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Optimal Liquidation of Assets in the Presence of Personal Taxes: Implications for Asset Pricing

GEORGE M. CONSTANTINIDES and MYRON S. SCHOLES*

I. Introduction

WITH A TAX on realized capital gains, investors realize capital losses and defer capital gains. However, if individuals realize capital gains to consume or for other reasons (such as a cash stock merger) they may defer capital gains tax payments by hedging. The hedge produces a capital loss in one tax year and an equal capital gain in the next tax year. If the hedge is operational, capital gains tax is deferred until death and the capital gains tax is avoided entirely.¹ With this idealized hedge, the effective marginal tax rate on both short-term and long-term capital gains and losses is zero. Also, investors are indifferent to changes in the actual capital gains rate.

Call options and commodity futures contracts are used to construct these hedges. Without transactions costs (no bid-ask spreads and brokerage commission fees), perfect hedges can indeed be constructed and used to defer capital gains tax, despite IRS regulations designed to discourage their use. Transactions costs may dissipate the hedge's tax benefits, and therefore hedges reduce, but do not eliminate, the capital gains tax's effect on consumption and investment.

The loophole in the tax code which enables tax deferral, at least in the absence of transactions costs, arises from the institutional structure of both the options exchange and the commodity futures exchange. A contract writer is not matched with a particular contract buyer. For example, if a call price falls, the call buyer can sell the call, realize a capital loss, and immediately buy a new call. The call writer is not forced to realize a corresponding capital gain. There is a net tax gain at the government's expense. Note that the net tax gain would be zero if the buyer and writer were matched. A tax gain for the buyer would be canceled by a tax loss for the writer.

With a blunted but not eliminated capital gains tax, the capital gains tax's effect on optimal investor behavior is quite complex. Under the simplifying assumption that short-term and long-term rates on capital gains and losses are equal, the optimal liquidation policy is to realize capital losses and defer capital gains. With this "lock-in" effect, it is important to study welfare effects of given

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¹ Even if the current IRS regulations were to change so that beneficiaries pay tax on unrealized capital gains, the marginal tax rate on capital gains is still reduced by deferring the tax payment to a future date.

changes in anticipated inflation, the capital gains tax rate, dividends and stock variability.

II. Illustration of a Simple Hedge

Let us outline a simple hedge, allowing capital gains' deferral from date $t = 1$ to date $t = 2$. Assume $t = 1$ is December, the year's last tax month, and $t = 2$ is January in the next tax year. The months were picked to defer capital gains tax realization from one year to the next. If the hedge works for one year, the investor can defer capital gains tax forever. To illustrate the argument make the simplifying assumption that the proportional stock price change, P_t/P_{t-1} , follows a binomial process with realizations u and d , $0 \leq d \leq u$. Without loss of generality, assume that the stock price at date zero equals one, i.e., $P_0 = 1$. It is not necessary to assume agreement on the probability of each state's occurrence. Also, it is not necessary to assume that the proportional stock price changes, P_t/P_{t-1} , follow a random walk. The probability of the realization $P_2/P_1 = u$ can depend on the value of the earlier realization, P_1/P_0 .

Define $C(P_t, t; E)$ to be the price at date t , $t \leq 2$, and stock price P_t , of an American call option with exercise price E and maturity date 2. It is assumed that the option's bid price equals the option's ask price. Also, assume the call writer can use call sale proceeds. In practice, the writer pledges collateral, stocks, bonds, or Treasury bills, as margin.

Two call options are used in the hedge, both with maturity date 2 and with exercise prices E_1 , E_2 , respectively, where

$$d^2 < ud \leq E_1 < E_2 < u^2. \quad (1)$$

Irrespective of the investor's marginal tax rate, the following relationships must hold to prevent arbitrage opportunities:

$$C(d, 1; E_i) = 0, \quad i = 1, 2; \quad (2)$$

$$C(ud, 2; E_i) = 0, \quad i = 1, 2; \quad (3)$$

$$C(d^2, 2; E_i) = 0, \quad i = 1, 2; \quad (4)$$

$$\frac{C(u, 1; E_1)}{C(1, 0; E_1)} = \frac{C(u, 1; E_2)}{C(1, 0; E_2)} \quad (5)$$

and

$$\frac{C(u^2, 2; E_1)}{C(1, 0; E_1)} = \frac{C(u^2, 2; E_2)}{C(1, 0; E_2)}. \quad (6)$$

