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Author(s): Alan Kraus and Gordon A. Sick
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Distinguishing Beliefs and Preferences in Equilibrium Prices

ALAN KRAUS and GORDON A. SICK*

I. Introduction

MODELS OF SECURITY market equilibrium based on expected utility maximization require that something further be assumed about preferences or beliefs, or both, in order to derive any sort of interesting results. Beliefs can be represented by probability statements about returns while preferences include the degree of risk aversion with respect to timeless gambles and, for multiperiod models, the degree of intertemporal impatience.

Since restrictions on beliefs and restrictions on preferences have both shown themselves to be productive routes to theories of security market equilibrium, the question naturally arises as to whether one can distinguish the effects of beliefs and preferences in observed prices. It is certainly not obvious that this can ever be done. Since individual agent optimality conditions involve the product of probability and marginal utility, it may be that any set of equilibrium prices that are consistent with some combination of beliefs and preferences could also have resulted from different beliefs combined with different preferences.

One can certainly think of situations in which the question of distinguishing inferences about beliefs from inferences about preferences is relevant. For example, it has been observed that the premium between yields on risky and riskless debt varies with the business cycle: high during recession, low in prosperous times. Can one infer with confidence that this is due wholly to varying pessimism/optimism about business conditions? Or is it also consistent with variations in risk aversion through the business cycle (for whatever reason such behavior should exist)? Similarly, one can ask whether low stock prices during a depression are the result of major shifts in probability beliefs alone or also reflect widely shared increases in risk aversion.

A number of authors (*eg.* Green (1973), Rothschild and Stiglitz (1976), Grossman (1976, 1977), Kreps (1977), Jordan and Radner (1977), Radner (1977), Allen (1979)) have considered rational expectations equilibria in which prices are signals of probability beliefs. In a previous paper (Kraus and Sick (1979)) we presented a model in which beliefs are homogeneous but agents may differ in a particular utility parameter and showed that, nonetheless, equilibrium prices must "reveal" the aggregate values of the preference parameter so that all agents in the economy effectively become equally informed by observing market prices.

Our purpose here is to make some initial steps in analysing conditions that

* University of British Columbia and Yale University, respectively.

allow or prevent separate inferences about beliefs and preferences in the context of a simple rational expectations equilibrium. Even in this highly simplified situation we shall see that both cases exist—the case in which equilibrium prices "reveal" a unique combination of beliefs and preferences as well as the case in which different beliefs-preferences combinations are consistent with the same equilibrium price structure.

Our model is a simple exchange economy with single consumption goods in which all agents possess time-additive negative exponential utility and normal probability beliefs. We assume that agents jointly possess information about beliefs and preferences that each agent individually tries to infer from observing prices. The question of whether prices allow such inferences is the question of whether a rational expectations equilibrium exists that reveals to the uninformed agents the information possessed by the informed agents.

In the following section, we develop a two-date version of the model in which the information to be inferred from prices is an aggregate risk aversion parameter and a single item regarding beliefs. The question is whether these two items can be inferred from the two prices in the economy. This is considered in Section III of the paper. Section IV extends the analysis to a three-date model in which three prices are available and the value of an aggregate impatience factor as well as a risk aversion parameter and a belief parameter must be inferred by the uninformed agents. This is considered in Section V. Section VI offers a brief summary and conclusions.

II. A Two-Date Model

In this section we hold fixed the rate of pure time preference (impatience) and consider whether differences in a belief variable (the future mean return on the risky asset) can be distinguished from differences in an aggregate risk aversion parameter. There are two dates. Date 1 wealth must be allocated between date 1 consumption, purchase of a riskless bond maturing at date 2 and purchase of a fractional share of a risky asset that pays an uncertain liquidating dividend at date 2. All date 2 wealth is consumed.

