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On The Dynamic Behavior of Prices in Disequilibrium

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SATISFIED AS WE MAY BE with the overall efficiency of the market system and with the tenets of the perfect market model, we all viscerally know that were we down on the market floor we would certainly react to a multitude of apparent price discrepancies. Indeed, it is intuitively inconceivable that a man-made institution (such as the market) could be so mechanically perfect that all such discrepancies would be totally annihilated before they can be observed. Accordingly, we would expect floor traders and other professionals to be speculating abundantly on what they perceive to be the direction in which the market is going. Almost surely, such behavior has an effect on the dynamics of stock prices. A financial theory that cavalierly ignores this component in the determination of prices would be regrettably deficient.

I. Introduction

Theoretical studies of security prices have traditionally concentrated on fundamentals, e.g., the relationship between a security's anticipated cash flows and its price, given the time and risk preferences of the investors bidding for it. Recent research on the microstructure of security markets draws attention to the role of the market's organization in the determination of security prices. It is now increasingly recognized that institutional factors—such as the brokers' handling of investors' orders, the management of the limit order book, or the existence of designated specialists entrusted with an affirmative obligation to maintain "price continuity"—affect the speed of the prices' adjustment to changing conditions. This study extends the scope of the analysis one step further. It takes explicit consideration of the investors' reaction to the limitations of existing markets. When traders attempt to exploit market "imperfections", and act on their perception of the current price trend, their actions can cause phenomena which are totally unrelated to economic fundamentals, and are thus unpredictable by fundamental analysis.

For economists, who view the market as an instrument for resource allocation, the platonic ideal of a market price is a price that instantaneously equilibriates the notional supplies and demands of all traders. These *equilibrium prices* guarantee an efficient allocation of resources, and are fully determined by the current state of the economic environment, i.e., the agents' endowments, prefer

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ences, and information about exogenous variables that can affect eventual outcomes. As the environment evolves through time, equilibrium prices must change to reflect the current conditions. But a persistent equilibrium in a continually changing economy is almost paradoxical: prices must respond instantaneously to current changes in the environment, while supply and demand must be always perfectly balanced. When price movements are forced by supply and demand imbalances which may take time to clear, a nonstationary economy must experience at least some transient moments of disequilibrium. Observed prices will then depend not only on the "state of the environment", but also on "the state of the market".

For agents trading in capital assets, the market is an instrument for profit-motivated investments. Given that prices and returns depend on the state of the market, investors will base their orders not only on their information about the state of the environment, but also on their estimates about the future state of the market. Investors' orders will thus differ from their notional "Walrasian" demands, and this can make prices deviate further from the Walrasian equilibrium. It may then be rational for traders to place even higher emphasis on the state of the market, and so forth. It is not inconceivable that in some circumstances observed prices will be dominated by the dynamics of the market process and will be almost unrelated to the current state of the environment.

Such phenomena make awkward the traditional presumption of equivalence between the Walrasian equilibrium price and the observed market price. The impressive body of empirical work on stock market efficiency is motivated by the possible discrepancy between these two notions of price: it attempts to test the marginal profit potential of information on the states of the environment and/or market. Recent research in financial economics has sought to draw explicit theoretical and empirical distinctions between the two notions of price. In a recent paper, Beja and Goldman (1979), we elaborate on the market factors that bound the rate of price adjustment and study the potential role of market-making specialists in improving the price signal. Garbade and Lieber (1977) have shown the empirical relevance of distinguishing between the (observable) transaction prices and the underlying (observable) equilibrium values. Similar distinctions were made by Cohen, Maier, Schwartz, and Whitcomb (1977) in a study of security return's variance and serial correlation; and by Garbade and Silber (1979) in a comparison of the NYSE with the regional stock exchanges.

The persistence of a discrepancy between observed prices and equilibrium values is associated with a non-instantaneous price adjustment in an incessantly changing environment. Beja and Hakansson (1977a, 1977b), in their studies of a general class of dynamic market adjustment processes, point out that the instantaneous Walrasian adjustment imposes prohibitive demands on communication and computation, while (under realistic assumptions) a sequence of tatonnement iterations must take time and thus, it will typically *never* converge. Operating markets are therefore better described by general processes that admit a *finite* adjustment speed and that allow transactions at the current (non-equilibrium) market prices.

