Conditional Predictions of Bond Prices and Returns: Discussion
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DISCUSSION:

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Progress on the problem of the term structure since the 1940's has been meagre. In contrast, the work of the present authors, and the related work referenced in their paper, represents a highly promising line of thought. In these models, as the authors explain, the value of a bond of any maturity is expressed as a deterministic function of a number of state variables. In the present paper, the state variables are taken to be two interest rates—a short rate and a long rate. The authors suggest—and I fully agree with this—that models of this type may be of practical use in bond portfolio management, perhaps the most obvious example being performance measurement.

Before passing on to the details of the paper, we might consider for a moment the question of whether models of this kind are properly viewed as a "theory" of the term structure. After all, to put it simply, the theory of the term structure concerns the economics of the relationship between the long rate and the short rate. However, the Brennan and Schwartz model determines interest rates on intermediate maturity instruments *given* the long and short rates. Thus the present paper might be viewed as presenting a theory which reduces the dimensionality of the term structure problem, rather than providing a theory of the

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term structure as a whole. Nevertheless, this is significant progress, and there is, moreover, an important insight in the new approach which represents its main difference from the traditional literature. This idea is well expressed by Cox, Ingersoll and Ross (1978), who point out that "there must be a limit to how far one can go in maintaining that bonds of close maturities will not be close substitutes." This limit depends on the dimensionality of the state space describing the term structure. Among other things the present paper helps to decide whether two state variables are "enough."

As I do not have time to discuss every aspect of this compact paper I shall devote most attention to a part which is both innovative and crucial: the estimation and validation of a stochastic model of the long and short rates.[1] A *continuous time* model is needed for application of the arbitrage-based valuation equation while, clearly, data are available only at *discrete* intervals. The authors go to some considerable trouble to obtain and estimate an *exact* discrete version of the continuous model, but it is not clear that this level of sophistication is justified here. This is because the dynamic interaction of the variables is weak and the coefficients correspondingly small. The table below gives the coefficients of the discrete model derived from the continuous time model coefficients (i) using the exact method and (ii) using a first order approximation.[2]

	a_{11}	a_{12}	a_{21}	a_{22}
Exact Method	0.9321	0.0679	−0.0049	1.0056
First Order Approx.	0.9299	0.0701	−0.0051	1.0058
Standard Error	(0.0050)	(0.0050)	—	—

It is clear that these two sets of coefficients are "economically close" and, moreover, it seems unlikely that they could be distinguished statistically. Thus, at least in this case, little would be lost by taking the more straightforward linear approximation to the continuous model.

The discrete model is tested by generating conditional forecasts of long and short rates and comparing these with forecasts generated by a naive martingale model. In one sense (see Table 2) the results are disappointing: the RMS errors for the two sets of forecasts are very similar. (This is despite the fact that the comparison apparently uses the same set of data used to estimate the parameters. This gives an advantage to the estimated model, because even if the *true* process were a pure martingale, the estimated constant terms reflect the sample means of the two series.) On the other hand, the results should not be surprising because the estimated model is in fact quite close to a martingale. (As can be seen from the values of a_{21} and a_{22} in the table, the process for the logarithm of the long rate is very close *indeed* to a martingale.)

[1] See Brennan and Schwartz (1979). Here, and later in my comments I have had to rely on data in the authors' earlier paper and excluded from the current paper because (presumably) of space limitations. References are to papers cited by the authors.

[2] If the continuous model is $dy = By + b + wdz$, then the exact transition matrix for the discrete model is e^B and a first order approximation is given by $I + B$. The coefficients in the table correspond to the elements of A in equations (5) and (6). The standard errors are the standard errors of B and therefore are only approximately equal to the true standard errors of the coefficients in the table.

Brennan and Schwartz estimate their model in terms of the long and short rates. Their estimated continuous time version is

$$d \ln r = 0.0701 \ln l/r - 0.0042 + 0.0736\, dz_1$$
$$d \ln l = -0.0051 \ln r + 0.0058 \ln l + 0.006 + 0.0250\, dz_2 \tag{1}$$

where the correlation coefficient between dz_1 and dz_2 is 0.3747. If we assume for a moment that the coefficients of ln r and ln l are of equal size (say to 0.0055) and opposite in sign, we may subtract the first equation from the second to obtain the following equivalent pair:

$$d \ln l/r = -0.0646 \ln l/r + 0.0043 + \sigma_3\, dz_3.$$
$$d \ln 1 = +0.0055 \ln l/r + 0.006 + 0.0250\, dz_2 \tag{2}$$

Simple calculation shows that $\sigma_3 = 0.0683$ but that, interestingly, the correlation between dz_3 and dz_2 is close to zero, (actually: −0.01) thus providing an approximately canonical representation of the process estimated by Brennan and Schwartz. This representation is motivated by a suggestion made by Ayres and Barry (1979) that the long rate of the interest and the *spread* between the long and short rates are uncorrelated. Equation (2) is a logarithmic version of their hypothesis and is strongly supported by Brennan and Schwartz' estimates. A further advantage of model 2 is that it contains fewer parameters—indeed the remaining structural parameter in the second equation might safely be ignored.

Equation (2) says that the log of the long rate is very close to a random walk with a small drift and that the log of l/r regresses towards zero with independent disturbances. However, as the authors point out, the residuals from the model still exhibit serial correlation and this suggests that a further state variable may be needed *even to describe the long and short rates themselves.* (By definition, admissable state variables must be jointly Markov.)

Finally, I shall briefly mention two points from the remainder of the paper. First, in identifying the factor loadings it would be better to take account of coupon differences between bonds rather than attempting to standardize simply in terms of the final maturity date. (The distribution of coupon size across the maturity spectrum almost certainly changes over the sample period.) Second, and perhaps most important, in deciding whether the model is successful in explaining the prices of intermediate maturity bonds one is bound to ask: "Compared with what?" As a first step it would be helpful to compare the results with a naive model which also takes as given the long and short rates. One simple but reasonable possibility would be to assume an exponential form for the term structure.

If I have seemed to criticize certain aspects of the paper it is only because its virtues need no apologist. This is an exciting paper which makes the first serious attempt to implement a promising new approach to bond pricing. Much remains to be done, but much has been achieved.