r_{ft} = \gamma''_0 + \gamma''_1\beta_{imt} + \sum_g \delta_{ig}\gamma''_{2g}(d_{it} - r_{ft}) + \gamma''_3\delta_{i0} + \tilde{\epsilon}_{it}. \tag{36}$$

Results

Data on security returns (R_{it}) were obtained from the monthly return tapes provided by the Center for Research in Security Prices at the University of Chicago. The same service provides a value weighted index of all the securities on the tape, which was used as a proxy for the market return (R_{mt}). From January 1931 until December 1951, the monthly return on high grade commercial paper was used as the return on the riskless asset: from January 1952 until December 1978 the return on a Treasury Bill (with one month to maturity) was used for r_{ft}.

The dollar dividends paid to each available security in a given calendar year, divided by the end of year security price was obtained for each year from 1930 until 1977. A security which was available for less than 9 months in the calendar was discarded in that year. These past yields were ranked in ascending order in each year from 1931 to 1978. The five groups were formed by proceeding down this ranking until a fifth, two fifths, etc., of the market value of all those stocks at the beginning of the year were reached. The market value of a threshold security (whose value bridged the cut-off at the fifths of the total value) was allocated so as to keep group values equal: the threshold security itself was assigned membership to the next group for the purpose of the subsequent cross-sectional regressions.

The group 'membership' is maintained throughout a given year, and the monthly returns for each group portfolio is constructed (R_{gt}). This series is then available for each of the five groups, from $t = 1$ (January 1931) until $t = 576$ (December 1978).

Group betas (β_{igt}) and market betas (β_{imt}) are then obtained for $t = 61$ (January 1936) to $t = 576$ from a time series regression of security on the group portfolio and market, respectively: in this time series regression, 60 months of data prior to month t are used. These regressions provide estimates of S_i^2 and of the variance of the measurement error in beta. This data is then used to conduct the cross sectional regressions in each month from January 1936 until December 1978.

In Tables 1A and 1B, the estimates of the tax clientele model parameters (with group betas and market betas, respectively) are provided. The coefficients on the market betas tend to have higher t-values than the corresponding coefficients on the group betas, except for the fifth group. This may be attributable to the naive procedure used to classify stocks into the optimal portfolios for the tax clienteles. With the exception of group II, the coefficients on beta are approximately the same order of magnitude across groups. This is consistent with the prediction of

Table 1A

Pooled Time Series and Cross Section Estimates, 1936–1978, of Tax Clientele CAPM, Five Groups

GROUP BETAS

$$E(\tilde{R})_i) - r_f = \gamma_0 + \gamma_1\beta_{ig} + \gamma_2(d_i - r_f)$$

	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$
Group I	0.00709	0.00326	0.688
	(3.24)	(0.96)	(4.71)
Group II	0.00826	−0.00070	0.545
	(3.35)	(−0.21)	(5.52)
Group III	0.00206	0.00486	0.313
	(0.87)	(1.63)	(4.16)
Group IV	0.00026	0.00698	0.309
	(0.11)	(2.46)	(5.08)
Group V	0.00248	0.00585	0.044
	(1.39)	(2.28)	(1.13)

Table 1B

MARKET BETAS

$$E(\tilde{R}_i) - r_f = \gamma'_0 + \gamma'_1\beta_{im} + \gamma'_2(d_i - r_f)$$

	$\hat{\gamma}'_0$	$\hat{\gamma}'_1$	$\hat{\gamma}'_2$
Group I	0.00414	0.00478	0.699
	(1.79)	(1.79)	(4.60)
Group II	0.00637	0.00133	0.524
	(2.82)	(0.50)	(5.53)
Group III	0.00175	0.00527	0.318
	(0.79)	(1.98)	(4.29)
Group IV	0.00091	0.00659	0.319
	(0.44)	(2.46)	(5.28)
Group V	0.00275	0.00540	0.058
	(1.56)	(2.19)	(1.50)

t-statistics are in parenthesis under each coefficient.