Since the nominal riskless rate of interest is nonnegative, the no arbitrage condition implies:

$$C(1, 0; E_i) \leq C(u, 1; E_i) \leq C(u^2, 2; E_i), \quad i = 1, 2. \quad (7)$$

At date zero a hedge is constructed by (a) writing $C(1, 0; E_2)$ two-period calls

with exercise price E_1 ; and, (b) buying $C(1, 0; E_1)$ two-period calls with exercise price E_2 . The net cash flow at date $t = 0$ is zero because

$$C(1, 0; E_2)C(1, 0; E_1) - C(1, 0; E_1)C(1, 0; E_2) = 0.$$

At date $t = 1$, if $P_1 = u$, from equations (2)-(7) there is a capital loss $L(u)$ on position (a), where

$$L(u) = C(1, 0; E_2)[C(u, 1; E_1) - C(1, 0; E_1)] \geq 0; \quad (8)$$

and an equal capital gain $L(u)$ on position (b), because

$$\begin{aligned} C(1, 0; E_1)[C(u, 1; E_2) - C(1, 0; E_2)] \\ = C(1, 0; E_2)[C(u, 1; E_1) - C(1, 0; E_1)] = L(u). \end{aligned} \quad (9)$$

If, instead, $P_1 = d$ at date $t = 1$, there is a capital loss $L(d)$ on position (b), where

$$\begin{aligned} L(d) = C(1, 0; E_1)[C(1, 0; E_2) - C(d, 1; E_2)] \\ = C(1, 0; E_1)C(1, 0; E_2) \leq 0 \end{aligned} \quad (10)$$

and an equal capital gain on position (a) because

$$C(1, 0; E_2)[C(1, 0; E_1) - C(d, 1; E_1)] = C(1, 0; E_2)C(1, 0; E_1) = L(d). \quad (11)$$

Thus, irrespective of the realization $P_1 = u$ or $P_1 = d$ at date $t = 1$, the hedger has a loss on one "leg" of the hedge and an equal gain on the other "leg" of the hedge. In this sense, the hedge is perfect.

To explain how investors realize a capital loss at date $t = 1$ and an equal capital gain at date $t = 2$, assume at date $t = 1$ that the price is $P_1 = u$. They pay $C(1, 0; E_2)C(u, 1; E_1)$, closing position (a) and realizing capital loss $L(u)$. Immediately thereafter they write $C(1, 0; E_2)$ call options and receive $C(1, 0; E_2)C(u, 1; E_1)$ and leave position (b) intact. The net cash flow at date $t = 1$ is zero and the realized net loss is $L(u)$.

At date $t = 2$ both positions are closed. The net cash flow is zero, irrespective of the outcome $P_2 = u^2$ or $P_2 = ud$ because

$$-C(1, 0; E_2)C(u^2, 2; E_1) + C(1, 0; E_1)C(u^2, 2; E_2) = 0$$

by equation (6), and

$$-C(1, 0; E_2)C(ud, 1; E_1) + C(1, 0; E_1)C(ud, 1; E_2) = 0$$

by equation (3). The net capital gain is $L(u)$, irrespective of the outcome $P_2 = u^2$ or $P_2 = ud$ because

$$\begin{aligned} -C(1, 0; E_2)[C(u^2, 2; E_1) - C(u, 1; E_1)] \\ + C(1, 0; E_1)[C(u^2, 2; E_2) - C(1, 0; E_2)] = L(u) \end{aligned}$$

by equations (6) and (8) and

$$\begin{aligned} -C(1, 0; E_2)[C(ud, 2; E_1) - C(u, 1; E_1)] \\ + C(1, 0; E_1)[C(ud, 2; E_2) - C(1, 0; E_2)] = L(u) \end{aligned}$$

by equation (8). If $P_1 = u$, the hedge has zero net cash flows at dates $t = 0, 1, 2$ and there is a net loss $L(u)$ at date $t = 1$ and a net gain $L(u)$ at date $t = 2$. Likewise, if $P_1 = d$, the hedge has zero net cash flows at dates $t = 0, 1, 2$ and the hedge generates a net loss $L(d)$ at date $t = 1$ and a net gain $L(d)$ at date $t = 2$.

