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Empirical Studies in Portfolio Performance Using Higher Degrees of Stochastic Dominance

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MUCH OF THE THEORETICAL and empirical work in portfolio analysis in the past decade has been based on the principal of utility maximization where either (a) the investor's utility function is assumed to be quadratic with nonpositive second derivatives or (b) the probability functions are assumed to be normal.¹ If at least one of these conditions holds, it can be shown that choosing among risky assets on the basis of their mean and variance only is consistent with the Von Neuman-Morgenstern² utility maximization model. Thus, given the above assumptions, the incorporation of higher moments of a distribution and the adoption of alternative approaches to portfolio selection have largely been ignored in favor of the well-known Markowitz [21], [22] model.

However, an increasing number of writers have challenged the assumptions on which the Markowitz approach is founded. Fama [6], Breen and Savage [4], and others have shown that distributions of stock price changes are inconsistent with the assumption of normal probability functions.³

Other sets of utility functions pertaining to the investment models are cubic, logarithmic and power utility functions. After the pioneering works in risk aversion of Pratt [28] and Arrow [2] the positive sign of the third derivative of the utility function was seen as a necessary result of the assumption that investors' risk aversion decreases as wealth increases. The positive third derivative was first used in a moment investment model by Arditti in 1967 [1]. An exact preference ordering for risky portfolios using the first three moments of portfolio return can, in general, be determined only for an investor having a cubic utility function for wealth.⁴ Thus, the incorporation of fourth and higher moments of distribution of returns in this model have been ignored.

The geometric mean model is justified if the decision-maker has a logarithmic utility function.⁵ This again is another restrictive assumption on the investors' behavior.

This paper is a further development of my doctoral dissertation, which was submitted to the Graduate School of Business, The University of Alabama, in 1979. I am indebted for the advice and suggestions of my Dissertation Committee, Professor William H. Jean, who served as Chairman, and Professors Billy P. Helms, Terrence F. Martell, Donald L. Hooks, and Edward R. Mansfield. Also I would like to thank Professor Mya Maung and the referee of the *Journal of Finance* for their comments on a previous version of the paper. I remain responsible for any errors and ambiguities.

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¹ Although recently, Levy and Markowitz [18] argue that this is not necessary.

² See Harry Markowitz [21, 22].

³ For more recent treatments the reader is referred to [5] and [9].

⁴ For the major theoretical works on the third moments approach to portfolio analysis see [12] and [16].

⁵ See Latané [17].

In answer to the objections raised about the above investment models, a system of preference orderings based on the principles of stochastic dominance was developed and discussed by Quirk and Saposnik [29] and extended by Hadar and Russell [8], Whitmore [32], Hanock and Levy [10], and others.⁶

Stochastic dominance is an approach that utilizes the entire probability density function rather than a finite number of moments such as the mean and the variance. The stochastic dominance rules require no assumption about the mathematical form of the distributions and very little information on the investor's preferences.

The empirical studies in the area of stochastic dominance have encountered practical problems because of the lack of a definite ranking of portfolios in many instances.⁷ These problems have occurred in part due to the insufficiency of a theoretical basis for the assumption of the signs of the fourth and higher derivatives of the investor utility function. On the assumption that the investor's risk premium function is completely monotonic the signs of higher derivatives of the utility function were shown to alternate and the ranking rules for higher degree stochastic dominance were derived by Jean [14].

This study is an attempt to determine if empirically the ranking rules of higher degree stochastic dominance can be used to yield definite ranking portfolios and to shed further light on the issue of the efficient sets obtained by different investment models.

In Section I, the various degrees of stochastic dominance models are reviewed, and the necessary conditions for the existence of stochastic dominance are explained. Section II develops the empirical methodology employed. Section III and IV present the empirical results of this investigation with some concluding general remarks.

I. The Stochastic Dominance Model

This paper will use distributions of a variable X , which in the most general case is the investor's wealth level. Wealth varies over the positive finite range $(0, b)$. Two types of functions are defined over x , a probability function and a utility function. The continuous probability density function $f(x)$ corresponds to the outcome of an investor's investment choice. When a second investment choice is possible, the probability density function of that outcome is $g(x)$.