We begin with the following notation.

w_{it} = date t wealth of agent i (t = 1, 2)
c_{i1} = date 1 consumption by agent i
b_{i12} = agent i's investment at date 1 in riskless bond maturing at date 2
x_{i2} = agent i's investment at date 1 in risky asset
D_{12} = date 1 price of riskless bond
P_1 = date 1 price of risky asset
C_2 = aggregate date 2 dividend of risky asset

The date 1 budget constraint and date 2 wealth of agent i are, respectively,

$$w_{i1} = c_{i1} + D_{12}b_{i12} \div P_1x_{i2} \tag{1}$$

$$w_{i2} = b_{i12} + C_2x_{i2} \tag{2}$$

Market clearing conditions are

$$\textstyle\sum_i b_{i12} = 0 \tag{3}$$

$$\textstyle\sum_i x_{i2} = 1 \tag{4}$$

$$\textstyle\sum_i c_{i1} = C_1 \tag{5}$$

Conditional on the joint information of all agents, the distribution of C_2 is $N(\bar{C}_2, \sigma_2^2)$, where we assume that all agents agree from the start on the variance σ_2^2 so that information concerns only the mean $\bar{C}_2$. We can think of this information as representing either the beliefs of agents known from the start to be "informed", which uninformed agents try to learn from prices, or just the collective information of all agents that each agent tries to learn from prices, as in Grossman (1976).

Agent i's date 1 utility is

$$U_{i1}(c_{i1}, w_{i2}) = -\exp(-\theta_i c_{i1}) - \eta \exp(-\theta_i w_{i2})$$

where $\theta_i(>0)$ is agent i's coefficient of absolute risk aversion and η is an impatience parameter. Each agent is assumed to know initially his own θ_i and the common η. We define a "*fully informing rational expectations equilibrium*" (FRE) as an equilibrium in which each agent can infer from observed prices all information (among that possessed jointly by all agents) that affects demands. Since the only random variable in w_{i2} is C_2, if FRE exists then w_{i2} is normal and agent i's expected utility at date 1 is

$$E(U_{i1}) = -\exp(-\theta_i c_{i1}) - \eta \exp[-\theta_i E(w_{i2}) + \tfrac{1}{2}\theta_i^2 \mathrm{Var}(w_{i2})] \tag{6}$$

Substituting for c_{i1} from (1) and maximizing (6) w.r.t. b_{i12} and x_{i2}, the first order conditions for agent i are

$$D_{12}\exp(-\theta_i c_{i1}) = \eta \exp[-\theta_i E(w_{i2}) + \tfrac{1}{2}\theta_i^2 \mathrm{Var}(w_{i2})] \tag{7}$$

$$P_1\exp(-\theta_i c_{i1}) = \eta(\bar{C}_2 - \theta_i\sigma_2^2 x_{i2})\exp[-\theta_i E(w_{i2}) + \tfrac{1}{2}\theta_i^2 \mathrm{Var}(w_{iw})] \tag{8}$$

Therefore, agent i's investment in the risky asset if he knows or can learn the value of $\bar{C}_2$

$$x_{i2} = (\bar{C}_2 - P_i D_{12}^{-1})(\theta_i\sigma_2^2)^{-1} \tag{9}$$

Agents' utility functions have been chosen so that they satisfy the conditions for aggregation (see Rubinstein (1974) and Brennan and Kraus (1978)). Summing (9) over i and applying (4) yields the date 1 CAPM relation

$$P_1 = D_{12}(\bar{C}_2 - \theta\sigma_2^2) \tag{10}$$

where we define the aggregate risk aversion parameter

$$\theta \equiv (\textstyle\sum_i \theta_i^{-1})^{-1} \tag{11}$$

To obtain a second equilibrium relation we take the log of (7) and rearrange as

$$\theta_i^{-1}\ln(D_{12}\eta^{-1}) = c_{i1} - E(w_{i2}) + \tfrac{1}{2}\theta_i \mathrm{Var}(w_{i2}) \tag{12}$$

Summing over i and using relations shown above, we obtain

$$\ln(D_{12}\eta^{-1}) = \theta(C_1 - \bar{C}_2) + \tfrac{1}{2}\theta^2\sigma_2^2 \tag{13}$$

There are two initially unknown variables in this economy, $\bar{C}_2$ and θ, and two prices, D_{12} and P_1, from which to learn the unknown variables. In terms of the definition of a FRE, it is only necessary to be able to infer $\bar{C}_2$ from prices. However, unless both $\bar{C}_2$ and θ can be inferred from prices, it will be impossible to learn $\bar{C}_2$ when θ is unknown.