In this study, we examine the dynamical characteristics of a simple model of the market price process in a disequilibrium setting. Of course, a primary component of price change is the adjustment to correct imbalances between basic supply and demand. Our admittance of non-equilibrium trading provides incentive and opportunity for "speculation on the current price trend", apart from the speculators' assessment of the securities' fundamental prospects. Most importantly we analyze how variations in the market structure affect price signal quality.

II. Price Adjustment

Consider a security traded in an active market operating within an incessantly changing economy. Trades can take place either continuously or intermittently, but in the latter case transaction prices can still be viewed as discrete time samples of a continually changing price. Our basic premise is that price changes are forced by the *demands* of investors,[1] and that the *speed* of price adjustment is *finite*. This is formalized by the equation

$$\dot{P}(t) = H_t(D_t(P_t)) \tag{2.1}$$

where $P(t)$ is the logarithm of the security's price at time t, $D_t(\cdot)$ is the investors' excess demand function for the security at time t, and $H_t(\cdot)$ is a monotone increasing function which vanishes at 0.

The demands that investors choose to submit to the market place depend on the investors' perception of the market mechanism that transforms orders into trades. One (conceptual) mechanism that has been most extensively studied in the theory of economics and finance is the Walrasian auction, where all transactions are executed at "equilibrium prices". In contradistinction, we posit a market where transactions and price adjustments occur *simultaneously*. To use the familiar Walrasian demands and prices as points of reference, we write

$$D_t(P) = D_t^0(P) + d_t(P) \tag{2.2}$$

where D_t^0 is the aggregate demand (function) that the auctioneer would face if at time t one were to conduct a Walrasian auction in this economy. We call D^0 "fundamental demand", and $d_t(P_t)$ is simply the difference between the actual excess demand at time t and the corresponding fundamental demand. Let $W(t)$ denote the logarithm of the price that clears fundamental demand at time t, i.e., $D_t^0(W(t)) = 0$. Without loss of generality, we can write

$$D_t^0(P_t) = h_t^0(W(t) - P(t)) \tag{2.3}$$

where $h_t^0(\cdot)$ is a monotone increasing function which vanishes at 0. We call $W(t)$ "the security's underlying *equilibrium price* at time t". The notation makes explicit the possible time variability of equilibrium prices, as traders' preferences, endowments, and especially information generally change with time.

Since our market process does not conform with the Walrasian paradigm, the

[1] See Beja and Hakansson (1977a) for a general description of market processes where both price adjustments and transactions are determined on the basis of currently outstanding orders.

demands which drive prices differ from the fundamental demand by $d_t(P_t)$. In particular, consider the opportunities afforded by the existence of continual trading "out of equilibrium". The demands submitted to a Walrasian auction at a given price state each trader's desired position *if this price were equilibrium.* In a market process with non-equilibrium trading, on the other hand, the demands should also reflect the potential for *direct* speculation on price changes, *including the price's adjustment towards equilibrium.*[2] If a trader perceives a high positive trend in a security's price, his interest in holding it—at least for a short period—will typically exceed his fundamental demand. Generally, this will depend both on the trader's assessment of the current trend in the security's price and on the alternative opportunities contemporaneously available to him (through non-equilibrium trading in other securities). Let $\psi(t)$ denote the speculators' (average) assessment of the current trend in P(t), and let g(t) denote the opportunity growth rate of alternative investments in non-equilibrium trading with comparable securities. Disequilibrium speculation generates additional demand which contributes to the security's rate of price adjustment. This additional demand is assumed to be determined by $\psi(t) - g(t)$, i.e.,

$$d_t(P_t) = h_t(\psi(t) - g(t)) \tag{2.4}$$

where $h_t(\cdot)$ is a monotone increasing function which vanishes at 0. Combining (2.1)-(2.4), we get

$$\dot{P}(t) = H_t[h_t^0(W(t) - P(t)) + h_t(\psi(t) - g(t))] \tag{2.5}$$

Operationally, we shall use a first order approximation to equation (2.5), viz.

$$\dot{P}(t) = a[W(t) - P(t)] + b[\psi(t) - g(t)] + e(t) \tag{2.6}$$

where a and b are non-negative constants and $e(t)$ encompasses all "random errors" involved in the approximation, including possible market "imperfections" (such as the practice of rounding transaction prices to the nearest \$⅛). Note that the current attributes of the environment and the market are represented by the variables $W(t)$, $g(t)$, and $e(t)$, while the response constants a and b are assumed time invariant.