The distribution function $F(x)$ is the integral of $f(x)$, and the further integrals are $F_1(x)$, and in general $F_k(x)$, such that

$$F_1(x) = \int_0^x F(t) dt, F_2(x) = \int_0^x F_1(t) dt, F_k(x) = \int_0^x F_{k-1}(t) dt.$$

The second choice outcomes' distribution function and higher integrals are $G(x)$ and $G_k(x)$. Dominance conditions will always be stated so that the first alternative (F) is preferred to the second (G). Preference is always to be understood in terms of the weak form.

⁶ For more theoretical extensions see [7], [23], and [30].

⁷ For major empirical works see [19], [24], [25], [26], and [27].

$U(x)$ is the investor's utility function for wealth and assumed to be continuous and differentiable to any degree. The k th derivative of that function is $U^k(x)$. Using the above notation, the principles of *SD* can be stated as follows:

First Degree Stochastic Dominance (FSD). The probability function $f(x)$ is said to dominate the probability function $g(x)$ by *FSD* if, and only if, $F(x) \leq G(x)$ for all values of x with strict inequality for at least one value of x . For proof see Quirk and Saposnik [29], Hadar and Russell [8], and Hanock and Levy [10].

Second Degree Stochastic Dominance (SSD). The probability function $f(x)$ is said to dominate the probability function of $g(x)$ by *SSD* if, and only if, $F_1(x) \leq G_1(x)$ for all values of x with strict inequality for at least one value of x . For proof see Fishburn [7], Hadar and Russell [8], Rothschild and Stiglitz [30], and Hanock and Levy [10].

Third Degree Stochastic Dominance (TSD). The probability function $f(x)$ is said to dominate the probability function $g(x)$ by *TSD* if, and only if, $F_2(x) \leq G_2(x)$ for all values of x with strict inequality for at least one value of x , and the arithmetic mean of the first investment must be at least as large as the mean of the second investment which can be expressed in integral terms as $F_1(b) \leq G_1(b)$. For proof see Whitmore [32].

Nth Degree Stochastic Dominance (NSD). The probability function $f(x)$ is said to dominate the probability function $g(x)$ by *NSD* if, and only if, $F_{n-1}(x) \leq G_{n-1}(x)$ for all values of x with strict inequality for at least one value of x , and $F_k(b) \leq G_k(b)$ for $k = 1, 2, \dots, n - 2$. For proof see Jean [14].

These *SD* criteria are consistent with the principle of expected utility maximization under very reasonable assumptions concerning the form of the utility function. *FSD* requires only that the investor's utility function have a nonnegative first derivative ($U^1(x) \geq 0$), therefore, it allows risk preference, risk indifference, or risk aversion. *SSD* eliminates risk preference with the additional restriction that the second derivative be everywhere nonpositive ($U^2(x) \leq 0$). *TSD* adds the requirement that the third derivative of the utility function with respect to return (x) be everywhere nonnegative ($U^3(x) \geq 0$). Finally, *NSD* requires that the higher derivatives of the utility function continue to alternate in sign (i.e., $(-1)^k U^k(x) \leq 0$ for all x and $k = 1, 2, \dots, n$).

Necessary Condition for SD Efficiency

The empirical study of determining a definite ranking of portfolio using the ranking rules of any degree stochastic dominance must consider five separate necessary conditions. Three of these conditions were shown by Whitmore [32]. The first is a condition on the lower bound for x in the two distributions. For F to dominate G in any degree stochastic dominance test it is necessary that x_2 be less than x_1 , where x_1 and x_2 are the lower bound of F and G respectively.

The second condition imposes a restriction on the mean of F and G . For any degree of stochastic dominance it is necessary that the mean of F be at least as large as the mean of G .

The third Whitmore condition is that:

$$(\sigma_G^2 - \sigma_F^2) + (E_F - E_G)(2b - E_F - E_G) \geq 0.$$

$$E_F \geq E_G.$$

Jean [15] showed that this condition can be rewritten as:

$$G_2(b) - F_2(b) \geq 0.$$

The fourth condition is the extension of the third condition and indicates that if any degree of stochastic dominance exists between the two alternatives in the finite distribution case then every pair of the sequence of definite integrals is ordered as shown by Jean [15]:

$$F_k(b) \geq G_k(b) \quad \text{for } k = 1, 2, \dots$$

The fifth condition results from Jean's [15] paper where he showed that any degree of stochastic dominance ranking results in a geometric mean ranking, and therefore a geometric mean ranking is a necessary condition for stochastic dominance.