Tax lawyers might argue that the Internal Revenue Service is likely to disallow the hedge's tax benefits because the transaction's economic purpose is to defer taxes, only. However, to evade detection, the hedge can be disguised. It is reasonable to assume that investors hold stock portfolios, including the stock on which they buy and write call options. Instead of holding a challengeable call option hedge and in addition long positions in stock, they may simply adjust the call proportions in the hedge to make the hedge's capital gains or losses equal to the stock's net capital gains or losses. Clearly, this alternative hedging procedure loses the "noneconomic transaction" taint attributable to option hedging alone.

Tax lawyers might have another objection. Assume that the realization at date $t = 1$ is $P_1 = u$. Closing position (a), realizing capital loss $L(u)$ and immediately writing the same number of call options with maturity date 2 and exercise price E_1 is a wash sale, unless thirty days elapse before again writing the calls. The tax loss is disallowed. But, the investor need simply write calls at date $t = 1$ with exercise price E_3 , where $ud \leq E_3 < u^2$, rather than exercise price E_1 . Call options with different exercise prices are substantially different assets and the wash sale rules no longer apply. To be exact, at date 1, it is not necessary to write $C(1, 0; E_2)$ call options with maturity date 2 and exercise price E_1 , because writing

$$\frac{C(1, 0; E_2)C(u, 1; E_1)}{C(u, 1; E_3)}$$

call options with maturity date 2 and exercise price E_3 is equivalent. Once again, the net cash flows at dates 1, 2 are zero, the net capital loss at date 1 is $L(u)$ and the net capital gain at date 2 is $L(u)$.

In constructing the original hedge, the investor writes calls with exercise price E_1 and buys calls with exercise price E_2 . Since this hedge helps all taxpayers defer their capital gains, who buys the calls that all investors write and who writes the calls that all investors buy? Naturally, the reverse hedge is as good as the original hedge. Both produce capital losses. To construct the reverse hedge, buy $C(1, 0; E_2)$ two-period calls with exercise price E_1 , and write $C(1, 0; E_1)$ two-period calls with exercise price E_2 . One hedger writes the calls which another hedger buys. Thus, supply equals demand and the no arbitrage conditions ensure that call prices satisfy equations (2)-(7).

To forestall another objection, assume the following events. At date 1, the stock and call prices have increased. The call writer sells to realize a capital loss. But the call buyer does not sell, deferring the capital gain. Who buys the writer's call? The answer is simple. Those realizing capital losses repurchase the calls, i.e., hedgers create their own demand. But this is a wash sale. Naturally, if there are two or more hedgers, they buy the calls that the others sell, avoiding wash sale restrictions, and resolve the problem.

The binomial process, which generates the proportional price changes, P_t/P_{t-1} , can be generalized to a multinomial process with n possible realizations, $n \geq 3$.

The generalized hedge uses n different options. Options on the same stock but with different maturities or different exercise prices are classified as substantially different assets for tax purposes. Since only a few listed options are traded on each stock, the hedge works if the process generating the proportional price changes can be fairly approximated by a multinomial process with a small number of realizations. In the next section this difficulty is resolved under the assumption that proportional price changes are generated by a Wiener process with drift.

III. A Hedge with Call Options in Continuous Time

Assume that proportional price changes are generated by a Wiener process with drift, i.e.,

$$\frac{dP}{P} = \mu dt + \sigma dw(t) \quad (12)$$

where μ, σ are constants and $dw(t)$ is the Wiener process $w(t)$ increment. At date t the hedger buys N_i call options with exercise price E_i and maturity date T_i , $i = 1, 2, 3$. N_i can be negative. The hedger writes $-N_i$ calls. For notational convenience define $C^i = C(P, t; E_i, T_i)$, $i = 1, 2, 3$. Choose each N_i such that the position's expected net gain or loss is zero over $[t, t + dt]$, i.e.,

$$\sum_{i=1}^3 \left[C_t^i + \mu P C_p^i + \frac{\sigma^2}{2} P^2 C_{pp}^i \right] N_i = 0 \quad (13)$$

and the position's variance of net gain or loss is zero over $[t, t + dt]$, i.e.,

$$\sum_{i=1}^3 C_p^i N_i = 0 \quad (14)$$

where use was made of Ito's Lemma. Equations (13) and (14) uniquely determine the ratios N_1/N_3 and N_2/N_3 .