Equilibrium relations (10) and (13) were derived under the assumption that every agent knows $\bar{C}_2$ or can learn it from observing prices. If the equilibrium that results from this assumption of full information is one in which every agent could indeed infer $\bar{C}_2$, then the equilibrium is a FRE. Therefore, the question we address in the next section is whether the equilibrium relations (10) and (13) can be solved uniquely for $\bar{C}_2$ and θ.

III. Conditions for a FRE in the Two-Date Model

To determine whether (10) and (13) imply unique solutions for $\bar{C}_2$ and θ, we consider the loci of $\bar{C}_2$, θ pairs satisfying each of these relations for fixed values of the other variables. Thus, consider (10) and (13) as representing $\bar{C}_2$ as a function of θ in each relation. However, since risk averse agents imply positive aggregate risk aversion in the economy, we are only concerned with positive values of θ.

In (10), $\bar{C}_2$ is linear in θ with positive slope σ_2^2. As regards (13), there are two cases depending on the sign of $\ln(D_{12}\eta^{-1})$. In both cases, $d\bar{C}_2/d\theta$ goes to $\sigma_2^2/2$ as $\theta \to \infty$ in (13). Note that this slope is less than that in (10).

The case when $\ln(D_{12}\eta^{-1}) < 0$ is shown in Figure 1. From the analysis above, it is clear that there is always exactly one pair of values $\bar{C}_2$, θ (with $\theta > 0$) satisfying (10) and (13) simultaneously in this case. When $\ln(D_{12}\eta^{-1}) > 0$, there are three possibilities depending on the relative magnitudes of $\ln(D_{12}\eta^{-1})$ and $(C_1 - P_1D_{12}^{-1})^2/2\sigma_2^2$, assuming the latter expression is positive, as shown in Figs. 2, 3 and 4.

When $\ln(D_{12}\eta^{-1}) > (C_1 - P_1D_{12}^{-1})^2/2\sigma_2^2$, the situation is as shown in Fig. 2 where no pair of values $\bar{C}_2$, θ satisfies both (10) and (13) with $\theta > 0$. When the inequality is reversed, there always exist two pairs $\bar{C}_2$, θ (with $\theta > 0$) satisfying (10) and (13) simultaneously, as depicted in Fig. 3. In both Figs. 2 and 3 a FRE does not exist, since agents cannot accurately infer $\bar{C}_2$ from observing prices.

Finally, Fig. 4 shows the situation when $\ln(D_{12}\eta^{-1}) = (C_1 - P_1D_{12}^{-1})^2/2\sigma_2^2$. Here, as in Fig. 1, exactly one pair $\bar{C}_2$, θ satisfies (10) and (13) with $\theta > 0$. It can be shown that the situation in Fig. 4 yields the solution $\bar{C}_2 = C_1$, so that this case implies that the expected value at date 1 of the date 2 aggregate dividend is just equal to the date 1 aggregate dividend. Furthermore, aggregate risk aversion in this case must satisfy $\theta = (C_1 - P_1D_{12}^{-1})/\sigma_2^2$.

Although the cases in Figs. 1 and 4 both imply the existence of a FRE, it is obvious that the latter case is a special circumstance that is not robust to small

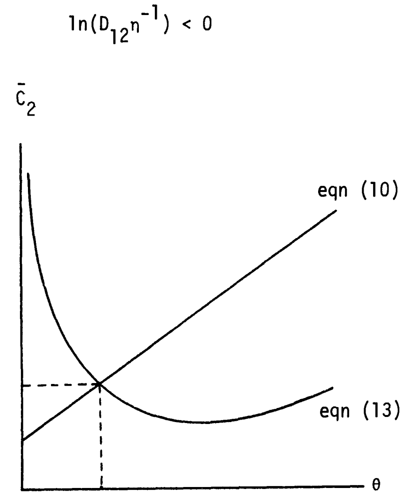

Figure 1.