A further condition for *SSD* and *TSD* but not for *NSD* is based on the semi-variance derived by Porter [25], Jean [13], and Bawa [3]. Porter [25] developed the theoretical relationship between second degree stochastic dominance and mean-semivariances (ES_h).⁸ His conclusion was that the ES_h efficient set is a subset of the *SSD* efficient set. Jean [13] and Bawa [3] have shown that every ES_h efficient set is a subset of *TSD* efficient set, that the *TSD* efficient set contains all ES_h efficient sets, and that dominance under *TSD* implies dominance under ES_h .

II. Method of Analysis

For our empirical comparisons of mean-variance (*EV*), mean-semivariance (*ES*), third moment (*TM*), geometric mean (*GM*), and stochastic dominance (*SD*) efficiency, one set of 500 and one set of 100 portfolios were analyzed. Each portfolio contained 20 securities with an equal investment in each security. These portfolios were generated randomly with replacement from the set of 685 stocks with complete data for the 1961-76 period.

The data used were taken from the University of Chicago Center for Research in Security Price Relative File, which contains monthly price, dividend, and adjusted price and dividend information for all securities listed on the New York Stock Exchange in the period January 1961-December 1976. The time series of holding-period returns were computed by adding the end of the month market value to the distributions (dividends) paid during the month and dividing the sum by the first of the month market value. No distinction was made between capital gain and ordinary income distributions, and no adjustment was made for sales or redemption fees. The risk-free security's monthly holding period return used as the reference point in calculation of the semi-variance was estimated to be 1.005.

We are now interested in determining a definite ranking for each set of portfolios using the ranking rules of higher degree stochastic dominance.

⁸ Semi-variance (ES_h) is defined as $E[\min(0, x - h)]^2$ where x is a random variable and h is a target value. See [21], [20], [25], and [11].

Specifically:

1. The relative magnitudes of the *SD*, *EV*, *ES*, *GM*, and *TM* efficient sets;
2. The relative effectiveness of *SSD*, *TSD*, and the *n*th degree stochastic dominance in reducing the size of the efficient set;
3. The nature and extent of other differences in the respective efficient sets;
4. The effect on the above decision criteria of switching from 500 portfolios to 100 portfolios.

III. Empirical Results

A. 500 Randomly-Constructed Portfolios

Our analysis indicates that of the 500 portfolios examined, eleven were efficient by the mean-variance (*EV*) rule, eleven by the mean-semivariance (*ES*), 55 by the third moment (*TM*), one by the geometric mean (*GM*), 72 by the second degree stochastic dominance (*SSD*), 16 by the third degree stochastic dominance (*TSD*), 15 by the fourth degree stochastic dominance (4th *SD*), and 13 by the fifth degree stochastic dominance (5th *SD*).⁹ The first degree stochastic dominance (*FSD*) efficient set was too large to be measured.¹⁰

One should note that the eleven efficient portfolio sets obtained by the mean-variance rule and the mean-semivariance rule are identical. This, as Markowitz pointed out indicates that the distributions of monthly returns for each portfolio should be either symmetric or characterized by the same degree of skewness.¹¹

There are substantial differences in the sizes of the various stochastic dominance efficient sets. The size of each stochastic dominance efficient set is related to the assumptions made with respect to the investors' utility functions. The stronger the utility restrictions, the smaller the number of portfolios included in the efficient set. This theoretical relationship can be discerned empirically in our data.

Of more importance, however, is the evaluation of the fifth degree stochastic dominance in the efficient set. Further consideration shows that the fifth degree stochastic dominance yields the definitive stochastic dominance ranking, as small an efficient set as can be derived using any degree of stochastic dominance. Table I presents the summary of the 13 portfolios in the fifth degree stochastic dominance efficient set. The five necessary conditions for any degree stochastic dominance as discussed in Section II were applied to the 13 efficient portfolios whose characteristics are summarized in Table I (note that the portfolios in Table I are ordered by the mean and thus the second condition for *SD* efficiency is satisfied), and yields the results shown in Table II.