Strictly speaking, $W(t)$ (and indeed also $g(t)$) could in general depend on the distribution of wealth among investors, and thus on the market mechanism which determines transaction prices and trades prior to time t. However, in what follows we shall take $W(t)$ as an *exogenous* representation of the economic *environment* (e.g., as applicable when Walrasian equilibrium values are not very sensitive to the endowments' *distribution* or when this distribution is not very sensitive to the particular mechanism within the range of mechanisms being considered). In particular, the model is consistent with a random walk in the logarithm W of equilibrium prices (as in Merton 1971), with or without a drift. Similarly, the opportunity rate $g(t)$ is taken as an environmental characteristic which can also change stochastically in time.

Looking for plausible determinants of $d_t(P_t)$, we note that the traders' specu-

[2] See Arrow and Hahn (1971), Ch. 13, and Beja and Hakansson (1977a).

lation on future *equilibrium* prices is a legitimate part of their fundamental demand. Speculation (through non-equilibrium trading) on the price's adjustment toward equilibrium must be primarily based on an assessment of the *state of the market,* rather than on the state of the environment. Typically, the assessment of the price trend will be based at least in part on recent price changes. But since each price change generally involves some random phenomena that have nothing to do with basic trends, traders will hesitate to place too much weight on single isolated items of data. These considerations suggest an *adaptive* process of trend estimation. Formally, we assume that demand for speculative non-equilibrium trading is based on trend estimates which are dynamically adaptive according to the following differential equation:

$$\dot{\psi}(t) = c[\dot{P}(t) - \psi(t)] \tag{2.7}$$

where the dots again denote time-derivatives and c is a positive constant. The equation says that the trend estimate ψ is adjusted upward when the price-change is higher than expected, and vice versa.

Of course, one need not take equation (2.7) as a literal description of each trader's learning behavior. Rather, ψ represents the speculators' *average* assessment of the current price trend. The important aspects here are that (1) trend estimates are at least partly determined by recent price changes and (2) detection of the current trend is generally less than perfect, marred as it is with time delays which make the data never fully up-to-date. Higher values of c in equation (2.7) correspond to speculators' behavior that is more flexibly adaptive to current data. We have noted above, however, that because of the positive time delays that typically apply to an individual speculator's learning, and because of the random errors involved, exceedingly high values of c need not be preferable to traders. Given the stylized nature of our model a filtering methodology could improve on trend estimates. Such ultra sophisticated speculation would complicate our analysis and probably add little to our understanding of local price behavior. Henceforth, we refer to such speculation on the trend (synonymously) as trend speculation, professional speculation, disequilibrium speculation. Those who participate in such speculation are trendists, professionals.

The well known arguments against *any* use of price history are theoretically based on equilibrium considerations, which clearly do not apply to our non-equilibrium context.[3] Empirically, the present model may offer an alternative framework for testing the information value of past prices. In any event, the adaptive model of (2.7) is not suggested as normatively "optimal". Since trend following is apparently far from extinct, we believe this model to be a simplified but highly plausible *description* of existing markets. The main concern in this study is with the implications for the dynamic behavior of prices when both fundamental demand and speculation based on recent price movements are present in the market.

[3] See Samuelson (1965) and Fama (1965).

III. Dynamic Behavior—Stability and Oscillation

Our description of the market system concentrated on the forces which determine price changes. Given the current values of the system's "state variables"—the price P and the estimated trend ψ—changes in these variables are determined up to some random noise e by the current state of the environment, that is the fundamental value W and the opportunity rate g. A "solution" of the differential equations (2.6)-(2.7) is a statement of the whole time path of the state variables induced by the time path of the exogenous variables from any possible initial conditions.