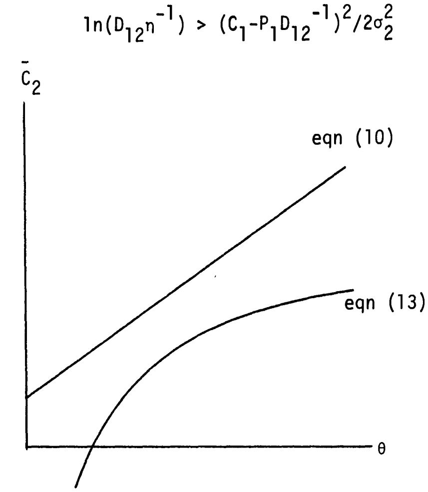

Figure 2.

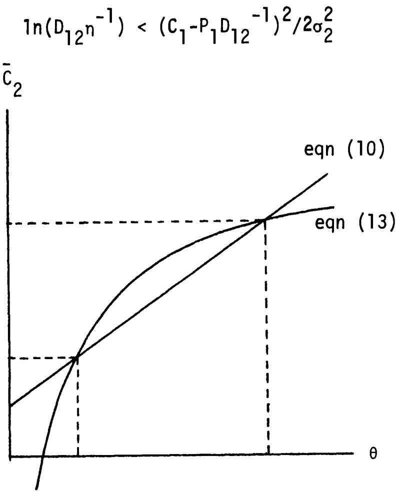

Figure 3.

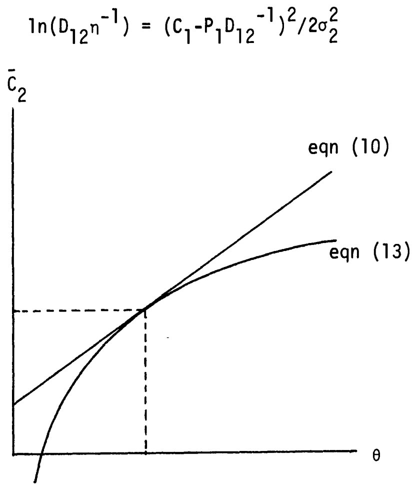

Figure 4.

changes in the parameters of the economy. In general,

$$\text{A FRE exists if and only if} \quad D_{12} < \eta. \tag{14}$$

Note that all agents know at date 1 whether or not this condition prevails since η is known and D_{12} is observed.

The condition for a FRE is that the date 1 price of a riskless dollar to be received at date 2 is less than the rate of impatience. Considering (13), this condition is equivalent to

$$\bar{C}_2 > C_1 + \tfrac{1}{2}\theta\sigma_2^2 \tag{15}$$

when date 2 social consumption is riskless ($\sigma_2 = 0$), we observe that $D_{12} < \eta$ requires $\bar{C}_2 > C_1$ so that an increased supply of date 2 consumption drives its date 1 price, D_{12}, below the rate of impatience. With risky date 2 aggregate social consumption, the condition for a FRE is that the date 1 certainty equivalent of date 2 aggregate consumption, $\bar{C}_2 - \tfrac{1}{2}\theta\sigma_2^2$, exceed the supply of date 1 aggregate consumption, C_1.

IV. Extension to a Three-Date Model

Our principal objective in extending the analysis to a three-date economy is to be able to incorporate a term structure in bond prices. To do this, we extend the model back in time one period to include date 0 and assume that agent i is endowed at date 0 with a fraction x_{i0} of the risky asset, which pays an aggregate dividend C_t at dates $t = 0, 1, 2$. Wealth w_{i0}, consumption c_{i0}, risky investment x_{i1} and riskless bond investment b_{i01} and b_{i02} are added to the model. Note that both one-period and two-period riskless bonds are available at date 0. Date 0 prices of the risky asset and the two riskless bonds are P_0, D_{01} and D_{02}, respectively.