The no arbitrage condition implies

$$\sum_{i=1}^3 C^i N_i \geq 0. \quad (15)$$

If $\sum_{i=1}^3 C^i N_i < 0$ investors could, irrespective of marginal tax brackets, set up this hedge at date t and receive positive net cash inflow $-\sum_{i=1}^3 C^i N_i > 0$. At date $t + dt$ investors would close the hedge positions and pay $-\sum_{i=1}^3 C^i N_i > 0$. Effectively, investors would have the use of $-\sum_{i=1}^3 C^i N_i$ over $[t, t + dt]$ at zero interest. Note that the net capital gain at $t + dt$ is zero. Equation (15) ensures that this money-pump does not exist.

If (N_1, N_2, N_3) is a solution to equations (13) and (14), so is $(-N_1, -N_2, -N_3)$, i.e., the reverse hedge. The no arbitrage condition with the reverse hedge implies

$$-\sum_{i=1}^3 C^i N_i \geq 0. \quad (16)$$

Equations (15) and (16) together, imply

$$\sum_{i=1}^3 C^i N_i = 0. \quad (17)$$

With this hedge there are zero net cash flows at both t and $t + dt$ and zero net capital gain or loss at date $t + dt$.

But with this hedge capital gains taxes can be deferred from date $t + dt$ to a future date. Equation (17) implies that N_1 , N_2 , N_3 cannot all have the same sign. Without loss of generality assume that $N_1 > 0$, $N_2 < 0$ and $N_3 < 0$. Therefore if $P_{t+dt} < P_t$, realize loss L on the N_1 calls, where

$$L = [C^1(P_t, t; E_1, T_1) - C^1(P_{t+dt}, t + dt; E_1, T_1)]N_1. \quad (18)$$

Tax gain realization on the $-N_2$ and $-N_3$ calls is deferred. If, on the other hand, $P_{t+dt} > P_t$, realize loss $-L$ on the $-N_2$ and $-N_3$ calls and defer tax gain realization $-L$ on the N calls.

IV. Transactions Costs and the Effectiveness of the Hedge in Deferring Capital Gains

For the hedge in continuous time, the expected loss, $\bar{L}$, at date $t + dt$ is

$$\begin{aligned} \bar{L} &= E[|C^1(P_{t+dt}, t + dt; E_1, T_1) - C^1(P_t, t; E_1, T_1)|]N_1 \\ &= E\left[\left|\left\{C_t^1 + \mu PC_p^1 + \frac{\sigma^2}{2} P^2 C_{pp}^1\right\} dt + \sigma PC_p^1 dw\right|\right]N_1 \\ &= \sqrt{\frac{2}{\pi}} PC_p^1 N_1 \sigma dt^{1/2} + o(dt^{1/2}) \end{aligned} \quad (19)$$

because $E[|dw|] = \sqrt{2/\pi} dt^{1/2}$. The expected loss per dollar invested in buying N_1 calls is

$$\frac{\bar{L}}{C^1 N_1} = \sqrt{\frac{2}{\pi}} \frac{PC_p^1}{C^1} \sigma dt^{1/2}. \quad (20)$$

The investor must make many transactions. By equation (17), $|C^2 N_2 + C^3 N_3| = C^1 N_1$. The transactions volume at date t is $2C^1 N_1$. The one-way transactions volume at date $t + dt$ is $\sim 2C^1 N_1$ and at date $t + 2dt$ is $\sim 2C^1 N_1$. In total, the one-way transactions volume is $6C^1 N_1$. With these transactions the investor defers tax on capital gains $\bar{L}$. If the investor invests in riskless, tax-exempt bonds yielding r , the after-tax gain from deferral is $r\tau\bar{L}$, where τ is the marginal tax rate on capital gains and losses. If short-term capital gains are deferred, the marginal tax rate equals the ordinary income tax rate.