This view of the system's behavior associates the dynamic behavior of observed prices with the dynamics of the environment. Notice, however, that some important dynamic characteristics of the system are generated endogenously and determined primarily by the structural constants a, b and c. For example, consider the consequences of a totally random instantaneous price increase. Ceteris paribus, this will subsequently induce a lower fundamental excess demand and a higher assessment of the current price trend. The two effects have conflicting implications for subsequent price changes. The dynamic behavior they induce will depend on the market impact of fundamental demand, on the market impact of demand based on "the professionals" assessment of the current price trend, and on the extent of the trendists flexibility in adjusting their estimates on the basis of current data.

This section is devoted to a study of the system's dynamic behavior. We first present the solution to the basic differential equations, thus linking the time path of price to the time path of the environment in general and to the time path of the fundamental (equilibrium) value in particular. We then show how instabilities and oscillations can arise endogenously, and study the relationship of these phenomena to the system's parameters.

3.1 *The Time Path of Prices*

In presenting the solution to the basic differential equations, there is an unavoidable tradeoff between exceedingly lengthy expressions and the introduction of additional notation. In what should hopefully be a reasonable compromise, we rewrite equations (2.6)-(2.7) as

$$\dot{X}(t) = MS(t) + \gamma f(t) \tag{3.1}$$

where

$$X(t) = \begin{pmatrix} P(t) \\ \psi(t) \end{pmatrix} M = \begin{pmatrix} -a & b \\ -ac & -c(1-b) \end{pmatrix} \gamma = \begin{pmatrix} 1 \\ c \end{pmatrix}$$

and $f(t) = aW(t) - bg(t) + e(t)$. Then it can be directly verified that[4]

$$X(t) = \Phi(t)X(0) + \int_0^t \Phi(t-s)\gamma f(s)\, ds \tag{3.2}$$

[4] See Gikhman and Skorokhod (1972).

where $\Phi(t)$ is the corresponding "fundamental matrix".

Equation (3.2) gives an explicit expression for the time path of the price logarithm $P(t)$ and its estimated trend $\psi(t)$ for any starting state $P(0)$, $\psi(0)$ and any realization of the time paths of $W(t)$, $g(t)$ and $e(t)$.

3.2 *Stability and Oscillation*

The basic dynamic properties of stability and endogenously generated oscillations are fully reflected by the eigenvalues λ. If the fundamental matrix $\Phi(t)$ contains exponentials with non-negative coefficients of t, then it is evident from (3.2) that the state of the system at any given time will have a non-negligible (and even magnified) effect on the system for an indefinite future. On the other hand, if all exponentials in $\Phi(t)$ have (strictly) negative coefficients of t, the values of all variables at time t will have no more than negligible effect on the state of the system at dates τ sufficiently larger than t, and the system is said to be "stable".

PROPOSITION 1. *The system is stable if and only if* $a > c(b - 1)$.

Endogenously generated oscillations are evident in the sine and cosine waves of $\Phi(t)$, which exist whenever λ has an imaginary part. This can be stated as

PROPOSITION 2. *The system is oscillatory if and only if* $(1 - \sqrt{a/c})^2 < b < (1 + \sqrt{a/c})^2$.

The dynamic properties associated with any combination of the system's parameters a, b and c, as depicted in propositions 1 and 2, are summarized in Fig. 1. The properties associated with each of the regions in this diagram are intuitively illustrated below in Fig. 2 with the price time path under the idealized conditions when $W(t) = W$, $g(t) = 0$ and $e(t) = 0$ for all t, and $\psi(0) = 0$ but $P(0) < W(0)$. In this case the relevant part of (3.2) reduces to

$$P(t) = W + \{P(0) - W\}[(\lambda^+ + a)e^{\lambda^- t} - (\lambda^- + a)e^{\lambda^+ t}]/(\lambda^+ - \lambda^-). \qquad (3.3)$$

The notation of unstable markets may perhaps sound somewhat fantastic and unduly alarming. Nevertheless, the stories behind regions IV and V in Fig. 2 are strikingly similar to Galbraith's (1972) accounts of the market atmosphere in the twenties and of the events leading to the great crash of 1929.