Thus, Table II revealed that the fifth degree stochastic dominance provides

⁹ Size of the efficient set as a percentage of the total number of portfolios examined are 2.2, 2.2, 11, .2, 14.4, 3.2, 3 and 2.6 respectively.

¹⁰ The *FSD* efficient set would include virtually all of the 500 portfolios investigated. Since the efficiency of our algorithms is a function of the frequency with which portfolios are eliminated, we could not run a complete test of the *FSD* rule on all 500 portfolios with monthly return within a reasonable time period.

¹¹ See Markowitz [21, page 194].

Table 1
Summary Results of 13 Portfolios Efficient by 5th Degree
Stochastic Dominance from 500 Portfolios

Portfolio #	Mean	Geometric Mean	Lower Bound
169	1.0140485	1.0125871	.8680
112	1.0127797	1.0113990	.8114
96	1.0127125	1.0110913	.8595
259	1.0125125	1.0112678	.8697
457	1.0116479	1.0100914	.8815
121	1.0115391	1.0104461	.8905
278	1.0114938	1.0103495	.8914
421	1.0111859	1.0103808	.8726
281	1.0108276	1.0105453	.8840
206	1.0103563	1.0093133	.8959
433	1.0099578	1.0089555	.8942
232	1.0096078	1.0084043	.8981
156	1.0075401	1.0064759	.9028

Table II
Results of Mean, Lower Bound, Geometric Mean, and $F_k(b)$ Conditions Applied to the 13 Portfolios
in the 5th SD Efficient Set

		Portfolio J												
Portfolio I		169	112	96	259	457	121	278	421	281	206	433	232	156
	169		C_2	C_2	A	A	A	A	A	A	A	A	A	A
	112			A	A	A	A	A	A	A	A	A	A	A
	96				A	A	A	A	A	A	A	A	A	A
	259					A	A	A	A	A	A	A	A	A
	457						A	A	B	A	A	A	A	A
	121							A	C_3	B	A	A	A	A
	278								B	B	A	A	A	A
	421									A	A	A	A	A
	281										A	A	A	A
	206											C_2	A	A
	433												A	A
	232													A
	156													

A = Indicates portfolio i can not dominate portfolio j due to lower bound necessary condition.
B = Indicates portfolio i can not dominate portfolio j due to geometric mean necessary condition.
C = Indicates portfolio i can not dominate portfolio j due to $F_k(b)$ necessary condition with the subscript to refer to which k.

Notice that due to the necessary condition of the ranking of the means, the left diagonal of Table 2 is ruled out.

the definitive stochastic dominance ranking results. Therefore, in this case no need exists for examining any higher degree stochastic dominance.

Further considerations concerning the evaluation of other differences in the efficient sets show that the eleven mean-variance efficient portfolios are also efficient by second degree stochastic dominance. Thus, the second degree stochastic dominance set contains 100 percent of the mean-variance set. Since this same eleven is only 15 percent of the total membership of the second degree stochastic dominance efficient set, we conclude that the major difference in mean-variance and second degree stochastic dominance efficiency is that the mean-variance set excludes a number of portfolios that are included in the second degree stochastic dominance efficient set. Similar conclusions also hold for the comparison of the mean-variance and the third, fourth, and fifth degree stochastic dominance respectively. The third degree stochastic dominance efficient set includes seven portfolios (64 percent) and the fourth degree stochastic dominance and fifth degree stochastic dominance efficient set each includes six portfolios (55 percent) of the eleven mean-variance efficient portfolios. In all these cases, more than 50 percent of the total portfolios that are stochastic dominance efficient are *EV* inefficient. Moreover, empirical results indicate that the third moment efficient sets include eight portfolios (73 percent) out of the eleven mean-variance or mean-semivariance efficient portfolios.

Evaluation of other differences between third moment efficient sets and second degree stochastic dominance through fifth degree stochastic dominance efficient sets shows that the third moment efficient set includes 18 portfolios (25 percent) out of 72 second degree stochastic dominance efficient portfolios. In addition, the third moment efficient set includes eleven portfolios (69 percent) out of 16 portfolios, ten portfolios (67 percent) out of 15 portfolios and nine portfolios (69 percent) out of 13 portfolios in the third, fourth and fifth degree stochastic dominance efficient sets respectively.