Table 2

Differences Between Adjacent Group Tax Brackets

	$\gamma_{21} - \gamma_{22}$	$\gamma_{22} - \gamma_{23}$	$\gamma_{23} - \gamma_{24}$	$\gamma_{24} - \gamma_{25}$
Group Betas	0.143	0.231	0.004	0.265
	(0.80)	(1.94)	(0.04)	(4.01)
Market Betas	0.175	0.206	−0.002	0.260
	(0.96)	(1.76)	(−0.02)	(3.94)

t-values are in parenthesis under each coefficient.

Table 3

Pooled Time Series and Cross Section Estimates of Tax Clientele CAPM, 1936–1978

$$E(\tilde{R}_i) - r_f = \gamma_0'' + \gamma_1''\beta_{imt} + \sum_g \delta_{ig}\gamma_{2g}(d_{it} - r_{ft}) + \gamma_3''\delta_{i0}$$

	$\hat{\gamma}_0''$	$\hat{\gamma}_1''$	$\hat{\gamma}_{21}''$	$\hat{\gamma}_{22}''$	$\hat{\gamma}_{23}''$	$\hat{\gamma}_{24}''$	$\hat{\gamma}_{25}''$	$\hat{\gamma}_3''$
Panel A	0.00243	0.00513	0.373	0.501	0.386	0.325	0.042	
	(1.64)	(2.21)	(3.26)	(4.79)	(5.31)	(5.22)	(0.97)	
Panel B	0.00317	0.00423	0.449	0.483	0.372	0.324	0.044	0.00336
	(2.14)	(1.91)	(3.32)	(4.72)	(5.23)	(5.24)	(1.04)	(2.29)

t-statistics are in parenthesis under each coefficient.

condition (C) discussed in Section 2. The coefficients on the excess yield variable decline from group I to group III, and from group IV to group V. The implied tax brackets on Groups III and IV are very close. The average difference between monthly implied tax rates for adjacent groups, and the associated *t*-values are given in Table 2.

These results are consistent with the tax clientele CAPM, and with the prior empirical findings of Elton and Gruber (1970) and Litzenberger and Ramaswamy (1979).

The intercept terms are significantly positive for groups I and II and positive but insignificant for the other groups. Possible explanations for the average positive intercept are the misspecification of the group and market portfolios (see Mayers (1972), Sharpe (1977) and Roll (1977)), or that the true betas follow a mean reverting process. A possible explanation for the higher than average intercepts on groups I and II is that the implied coefficient on "$-r_f$" may be less than that on d_i for these groups: this would be consistent with the implication of the tax clientele model when a tax exempt bond exists.

The results of the econometric model (35) with the coefficient on market beta constrained across groups are given in Table 3A. The coefficient on the market beta is significantly positive, suggesting that condition (C) is a reasonable first order approximation. The implied tax rate for group I is now less than that for group II: however, in many of the early months group I contained very few dividend paying stocks. To account for the effect of non-dividend paying stocks the econometric model was fitted with a dummy variable, as in relation (36). These results are also presented in Table 3. The coefficient on this dummy variable is significantly positive, indicating that the required return on non-dividend paying stocks is higher than would be predicted by the tax clientele

CAPM. This is consistent with the Blume (1979) results. In a capital market where short selling is restricted, non-dividend paying stocks would have to pay a premium to attract (unrestricted) investors to absorb the stock, which some investment units may be prohibited from acquiring. This effect is analogous to the effect of institutional restrictions on portfolio and bond prices that was developed by Glenn (1976). Note that the introduction of the dummy variable does increase the estimate of the group I tax bracket, though it is still less than that of group II.

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DISCUSSION

PATRICK J. HESS*: Litzenberger and Ramaswamy (hereafter LR) are interested in providing a theoretical rationale and empirical tests for tax induced clientele

* Ohio State Univesity. I am thankful to S. Buser and G. Hite for their insightful comments.