It is interesting to see in Fig. 1 how the dynamic characteristics of the system change with the market parameters a, b, c. We first note that the dynamics are not fully determined by the *relative* values of the market impacts a and b of the fundamental and extrapolatory demands, respectively, but by their *joint* values. Increasing the market impact of the trendists will make the system oscillate and then become unstable, as expected. The effects of changes in c are somewhat more surprising. More flexible trendists, who are quicker to detect trend changes, are characterized by high c. It is interesting that the effect of higher c on the system's stability and oscillatory characteristics is exactly equivalent to *lower* values of a. Increasing c may induce oscillations, but for sufficiently high c the system is non-oscillatory. The system is never unstable if $b < 1$. When c is small,

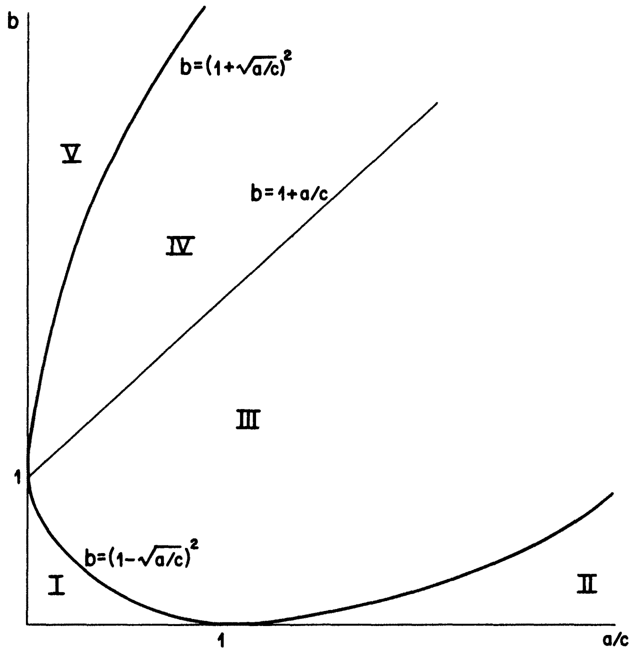

Figure 1.

the system can be stable even with $b > 1$. However, for sufficiently high c the stability is determined by whether b is greater than or less than 1.

A system governed exclusively by fundamental demand (i.e., $b = 0$) is inherently stable and has no endogenously generated oscillations. But when trendists have *some* market power ($b > 0$), increasing the impact of 'a' of fundamental demand may induce oscillations in an initially non-oscillatory system. However, no matter what the speculators' impact b and their speed of adjustment c may be, when 'a' is sufficiently large fundamental demand dominates and the system is indeed stable and nonoscillatory.

IV. The Rate of Convergence to Equilibrium

The theoretical significance of equilibrium prices derives to a large extent from their potential as efficient signals for resource allocation. But for these efficient signals to be operationally effective, equilibrium values must be at least approximately observable in actual market prices. Indeed, this has been suggested as a conceptual measure of "stockmarket efficiency".[5] A faster convergence to equilibrium can accordingly be considered an improvement of the signaling accuracy of prices. This section is devoted to the determinants of the system's rate of convergence toward equilibrium.

Two relevant observations can be called from the previous section. First, that excessive speculation on the price-trend causes oscillations and even instabilities.

[5] See Fama (1970).

These endogenously generated phenomena—which have nothing to do with the state of the environment—spoil the signaling potential of prices. Second, study of equation (3.2) indicates that an arbitrarily close approximation of equilibrium values by observed prices requires an arbitrarily high impact of fundamental demand (arbitrarily high 'a'). These two observations may lead one to associate higher values of 'a' with faster convergence to equilibrium, and higher values of b with greater divergence from equilibrium. However, market interaction between the two basic forces introduces subtle variations in their respective effects on the process of price convergence. For example, when speculators identify an adjustment trend and act on the basis of their assessment, their action accelerates the price adjustment initiated by fundamental demand (i.e., 'b' augments 'a'). On the other hand, a very strong fundamental pressure for adjustment toward equilibrium may provoke a chain of speculative activities that causes price overshooting and ultimately hampers the convergence to equilibrium.