Initial wealth, date 0 budget constraint, and date 1 payoffs are given by, respectively,

$$w_{i0} = (C_0 + P_0)x_{i0} \tag{16}$$

$$w_{i0} = c_{i0} + P_0 x_{i1} + D_{01}b_{i01} + D_{02}b_{i02} \tag{17}$$

$$w_{i1} = b_{i01} + D_{12}b_{i02} + (P_1 + C_1)x_{i1} \tag{18}$$

Market clearing conditions for date 0 are

$$\textstyle\sum_i b_{i01} = \sum_i b_{i02} = 0 \tag{19}$$

$$\textstyle\sum_i x_{i0} = \sum_i x_{i1} = 1 \tag{20}$$

$$\textstyle\sum_i c_{i0} = C_0 \tag{21}$$

In order to develop the conditions for a FRE, we again begin by deriving equilibrium conditions assuming all agents are fully informed and then investigate whether these relations are such that initially uninformed agents can infer the desired information from equilibrium prices. As in the two-date model, we assume agents are initially uninformed about the aggregate risk aversion parameter θ. In addition, we can incorporate uncertainty of a particular sort about the impatience parameter η. Agent i, who is not large enough to affect the equilibrium, knows that all other agents share a common impatience parameter (or that the equilibrium can be described as if this were true) but does not know the value of this parameter.

To describe beliefs about future aggregate dividends on the risky asset, we first define a variable representing the difference between the date 1 expected value of the date 2 aggregate dividend and the date 1 aggregate dividend:

$$\Delta \equiv \bar{C}_2 - C_1 \tag{22}$$

We assume that date 0 information, which may or may not be initially possessed by a group of "informed" agents but which, in any case, other agents try to learn from observing prices, includes the value of Δ. However, we do assume that both $\bar{C}_2$ and C_1 are random variables at date 0 and that only their difference Δ is non-stochastic. The aggregate dividend will follow a Markov process where the information Δ represents the expected change in production. This assumption is made to ensure a tractable derived utility function for date 1 wealth, since it makes D_{12} non-stochastic. The date 1 information structure is thus slightly different from the model in the previous section in that, if Δ has been revealed by prices at date 0, $\bar{C}_2$ is known to all at date 1 and does not need to be revealed by date 1 prices. All agents know from the start that C_1 is distributed $N(\bar{C}_1, \sigma_1^2)$, while the distribution of C_2 will, as in the previous model, be known at date 1 to be $N(\bar{C}_2, \sigma_2^2)$ if a FRE exists.

To summarize the state of date 0 information, the three items which initially uninformed agents can learn if a FRE exists include θ, η and Δ. Information about these variables is imparted by equilibrium values of P_0, D_{01} and D_{02}. Although the particular information structure we have assumed is somewhat contrived, it possesses the virtue of making the problem much more tractable than under alternative assumptions and preserves the basic objective of a setting in which risky asset prices and a term structure of bond prices may convey information about the fundamental factors of risk aversion, impatience and beliefs.

Our strategy to discover if a FRE exists is, as before, to assume complete information, solve for equilibrium prices and check whether it is indeed possible to become informed by observing these prices. We first establish the conditions for individual optimality. Agent i's date 0 utility is

$$U_{i0}(c_{i0}, c_{i1}, w_{i2}) = -\exp(-\theta_i c_{i0}) - \eta \exp(-\theta_i c_{i1}) - \eta^2 \exp(-\theta_i w_{i2}) \tag{23}$$

If a FRE exists, (13) will prevail at date 1. This can be written as

$$\ln(D_{12}) = \ln(\eta) - \theta\Delta + \tfrac{1}{2}\theta^2\sigma_2^2 \tag{24}$$

Therefore, if a FRE exists, the date 1 price of a riskless bond maturing at date 2 is known at date 0. Therefore, to avoid riskless arbitrage in the term structure of bond prices, we must have

$$D_{01}D_{12} = D_{02} \tag{25}$$

This allows us to set $b_{i02} = 0$ in w_{i0} without loss of generality, since a two-period bond is perfectly duplicated by rolling over successive one-period bonds.