Let k be the proportional transactions cost rate per dollar of round-trip transactions. The tax benefit exceeds the transactions cost provided

$$3C^1 N_1 (1 - \tau)k \leq r\tau\bar{L}. \quad (21)$$

With the aid of equation (20) this simplifies to

$$\text{transactions costs rate} \equiv k \leq \sqrt{\frac{2}{\pi}} \frac{r\tau\sigma dt^{1/2}}{3(1-\tau)} \frac{PC_p^1}{C^1} \quad (22)$$

If the annual yield on a riskless, tax-exempt bond were $r = .08$, the time interval dt one month, the monthly stock return's standard deviation $\sigma dt^{1/2} = .15$, the time to the option's maturity 6 months and the option's exercise price were equal to the stock price, then equation (22) implies

$$k \leq 0.006\tau/(1-\tau). \quad (23)$$

Obviously, a small transactions cost is sufficient to dissipate the hedge's tax benefits.

The expected tax benefit exceeds $r\tau\bar{L}$ as investors near death. For upon death, beneficiaries start with a new tax basis which eliminates the capital gain. In particular, assume that death is a Poisson arrival with force λ i.e., the expected lifetime of the investor is λ^{-1} . Then the hedger's expected tax benefit is $\{\lambda + (1 - \lambda)r\}\tau\bar{L}$ and exceeds the transactions cost provided

$$3C_1N(1-\tau)k \leq \{\lambda + (1-\lambda)r\}\tau\bar{L}. \quad (24)$$

It is oftentimes claimed that commodity futures contracts are riskier than common stocks and call options on stock and, therefore, given a dollar volume of transactions, a hedge constructed with commodity futures contracts has a higher expected capital loss $\bar{L}$, than a hedge constructed with call options. If this were indeed the case, it is possible that creating hedges with futures contracts might produce a positive gain.

To demonstrate the futures contract hedge, assume continuous trading in the futures market, and assume that the commodity spot price is described by a diffusion process and the spot price is the only relevant state variable. The investor buys N_i futures contracts for delivery at date T_i , $i = 1, 2, 3$. A negative value of N_i implies that the investor writes $-N_i$ futures contracts. The numbers N_i are chosen such that the hedge's expected net gain or loss and variance is zero over $[t, t + dt]$. As in the last section, the no arbitrage condition implies that the net cash flow at date t is zero.

Since the net cash flow at date t is zero, it follows that N_1, N_2, N_3 cannot all have the same sign. Without loss of generality set $N_1 > 0, N_2 < 0, N_3 < 0$ ². For a futures price increase at date $t + dt$, gains on long contracts would be offset by losses on the short contracts. For a futures price decrease, the converse is true. Loss is realized on one hedge "leg" and gain deferred on the other. Thus the investor realizes a net tax loss at date $t + dt$. A new perfect hedge is constructed by using the winning "leg" and other futures contracts. At date $t + 2dt$ all positions are closed and the realized net tax gain equals the net tax loss realized at date $d + dt$. Tax realization at date $t + dt$ is deferred to a future date.

The expected tax capital loss at date $t + dt$ is determined as follows. Let F_1 be the futures price per contract at date t , of the commodity to be delivered at date

² The hedge is termed a "butterfly" if $N_1/2 = -N_2 = -N_3$, where $T_2 < T_1 < T_3$. The actual contract proportions which make the hedge riskless deviate slightly from the proportions of the butterfly hedge.

T_1 ; and let $F_1 + dF_1$ be its price at date $t + dt$. The futures contract becomes a forward contract at date $t + dt$ with price $e^{-r(T_1-t)} dF_1$, where r is the yield of a riskless, tax-exempt bond.³ Therefore the expected capital loss at date $t + dt$ is

$$\bar{L} = e^{-r(T_1-t)} N_1 E[|dF_1|] \quad (25)$$

where $E[]$ is the expectation operator.

The dollar volume of trading in futures contracts is $2N_1F_1$ at date t , and approximately $2N_1F_1$ at dates $t + dt$ and $t + 2dt$. If the round-trip transactions cost per contract is KF_1 the total transactions cost is $3KN_1F_1$. Assuming that the date $t + dt$ is December and the date $t + 2dt$ is January of the following tax year, the tax benefit is $r\tau\bar{L}$ and exceeds the after-tax transactions cost provided

$$K \leq \frac{e^{-r(T_1-t)}}{3(1-\tau)} E[|dF_1|/F_1] r\tau. \quad (26)$$

If dF_1 is normally distributed and the mean is small compared to the standard deviation, equation (26) simplifies to

$$L \leq \sqrt{\frac{2}{\pi}} \cdot \frac{r\tau}{3(1-\tau)} \text{std}(dF_1/F_1). \quad (27)$$