We study these phenomena by an analysis of the path of convergence to a fixed W, with $\psi(0) = 0$ and $P(0) \neq W$, as done in the previous section (see Fig. 2

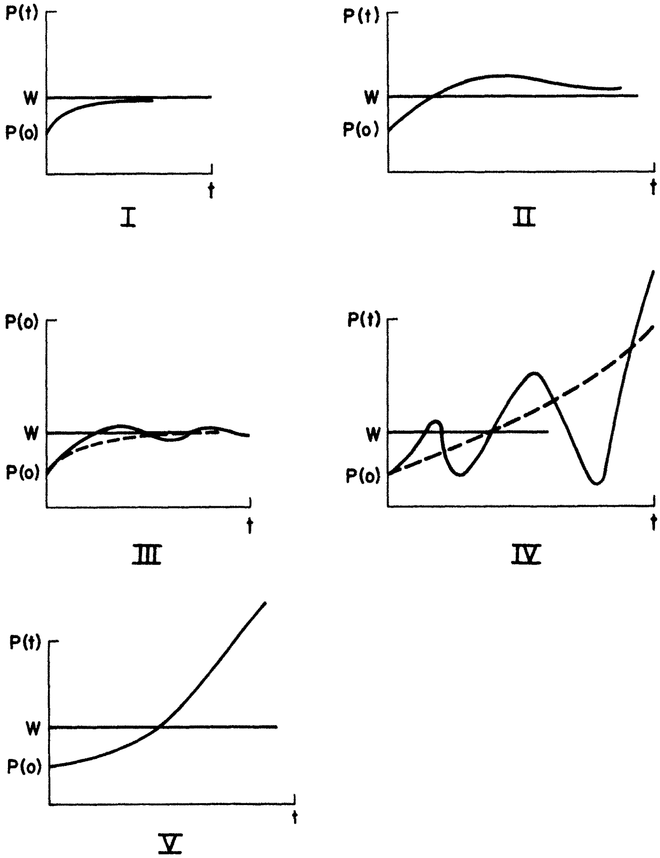

Figure 2.

and equation (3.3)). It will be convenient to transform equation (3.3) into

$$\eta(t) = \frac{P(t) - W}{P(0) - W} = \frac{(\lambda^+ + a)e^{\lambda^- t} - (\lambda^- + a)e^{\lambda^+ t}}{\lambda^+ - \lambda^-} \tag{4.1}$$

$\eta(t)$ is a measure of the average (relative) *residual deviation* of observed prices from equilibrium values. Given $P(0)$, lower values of the "squared deviation" $[\eta(t)]^2$ occur when $P(t)$ is closer to W.

The *initial* trend in the squared residual deviation is determined *exclusively* by fundamental demand. This is apparent in the time derivative of $[\eta(\mathrm{t})]^2$ at $t =$ 0, viz:

$$\left.\frac{d[\eta(t)]^2}{dt}\right|_{t=0} = -2a \tag{4.2}$$

A higher impact 'a' of fundamental demand causes a faster initial convergence toward equilibrium. When $b = 0$ the residual deviation is $\eta(t) = e^{-at}$, and the squared residual deviation is inversely related to 'a' for all t. However, the trendists' response can transform this effect, and the consequences of an increase in 'a' need not be uniform in time. The next proposition shows that an increase in the market impact of fundamental demand can in some conditions *increase* the squared residual deviation $[\eta(t)]^2$ for "almost all t".