Discussion of several other significant points is facilitated by the use of Porter's [24] *EV/SD* graphical analysis. The intent here is simply to provide a visual comparison of the various sets rather than to set up the mean-variance (*EV*) location as a standard of validity.

Examination of Figure 1 discloses that the number of conflicts between mean-variance and third, fourth, and fifth degree stochastic dominance, respectively, is a decreasing function of the magnitude of the mean. That is, in the upper range of mean and variance the four sets are virtually identical. Most of the third, fourth, and fifth degree stochastic dominance efficient portfolios that are mean-variance inefficient occur in the middle range of the mean and variance. Most of the portfolios that are mean-variance efficient but not third or fourth or fifth degree stochastic dominance efficient occur in the lower range of the mean and variance. With the exception of second degree stochastic dominance the results derived from our study tend to confirm Porter's study of 1973 [24].

Another point to notice is that there appears to be no uniform pattern for location of stochastic dominance efficient sets. Thus, this result stands in contrast to that of Porder [24] in which he pointed out that "no second or third degree stochastic dominance efficient portfolio lies far from the *EV* efficient frontier."

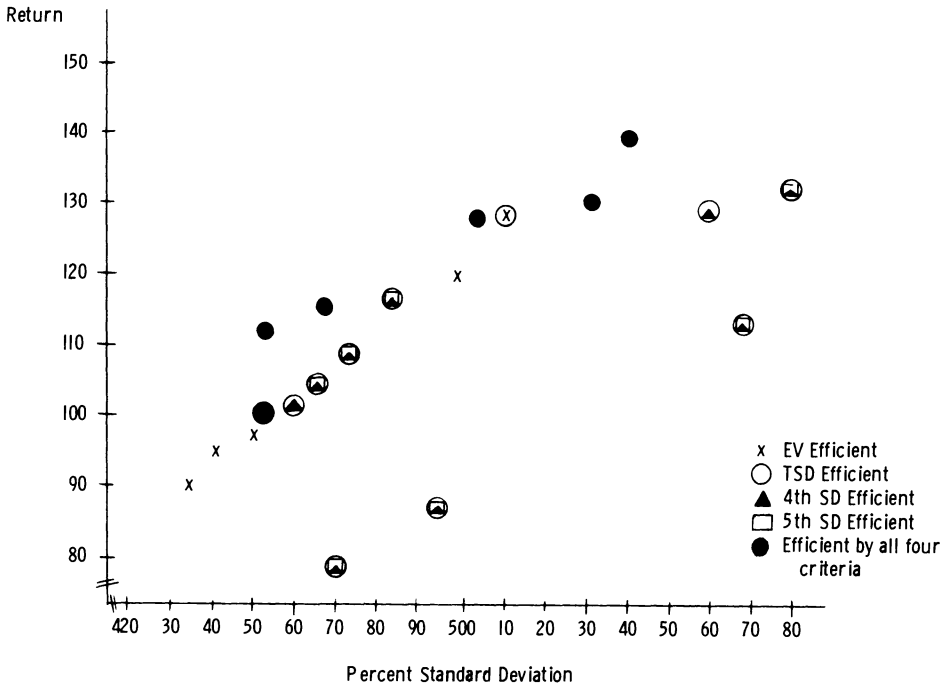


Figure 1. EV, TSD, 4th SD, and 5th SD Efficient Sets—500 Portfolios

To summarize the results from the 500 portfolios, with some exceptions a basic similarity of the various efficient sets exists. The most significant difference appears to be in their respective sizes. The fifth degree stochastic dominance test yields the definitive stochastic dominance ranking. Compared to other sets, the second degree stochastic dominance sets include most of the mean-variance sets whereas the fifth degree stochastic dominance sets include the least of the mean-variance sets.

B. 100 Randomly-Constructed Portfolios

The results from applying the various investment models to the 100 portfolios are given in Table III above.

As is the case in the 500 portfolios, the stronger the utility restrictions, the smaller the number of portfolios included in the stochastic dominance efficient set as shown in Table III. In addition, compared to the 500 portfolios, the size of the efficient set as a percentage of the 100 portfolios examined in all the investment models considered has increased.

