Tests of the Black-Scholes and Cox Call Option Valuation Models: Discussion
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DISCUSSION

STEVEN MANASTER*: MacBeth and Merville (hereafter MM) test the Cox call option model for constant elasticity of variance diffusion process against the constant variance option model of Black and Scholes (hereafter BS). Given that the Cox model includes the BS model as a special case, it is clear that the Cox model must explain observed option prices at least as well as the BS model. The empirical observation that stock return variances move inversely with stock price almost guarantees that the Cox model, for reasonable parameter values, will perform better than the BS model. Therefore, it is not surprising that, on the basis of their tests, MM conclude that the Cox model fits the market data better than the BS model. Rather than discussing MM's conclusions, in these comments I should like to address two related questions; first, what is the most useful method to estimate the Cox model parameters δ and θ; and second, is the extra effort required to estimate these parameters justified by the improved results relative to the BS model.

The principle benefit of the Cox model relative to the BS model is noted by MM. They correctly explain that if the variance rate (σ^2) changes with stock price changes, the BS model will require constant ad hoc adjustments in the value of σ^2 employed in the BS formula. The Cox model, on the other hand, automatically adjusts for changes in the variance rate. The principle cost associated with the Cox model is that two parameters, δ and θ, must be estimated rather than σ alone.

Under either the Cox or the BS assumptions the parameters, δ, θ and σ define characteristics of the underlying stock. In principle their values can be estimated from stock price and return data alone; no reliance on option prices nor on a specific valuation formula is required. In practice however, researchers have assumed a particular option valuation formula and used observed option prices to calculate implied parameter values. The implied parameters are constructed so that prices calculated from the formula closely fit the observed option prices. In using implied parameter values, care must be taken not to employ the same model and data twice, once to estimate the parameters and again to make inferences regarding the validity of the model.

MM estimate the parameters δ and θ using a three step procedure. First, they

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use the Cox model and search for integer values of θ for which the implied values of δ are most nearly constant for each stock. Second, they use the BS model and their previously reported methods to obtain implied values of σ. These implied values, $\hat{\sigma}_t$, vary over time and are recalculated for every market day in 1976. Finally, using the $\hat{\theta}$ and the $\hat{\sigma}_t$ estimates, on each day, a new value of $\hat{\delta}_t$ is calculated for each stock. The value $\hat{\delta}_t$ is calculated so that the following equation is satisfied.

$$\hat{\sigma}_t = \hat{\delta}_t S_t^{(\hat{\theta}-2)/2} \tag{1}$$

In equation (1) S_t represents the underlying stock price at time t.

If the Cox model is the correct model and if $\hat{\delta}_t$ and $\hat{\theta}$ are equal to the true parameters δ and θ, then MM are correct in stating that the right hand side of equation (1) represents the stock return's instantaneous standard deviation at time t. However, under these conditions (Cox model with $\theta \neq 2$) the BS model is not the correct model and the BS implied standard deviation, $\hat{\sigma}_t$, will not equal the true instantaneous standard deviation, σ_t. Therefore, even if the Cox model is valid, MM's interpretation of equation (1) is not correct and their estimates of $\hat{\delta}_t$ may differ systematically from the true δ.

Furthermore by using the BS implied parameter $\hat{\sigma}_t$ to estimate $\hat{\delta}_t$, MM assure that the best information available from the BS model is impounded in Cox model's $\hat{\delta}_t$. They then test the two models with the same data as was used to estimate the parameters. As expected the Cox model wins. It is easy to speculate that the results of the MM tests are not sensitive to the values of $\hat{\theta}$ (for $\hat{\theta} < 2$) since equation (1) can be solved for δ for any value of $\hat{\theta}$. This is why MM can successfully employ what they refer to as "ball park value for θ."

I suspect that results even more favorable to the Cox model could be achieved if θ and δ were estimated jointly. Finally by reestimating δ_t daily from equation (1), MM have chosen not to test an important feature of the Cox model relative to the BS model. In principle the Cox model parameters δ and θ should not require adjustments as frequently as the BS parameter σ. Since MM readjust all the parameter estimates daily, this potentially valuable feature of the Cox model is not evaluated. MM are aware that their parameter estimation technique and test procedure guarantee that on each day their calculated BS price and their calculated Cox price both will be equal to the market price of at the money options. They test in this fashion in order to focus on the differences between the two models for far in and far out-of-the money options, leaving issues "such as stationarity of model parameters for future research."

I would not like to leave the impression that I object to the daily reestimation of $\hat{\sigma}_t$ and $\hat{\delta}_t$, but I would like to know whether the Cox model continues to fit the observed data better than the BS model for the day or week following the parameter estimation. If in post-estimation testing the BS model performed as well as the Cox model, MM's conclusions would require modification. I would also have more faith in the MM conclusions if they had put their assertions to a market test. The test I envision would answer the following question: Would a trader who bought or sold options, depending on whether the market price was below or above the Cox calculated price, outperform a trader who followed a similar strategy using the BS model? the answer to this question would indicate in dollars and cents the value of the Cox model as a substitute for the BS model.

I urge MM to extend their work. Specifically, I recommend that δ and θ be estimated jointly and that a post-estimation comparison of the BS model with the Cox model be performed.