The typical estimated annual standard deviation of futures price percentage changes is .4, as shown for a wide range of commodities in Bodie and Rosansky (1979, Table 8). Using this estimate and assuming that dt is one month, $r = .08$ and $T_1 - t = 6$ months, equation (27) simplifies to

$$K \leq 0.002\tau/(1-\tau). \quad (28)$$

The commission fee on futures contracts is typically small (about \$50 per \$20,000 contract), but even a small spread between bid and ask prices would be sufficient to dissipate the hedge's tax advantages. Breeden (1978) reached a similar conclusion analyzing hedges using gold and silver futures contracts.

If the time interval dt is increased from one month to one year, the standard deviation of dF_1/F_1 increases by approximately a factor $\sqrt{12}$ and the hedge's efficiency increases accordingly. Yet the hedge is no longer perfect. There is undiversified risk which may be eliminated by more hedges but at greater transaction costs. An investor cannot effectively eliminate the tax on realized capital gains, hedging with either call options or commodity futures contracts. The transactions costs would be prohibitive. However, the futures market folklore contains many stories about tax spreading. For a tax hedge to work the hedger must be lucky. Or, absent luck, the hedger must negotiate the trades. A recent Revenue Service ruling (Rev. Rul. 77-185) disallowed losses on a series of simultaneous purchases and sales of silver futures contracts with different delivery dates, used to minimize tax on short-term capital gain from another asset's sale. Obviously, for the hedge to work, investors must negotiate a very low commission. Also if price can be negotiated, this will help. It is difficult to separate a negotiated trade series from a market series. This difficulty, we surmise, led to the Revenue

³ See Black (1976) for the pricing of forward contracts.

Service ruling. If transactions costs can be negotiated, the hedge will defer capital gains at a profit. But, regular trading costs are of about the same magnitude as the tax saving.

V. Optimal Liquidation of Assets in the Presence of Capital Gains Tax: The Lock-in Effect

Consider a stock position, with a cost basis $\hat{P}$ and a current price $P(t)$, where $P(t) \geq \hat{P}$. Assume the investor intends to hold the stock as an investment until some later date T . Under the simplifying assumption that the marginal tax rate on short-term capital gains and losses equals the marginal tax rate on long-term capital gains and losses, it is straightforward to prove that it is optimal to realize capital losses and defer the realization of capital gains. Specifically, if $P(t) > \hat{P}$, retain the stock and defer the realization of capital gain $P(t) - \hat{P} > 0$. If $P(t) < \hat{P}$ sell and realize the capital loss $\hat{P} - P(t) > 0$, and repurchase the share immediately at the market price $P(t)$.⁴

If marginal tax rates on net short-term capital gains and losses are higher than marginal tax rates on net long-term capital gains or losses, the optimal liquidation policy is complicated. It is no longer necessarily true that it is optimal to defer realizing capital gain. If stock were held for more than a year and if $P(t) - \hat{P} > 0$ but $P(t) - \hat{P}$ is sufficiently small, it may be optimal to sell the stock, realize a capital gain and immediately repurchase the stock. If the price falls within the next year, short-term capital loss rather than long-term capital loss is realized. This may result in a higher tax deduction than not selling. If the short-term tax rate exceeds the long-term rate, it remains true, however, that it is optimal to realize capital loss, if the intention is to hold the stock until death, for beneficiaries are currently granted a tax basis equal to the stock price at death.

For a portfolio, the differing short-term and long-term tax rates further complicate the optimal liquidation policy. The following example illustrates the policy's complexity. Assume a realized \$1,000 short-term capital loss. If a \$100 long-term capital loss on another asset is realized, the effective tax rate is the long-term rate. If, instead, the \$100 is a long-term capital gain on another asset, the long-term gain offsets \$100 of the \$1,000 short-term capital loss. Consequently, the effective tax rates on the long-term gain is the short-term rate. Effective tax rate may be different on long-term gains than on long-term losses. Rates depend on the decision to realize gains and losses on other assets. This (and other) tax code peculiarities make the optimal liquidation policy nonseparable across different assets. Optimal liquidation policy in the presence of personal taxes becomes exceedingly complex.