Substituting from (1), (2), (9) and (10) in (8), we can derive the following relation for $-\theta_i c_{i1}$ from date 1 optimality.

$$-\theta_i c_{i1} = D_{12}(1 + D_{12})^{-1}[-\ln(D_{12}\eta^{-1}) - \tfrac{1}{2}\theta^2\sigma_2^2 - D_{12}^{-1}\theta_i w_{i1}] \tag{26}$$

Taking expectations in (23), applying (7) and (26) for w_{i2} and c_{i1}, date 0 expected utility can be written as

$$E(U_{i0}) = -\exp(-\theta_i c_{i0}) - \Phi E \exp[-\theta_i(1 + D_{12})^{-1} w_{i1}] \tag{27}$$

where we define

$$\Phi \equiv \eta(1 + D_{12})(\eta D_{12}^{-1})^{D_{12}/1+D_{12}} \exp[-\tfrac{1}{2} D_{12}\theta^2\sigma_2^2(1 + D_{12})^{-1}] \tag{28}$$

Using (17) for c_{i0} and (18) for w_{i1} and maximizing (27) w.r.t. b_{i01} and x_{i1}, the date 0 first order conditions for agent i are

$$D_{01}\exp(-\theta_i c_{i0}) = (1 + D_{12})^{-1}\Phi E \exp[-\theta_i(1 + D_{12})^{-1} w_{i1}] \tag{29}$$

$$\begin{aligned} P_0\exp(-\theta_i c_{i0}) = {} & (1 + D_{12})^{-1}\Phi[E(P_1 + C_1) \\ & - \theta_i(1 + D_{12})^{-1}\mathrm{Var}(P_1 + C_1)]E \exp[-\theta_i(1 + D_{12})^{-1} w_{i1}] \end{aligned} \tag{30}$$

From the date 1 CAPM relation (10),

$$P_1 + C_1 = (1 + D_{12})C_1 + D_{12}(\Delta - \theta\sigma_2^2) \tag{31}$$

Since a FRE implies Δ is known at date 0, we can divide (30) by (29) and use (31) to calculate the moments of $P_1 + C_1$ to obtain the CAPM relation for date 0:

$$P_0 = D_{01}[(1 + D_{12})(\bar{C}_1 - \theta\sigma_1^2) + D_{12}(\Delta - \theta\sigma_2^2)] \tag{32}$$

Our second date 0 equilibrium relation is derived by taking the log of (29), summing over i and simplifying using (24), (31) and (32) to obtain

$$\ln(D_{01}\eta^{-1}) = \theta(C_0 - \bar{C}_1) + \tfrac{1}{2}\theta^2\sigma_1^2 \tag{33}$$

The equilibrium relations (32) and (33) along with (24) are a system of three equations relating the observed equilibrium prices P_0, D_{01} and D_{12} to the three variables revealed by these prices if the equilibrium is a FRE: θ, η and Δ. We note that this system displays a certain degree of separability, since Δ does not affect D_{01} (given θ and η) and η affects the ratio of P_0 to D_{01} only indirectly through its effect on D_{12}. The latter price, however, is directly affected by θ, η and Δ.

Given complete information about θ, η and Δ, equilibrium prices satisfy (24), (32) and (33). Therefore, the question of whether this equilibrium can indeed be sustained as a FRE, which we address below, is equivalent to asking whether this system implies unique values of θ, η and Δ given observed values of P_0, D_{01} and D_{12} (which is inferred from the term structure D_{01} and D_{02} by (25)).