The results of other differences in the efficient sets reveal that the seven mean-variance or mean-semivariance (the number and the type of portfolios in the efficient sets are identical) efficient portfolios are also efficient by second and third degree stochastic dominance. Thus, the second or third degree stochastic dominance set each contains 100 percent of the mean variance or mean-semivariance set. On the other hand, the fourth, fifth, and ninth degree stochastic

Table III
Size of the Efficient Sets for 100 Portfolios

		Efficiency Criteria						
<i>EV (%)</i> ^a	<i>ES (%)</i>	<i>TM (%)</i>	<i>GM (%)</i>	<i>SSD (%)</i>	<i>TSD (%)</i>	4th <i>SD (%)</i>	5th <i>SD (%)</i>	9th <i>SD (%)</i>
7 (7)	7 (7)	42 (42)	1 (1)	29 (29)	22 (22)	20 (20)	18 (18)	17 (17)

^a Size of efficient set as a percentage of the total number of portfolios examined.

dominance efficient sets each include six portfolios (86 percent) out of the seven mean-variance or mean-semivariance efficient portfolios. However, in all these cases more than 60 percent of the total portfolios that are stochastic dominance efficient are mean-variance or mean-semivariance inefficient.

Evaluation of other differences in the efficient sets shows that the seven mean-variance or mean-semivariance efficient portfolios are also efficient by the third moment rule. Thus, the third moment rule contains 100 percent of the mean-variance or mean-semivariance efficient sets. In addition, the empirical results show that the third moment efficient set in the 100 portfolio case includes 22 portfolios (76 percent) out of 29 portfolios, 18 portfolios (82 percent) out of 22 portfolios, 16 portfolios (80 percent) out of 20 portfolios, 15 portfolios (83 percent) out of 18 portfolios and 14 portfolios (82 percent) out of 17 portfolios in the second, third, fourth, fifth and ninth degree stochastic dominance efficient sets respectively.

Our analysis indicates that, unlike the case of the 500 portfolios, the fifth degree stochastic dominance did not provide the definitive stochastic dominance ranking result. Therefore, a need exists for examining higher degree stochastic dominance. Empirical tests show that, as is the case with the fifth degree stochastic dominance, the sixth, seventh, and eighth degree stochastic dominance each include 18 portfolios in the efficient set, and, thus, are not included in Table III. The ninth degree stochastic dominance, however, includes 17 portfolios in the efficient set and yields the definitive ranking result.

Table IV presents the summary of the statistical characteristics of the 17 portfolios in the ninth degree stochastic dominance efficient set. Examining the least value necessary condition and the geometric mean necessary condition for Table IV reveals the following results:

1. Portfolio 32 could dominate portfolio 67, 57;

Table IV

Summary Results of Portfolios Efficient by 9th Degree Stochastic Dominance from 100 Portfolios

Portfolio #	Mean	Geometric Mean	Lower Bound
7	1.0126317	1.0109699	.8084
74	1.0125203	1.0106975	.8277
88	1.0119005	1.0106735	.8549
1	1.0118677	1.0105135	.8627
32	1.0117620	1.0103594	.8692
75	1.0117453	1.0103886	.8667
52	1.0116984	1.0101952	.8694
67	1.0109974	1.0098448	.8677
57	1.0108105	1.0099283	.8640
8	1.0105100	1.0093024	.8729
33	1.0104271	1.0091312	.8790
89	1.0103464	1.0094060	.8714
60	1.0099802	1.0089295	.8814
22	1.0099479	1.0086684	.8863
30	1.0096656	1.0088850	.8821
11	1.0093990	1.0093808	.8805
61	1.0082104	1.0072093	.8885

2. Portfolio 75 could dominate portfolio 57;
3. Portfolio 52 could dominate portfolio 67, 57.

The $F_k(b) \leq G_k(b)$ for $k = 1, 2, \dots, n - 2$ necessary condition applied to the results above allows the conclusion that:

1. Portfolio 32 cannot dominate portfolio 67, 57, since the above condition will not hold for $k = 3$;
2. Portfolio 75 cannot dominate portfolio 57 for $k = 2$;
3. Portfolio 52 cannot dominate portfolio 67 and 57 for $k = 5$ and $k = 3$, respectively.

Thus, our analysis from Table IV and the above necessary condition indicates that the ninth degree stochastic dominance yields a definitive ranking.