According to the current tax code, unlimited short-term or long-term capital gains may be offset with short-term or long-term capital losses. If the capital losses exceed capital gains, \$3,000 of ordinary income can be shielded from taxation. If the \$3,000 exclusion limit is binding, and if this limit is expected to be

⁴ In practice the investor would have to wait thirty days before repurchasing the share, otherwise the IRS would term the transaction a "wash sale" and would disallow the capital loss deduction.

binding in subsequent years, the marginal tax rate on short-term and long-term capital gains and losses is zero. In the margin, this investor behaves like a tax-exempt investor. If the \$3,000 exclusion limit were binding for all investors, then equilibrium prices would be determined as if the marginal tax rate on capital gains were zero.

VI. Implications on Investor Welfare

Although riskless hedging using options and futures contracts can eliminate the capital gains tax, transactions costs blunt and may negate the strategy's effectiveness. With these roadblocks, capital gains tax does matter and renders the optimal liquidation policy complex, given the distinction between short-term and long-term capital gains and loss deduction limitations. To study the implications of capital gains tax on investor welfares make the simplifying assumption that all capital gains and losses are long-term and there are no loss deduction limitations. Investor welfare is used in the restricted sense that the investor does not benefit from the taxes collected by the government, so that an investor's welfare is maximized when the investor pays the least tax. Our discussion is within the partial equilibrium framework because there is an implicit assumption that relative prices are unaffected by changes in the tax rate, dividends and inflation. Welfare implications may be different in a general equilibrium framework. With the above caveats we discuss the effect of changes in inflation, capital gains tax rate, dividends and stock return variability on investor welfare. There is a fundamental asymmetry between capital gains and losses, in that gains are deferred and losses are realized. This asymmetry has important implications for investor welfare and equilibrium pricing.

If real prices remain unchanged, an increase in anticipated inflation decreases welfare because basis prices fall in real terms and the probability of realizing a capital loss decreases. Feldstein and Slemrod (1978) argue that inflation reduces investor welfare because nominal and not real capital gains are taxed. These arguments are appealing if there are no legislated capital gains tax cuts. But there have been cuts in the capital gains tax and there will be future tax cuts in response to inflation.

An increase in the capital gains tax rate increases the amount of ordinary income which a given capital loss can shield and therefore increases the welfare for those planning to hold their assets until the end of their lifetime. The welfare implications may be reversed for those who must liquidate their assets before the end of their lifetime. Feldstein, Slemrod and Yitzhaki (1978) estimated that the tax rate elasticity of the volume of realized capital gains exceeds one, so that a decrease in the capital gains tax rate increases the gross tax collected on realized capital gains.

The effect of dividends on the investor's welfare is ambiguous. If dividends are taxed at the investor's marginal tax rate on ordinary income, dividends decrease the investor's welfare irrespective of whether the investor is planning to sell the stock before or at the end of his lifetime. Miller and Scholes (1978) argue that there exist effective ways to defer tax on dividends until the end of the investor's

lifetime. Also, the marginal tax rate on dividends is lower than the marginal tax rate on capital gains for corporations and for investors with large interest expenses. Then dividends increase the investor's welfare because dividend payments increase the probability of realizing capital losses during a lifetime.

Assume that the rate of return, $\tilde{R}$, on some stock is generated by the market model

$$\tilde{R} = \alpha + \beta \tilde{R}_M + \gamma \tilde{\epsilon}$$

where $\tilde{R}_M$ is the return on the market portfolio, $(\tilde{R}_M, \tilde{\epsilon})$ are bivariate normal, $E(\tilde{\epsilon}) = 0$, $\text{cov}(\tilde{R}_M, \tilde{\epsilon}) = 0$ and $\gamma \geq 0$. An increase in the parameter γ , other things equal, increases the probability of a realized capital loss and, therefore, increases welfare for those already holding the stock.

Although the effects of changes in inflation, tax rates, dividends and stock return variability on welfare were described, how do these changes effect stock prices and expected stock returns? These are difficult questions. However, the effects of dividends and stock return variability are discussed in Constantinides (1979) in the context of an equilibrium model of asset pricing which includes a capital gains tax and which, unlike earlier models, explicitly incorporates the lock-in effect.

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