V. Conditions for a FRE in the Three-Date Model

The system of equations (24), (32) and (33) can be rewritten so as to permit us to apply similar arguments to those used in Section III. Eliminating η between (24) and (33) gives

$$\ln(D_{12}D_{01}^{-1}) = (\bar{C}_1 - C_0 - \Delta)\theta + \tfrac{1}{2}(\sigma_2^2 - \sigma_1^2)\theta^2 \tag{34}$$

If (32) and (34) imply unique solutions for θ and Δ, then a FRE exists because the solution for η will also be unique, as can be seen from (24) or (33). As before, we assume universal risk aversion and are only concerned with solutions for which $\theta > 0$.

Following the arguments in Section III, consider (32) and (34) as each representing Δ as a function of θ. In (32), Δ is linear in θ with positive slope $(1 + D_{12}^{-1})\sigma_1^2 + \sigma_2^2$. In (34), the shape of Δ as a function of θ is similar to that of $\bar{C}_2$ as a function of θ in (13), but where the different cases analogous to those shown in Figs. 1–4 depend on the magnitude of $(\ln(D_{12}D_{01}^{-1})$. Therefore, we conclude that the only robust condition for a FRE is when $\ln(D_{12}D_{01}^{-1}) < 0$. Thus,

$$\text{A FRE exists if and only if} \quad D_{12} < D_{01} \tag{35}$$

Letting r_{01} and r_{02} represent the yields to maturity on one-period and two-period discount bonds, respectively, at date 0, we have $D_{01} \equiv (1 + r_{01})^{-1}$ and $D_{02} = (1 + r_{02})^{-2}$. The condition $D_{12} < D_{01}$ in (35) is equivalent, using (25), to $D_{01} > D_{02}$. In terms of yields to maturity, $D_{01} > D_{02}$ can be written as $r_{01} < r_{02}$. In other words, we have established that a FRE exists in the three-date economy if and only if the term structure of yields at date 0 is upward sloping. If this condition is met, then information about beliefs (Δ) can be inferred from prices separately from information about preferences (θ and η). Otherwise, alternative beliefs cannot be distinguished by observing prices since difference beliefs are consistent with the same prices when preferences are not known.

VI. Summary and Conclusions

In the two-date model, initially uninformed agents use the prices of the risky asset and one-period riskless bond as sources of information about aggregate risk aversion (θ) and the expected date 2 aggregate dividend ($\bar{C}_2$). They are able to learn these items, so that the equilibrium is a FRE, when the price of the bond (D_{12}) is less than the rate of impatience (η).

In the three-date model, the sources of information are the prices of the risky asset, the one-period bond and the two-period bond (or a forward loan). Agents try to use these prices to learn the values of aggregate risk aversion, rate of impatience and expected future growth one period hence in social consumption (Δ). The condition for the equilibrium to be a FRE in this economy is that the term structure of yields be upward sloping.

Obviously, the assumptions describing the economies we have considered here do not lead us to regard our conclusions as having wide generality; nonetheless, we have been able to cast a bit of light on the important and interesting question of whether changes in beliefs can be distinguished from changes in preferences by observing prices. We have shown that there are at least some circumstances in which the answer is "yes", but whether or not the behavior in these special circumstances indicates tendencies that are present in more general situations remains to be investigated.

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DISCUSSION

BETH ALLEN*: The paper by Alan Kraus and Gordon Sick addresses a question which is of interest both to specialists in the theory of financial markets and to persons working in microeconomic theory. The ability to separate effects which are due to changes in agents' preferences from those which result from changes in traders' beliefs about probability distributions is important, especially if you believe, as I do, that preferences should be viewed as a fixed (albeit unobservable) primitive characteristic in the description of an economic agent, while probability beliefs tend to be subjective and to change, especially as one acquires information.

However, I do have some reservations about what the Kraus-Sick results tell us about markets under uncertainty. My remarks will focus on the themes of restrictions on parametric forms, the possibilities for recoverability of preferences from demands, and the invertibility of equilibrium price functions. I shall begin

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