PROPOSITION 3. When the system is stable and non-oscillatory, $a > c$, and $b >$ 0, increasing the impact 'a' of fundamental demand on price adjustment increases the squared residual deviations $[\eta(t)]^2$ *for all sufficiently large t.*[6]

In the market structure postulated in the previous proposition, increasing the impact of fundamental demand failed to decrease the (squared) residual deviations because of the delayed effects of the slowly adaptive speculators. Our next proposition demonstrates that increasing both the impact 'a' of fundamental demand and the rate of adjustment 'c' of the speculators' trend assessment does indeed accelerate the process. Let a_0 and c_0 be any fixed values, and let $\zeta(t; \alpha)$ be defined for $\alpha, t > 0$ as the value $\eta(t)$ of (4.1) when $a = \alpha a_0$ and $c = \alpha c_0$. The effect of a simultaneous change in both 'a' and 'c' in accelerating the system is given by

PROPOSITION 4. $\zeta(t; \alpha) = \zeta(\alpha t; 1)$.

The proposition shows that the residual deviation at time t with $\alpha > 1$ is equivalent to the deviation that would prevail at a later time in a system with α = 1. Increasing α is therefore an *acceleration* of the process. In convergent systems, acceleration is associated in some weak sense (but not uniformly) with "smaller" residual deviations.

The parameter 'c' represents the professionals' speed in detecting trend changes. It is interesting to note that an increase in 'c' alone—which in a sense

[6] It may seem paradoxical that there exists some a_0 such that for all $a > a_0$ an increase in a increases $[\eta(t)]^2$ for almost all t, while $[\eta(t)] \to 0$ as $a \to \infty$ for all $t > 0$. This is resolved when one notes that the convergence is not monotone and that (4.4) is negative on an interval $(0, T(a))$ where $T(a) \to \infty$ as $a \to \infty$.

improves the professionals' assessment—will *not necessarily* improve the price signal.

The trendists' demand is not based on the security's fundamental prospects. It is therefore not surprising that their presence in the marketplace can interfere with the convergence to equilibrium, as seen in various forms in Propositions 1, 2, and 3. But the trendists do respond to price adjustments originated by fundamental demand, and their effect on the system is thus not unrelated to the equilibrium values. Our last proposition identifies conditions in which the presence of non-equilibrium speculation *contributes* to the convergence to equilibrium and hence to the quality of the price signal. Furthermore, the contribution takes effect immediately and lasts through time.

16 PROPOSITION 5. When $c > a$ and $b < (1 - \sqrt{a/c})^2$, increasing b decreases $[\eta(t)]^2$, *uniformly* in t.

Our result shows that although excessive speculation is harmful to the signaling quality of prices, moderate speculation sometimes improves the price signal. This applies when $c > a$, i.e., when the speculators' response to changes in price movements is relatively fast, while the impact of fundamental demand on price adjustment is relatively slow. In this case, the existence of some (moderate) speculative activity makes prices closer to equilibrium than they would be if determined exclusively by fundamental demand. This holds even when fundamental demand is fully governed by the drive towards equilibrium, while the disequilibrium speculation is based on no more than chart reading.

V. On Equilibrium and Disequilibrium Speculation

The behavior which we have loosely termed "speculation on the price trend" played a central role in our analysis of the dynamic behavior of prices. The effects of speculators participating in a market has long been a matter of controversy in financial economics. Perhaps much of the intellectual muddle about the effects of speculation is attributable to the imprecise definition of the issues. Indeed, the mere attempt to define speculation is fraught with danger. Intuitively, we consider as speculation any position which is taken primarily on the basis of the trader's forecast of future prices when he believes that his assessments are not shared by the market at large. In this context, we can list three issues in the role of speculation in securities markets:

1) Do speculators aid in augmenting more information in *equilibrium* prices? (cf. Grossman and Stiglitz 1976, Goldman and Sosin 1979).
2) Still in the *equilibrium* framework, do speculators create a risk sharing mechanism by participating in the market? (cf. Cootner, 1960)
3) In a *disequilibrium* context, do speculators affect the relationship between market prices and equilibrium prices?

An investigation of the equilibrium questions (1) and (2) would seek to determine the benefits of speculation in achieving "better" *equilibrium* positions. However, by investigating the third inquiry, the disequilibrium question, we could achieve an understanding of when speculation would help make the equilibrium analysis relevant. That is, should speculators "force" market prices to converge to equilibrium prices (quickly) then equilibrium analysis would be very relevant; however, should speculators cause substantial price changes which are unrelated to the equilibrium prices, then results forthcoming from equilibrium analysis would be of questionable value.