The graphical results as shown in Figure 2 tend to confirm those derived using the 500 portfolios. The portfolios that are third, fourth, fifth, or ninth degree stochastic dominance efficient but not mean-variance efficient are even further away from the *EV* frontier than in the 500 portfolios. Again, most of these portfolios occur in the middle range of the mean and variance while in the upper range of means the five sets are virtually identical.

V. Summary and Conclusions

The results of the empirical tests presented in this study tend to support the following conclusions. First, the definitive stochastic dominance ranking result is achieved with a finite degree of stochastic dominance test. Indeed, the fifth degree was sufficient in the one case and ninth degree in the other case and these degrees certainly were small enough degrees to be practical in the usage of the n th degree stochastic dominance tool. Second, with the exception of first and second degree stochastic dominance, the differences between either the mean-variance or mean-

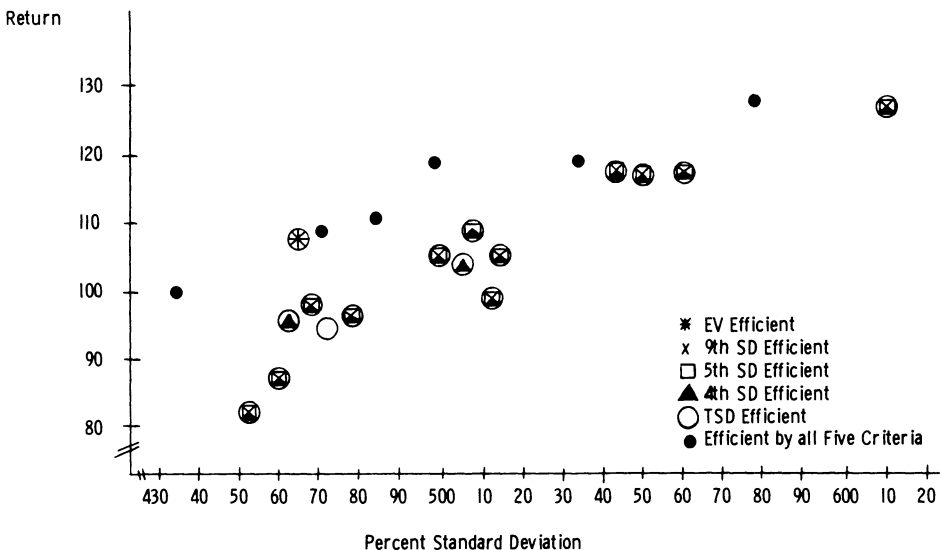


Figure 2. EV, TSD, 4th SD, 5th SD, and 9th SD Efficient Sets—100 Portfolios

semivariance and any degree of stochastic dominance efficiency were not as great as might have been expected. The most significant difference between either the mean-variance or mean-semivariance result is the tendency of stochastic dominance to eliminate from consideration the low return low variance (or semivariance) portfolios. Third, we could conclude that our results confirm both the studies of Porter and Gaumnitz [27] and Porter [24]. This latter result implies that the number of conflicts between mean-variance and stochastic dominance is a decreasing function of the magnitude of the mean. That is, in the upper range of mean and variance the two criteria are virtually identical. Most of the stochastic dominance efficient portfolios that are mean-variance inefficient occur in the middle range of the mean and variance. Most of the portfolios that are mean-variance efficient but not stochastic dominance efficient occur in the lower range of the mean and variance. Fourth, our empirical results from this study indicate that, as the number of portfolios in each set decrease, the size of the efficient set as a percentage of the number of portfolios examined in all the investment models increases.

In summary, the mean-variance rule, the mean-semivariance rule and the third moment rule require more restrictive assumptions about investor behavior, yet do not necessarily have greater resolving power than the stochastic dominance rules. The stochastic dominance rules allow for less restrictive and more realistic assumptions about the investor's utility functions. From this study it can be seen that the higher degrees of stochastic dominance rules yield a definitive ranking result and discriminate almost as often as the mean-variance or mean-semivariance rule when applied to raw data. Although the present results are encouraging, further empirical studies are necessary to verify that in general the *NSD* efficient set would be between the (one-dimensional) mean-variance efficient set and the (two-dimensional) mean-variance-third moment efficient set (but closer to the former).

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