The results in sections III and IV above relate to the third aspect of the role of speculators—their impact on the relationship between equilibrium prices and actual market prices. We have seen that speculative trading can accelerate the convergence to equilibrium, but in excess it can also cause price oscillations and instability. The quality of the price signal is determined by the mix of trendist speculation and fundamental demand, and by the trendists "flexibility" in assessing the current price trend.

The impact of trendist speculation on price movement depends on the extent that traders get involved in this activity. The extent of involvement will in turn depend on its profitability.[7] Without attempting a comprehensive analysis of the trendists' profits, we note that the activity is profitable during periods when the signs of $[\psi(t) - g(t)]$ and $[P(t) - g(t)]$ coincide: when they are positive, the professionals are long in fast growing assets, and when negative, the professionals are short in slowly growing (or falling) assets. In the idealized conditions of Fig. 2, with a stationary environment and no "random noise" in price changes ($e(t) = 0$), disequilibrium speculation would tend to be more profitable when price movements are monotone. If there is practically no trendist activity in this environment, monotonicity applies. The profit motive would then encourage trend speculation by professionals with flexible trend assessment (high c). When the professionals' involvement in speculation on the price trend is more massive, the higher values of b induce price oscillations. Monotone convergence no longer applies, and this eliminates much of the profit potential of trend speculation. In stochastically changing environments, the random fluctuations $e(t)$ not only increase the risks in trend speculation, but also further restrict the expected profits.[8] These considerations indicate that the profit motive may tend to induce a non-zero level of trendist activity, but keep it limited in impact so that the system's parameters lie in the stable regions I–III. Note, however, the limited power of the profit motive to control the system if speculators somehow get to be extremely active. With a high value of b the path in region V of Fig. 2 is again monotone, and high (or low) values of both $P(t)$ and $\psi(t)$ tend to persist. Trend speculation appears highly profitable—the profit motive will certainly not reduce the impact of speculators, and will only reinforce the instability of the system. In this case, stability can be restored only by active policy measures that discourage speculation. Such measures (e.g., taxes on "speculative profits") cannot distinguish between equilibrium and disequilibrium speculation. Consequently, the policy will not only reduce the impact coefficient b, but will also suppress the equilibrium speculation which is a legitimate part of the fundamental demand.

[7] The authors thank Carl Futia for his suggestions relevant to this path of inquiry.

[8] This happens because the random elements in observed prices induce a negative autocorrelation in price movements, cf. Beja and Goldman (1979) and Cohen, Maier, Schwartz and Whitcomb (1977).

This will alter the equilibrium prices and restrict the positive potential of equilibrium speculation in information dissemination and in risk sharing.[9]

To simplify the analysis and the exposition, disequilibrium speculation was represented in our system by a specialized model of adaptive trend forecasting (i.e., "exponential smoothing" of past trends). Clearly, some of the trading strategies used by today's trend followers employ different schemes. Yet, one basic difference between equilibrium and non-equilibrium speculation is that assessment of future *equilibrium* prices will typically be based on *fundamental* data, whereas forecasting the *non-equilibrium* adjustment is primarily *technical.* Recent research on the "weak form of stockmarket efficiency" has been interpreted as evidence that technical "trendism" does not pay.[10] If this interpretation is accepted by investors, it may be more effective than policy measures in restricting the impact of disequilibrium speculation and its negative effect on the efficiency of the price signal. But when the system is in the unstable region V, it may be extremely difficult to convince investors to disregard past trends, while it is precisely in this region that restricting speculation is most necessary.

Finally, it should be emphasized that our simple, stylized model is but a local aproximation of the real price dynamic. Obviously, many global forces come into play in the long run that could dramatically shift our parameter values. Accordingly, our analysis is intended as a guide to short term, not to long term price behavior.

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[9] At the very least, such measures will also tend to reduce a, and thus also make the convergence to equilibrium very slow even if such convergence is established.

[10] Cf. Fama (1970).

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