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Taxes and the Optimal Capital Structure of the Firm

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I. Introduction

IN THE PERIOD FOLLOWING Modigliani and Miller's correction paper [9], it was generally agreed that where capital markets are perfect (taxes notwithstanding) there is an advantage to debt financing. In reaching this conclusion, Modigliani and Miller disregarded the effect of personal income taxes on the behavior of investors. When Miller [8] introduced personal income taxes into the analysis, he was able to state that for a wide range of values of dividend, interest, and corporate tax rates "the gain from leverage vanishes entirely or even turns negative" (p. 267). In deriving this result Miller assumed the following: (1) all investors are taxed at the same rate; (2) the objective of the firm is to maximize its value; (3) no capital gains tax exists; (4) no bankruptcy risk exists; and (5) the firm pays all of its earnings in dividends. Later on in his paper Miller drops assumption (1) in order to reach the surprising result that there is no optimal capital structure for the individual firm but there is an economywide optimal capital structure. This paper argues that when individuals differ in the tax rates imposed on their interest income, value maximization is a meaningless dictum. Assumption (1) is therefore adopted throughout in order to investigate the impact of removing assumptions (3), (4), and (5). A short discussion of previous references to these assumptions is in order.

The impact of capital gains taxes on the optimal financial policies of the firm is usually overlooked by the students of these policies. This overlooking of capital gains taxes is usually justified (Miller, [8]; Litzenberger and Van Horne, [7]) by the deferral effect of these taxes: capital gains taxes are usually paid in the far future and their present value at the time of making the investment decision is therefore negligible. This argument is questionable on two grounds. Observe an earning-retaining firm. For the shares of this firm to have any value in the present they must appreciate in value over any holding period whatever its length. Now, if the present value of the tax on this gain is negligible, so must be the gain itself and hence these shares cannot have any economic value. From the empirical point of view the data fail to bear out this assumption. In 1975 the average taxpayer realized $3,121 in capital gains vs. $2,472 in dividend income,[1] i.e., capital

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[1] See table I.4, columns 17, 18, 23, and 24 in Individual Tax Returns, Publication 79 (5-78), Internal Revenue Service.

gains realized is of an order of magnitude of dividend income. Moreover, whereas 8.8 million taxpayers included dividend income in their returns, 5.0 included net capital gains and 2.5 included net capital loss. Because it is reasonable to assume that the first group of taxpayers overlaps the other two, it follows that 85 percent of all taxpayers realized a capital gain (or loss) during 1975, from which 66 percent realized a gain that exceeds their dividend income. These figures represent convincing evidence that investors turn over their portfolios quite frequently. It is essential, therefore, to investigate the impact of the taxation of capital gains on the profitability of debt financing.

Three aspects of bankruptcy are usually referred to in the literature: increased interest costs, loss of future tax deductibility, and the occurrence of bankruptcy costs. The first aspect: increased interest costs are usually discussed in the classical literature, Durand, [5]. According to this literature, as the debt-equity rises so does the probability of financial failure. Bondholders require higher compensation at the expense of stockholders and hence the detrimental effect of high debt-equity ratio on the wealth of these stockholders. This study shows that when taxes are introduced, increased compensation to bondholders may actually benefit stockholders. In regard to the loss of future tax deductibility, it is argued, Brennan and Schwartz, [3][2] that when the firm fails stockholders lose the tax shield awarded by the bond. This argument however, is flawed; true, the stockholders of the bankrupt firm lose the tax shield associated with interest payment, but they are also exempt from the interest payment itself. If they lose the shield they win the even bigger interest nonpayment. As to the impact of bankruptcy cost: these costs will be ignored throughout this paper. Haugen and Senbet [6] argued convincingly that these costs are inconsequential to the capital structure problem. We will therefore construct our analysis in a fashion which will allow us to ignore these costs.

Recently Litzenberger and Van Horne [7] have shown that the capital structure issue cannot be divorced from the dividend policy of the firm.[3] It is essential, therefore, to examine whether the removal of assumption (5)—that no earnings are retained—will influence the optimal capital structure of the firm. The plan of the study is as follows: first, the advantage of debt is derived for the no-default case. This is done separately for a dividend-paying firm (section II) and for an earning-retaining firm (section III). The analysis is then extended for the earning-retaining firm with bankruptcy (section IV). In section V it is argued that no statement regarding the optimal capital structure can be made if individuals vary in regard to their taxation rates.

II. Debt Financing by Dividend-Paying Firms: The No-Default Case

Observe a market where dividend income is taxed at a rate τ_d, interest income at a rate τ_b, capital gains at a rate τ_g and corporate earings at a rate τ_c. To make any

[2] Brennan and Schwartz employed a continuous time model and ignored personal taxes.

[3] The relationship between these two decisions was first discussed qualitatively by Black [1].

Table I

Cash Flows to Current Stockholders of a Levered Firm

	State 1: Firm is Liquid $(X - \alpha rI)(1 - \tau_c) - \alpha I > 0$	State 2: Firm is Illiquid $(X - \alpha rI)(1 - \tau_c) - \alpha I < 0$
From sale of stock	$(1 - \beta)p_1$	$(1 - \beta)[p_1 + (X - \alpha rI)(1 - \tau_c) - \alpha I]$
From dividends (after personal taxes)	$(1 - \beta)[(X - \alpha rI) \cdot (1 - \tau_c) - \alpha I](1 - \tau_d)$	0
Capital gains tax	$[(1 - \beta)p_1 - p_0]\tau_g$	$[(1 - \beta)]p_1 + (X - \alpha rI) \cdot (1 - \tau_c) - \alpha I] - p_0]\tau_g$

Legend: β is the dilution factor due to issuance of $(1 - \alpha)I$ in equity.
p_1 is the value of the shares at time 1 stripped of any prior liability.
p_0 is the price at time 0 of shares outstanding prior to issuance of new securities.
X operating earnings of the firm during the period.

discussion of capital gains meaningful, it is assumed that at a particular point of time, say time 1, all investors liquidate their stock holdings.[4] At time 1 individuals sell their shares, pay taxes, and are left with an after-tax amount available for consumption. Individuals determine the current price of future cash flows by employing an evaluation formula with after-tax cash flows as its argument. We shall assume that this evaluation formula is additive and homogeneous of degree one.[5] Last, it is assumed that both individuals and corporations can offset current losses against other profits.[6] Observe now a firm that is initially fully equity financed. The firm decides to finance a new project with a proportion α of debt and $(1 - \alpha)$ of equity. The amount needed for investment is I. The debt is issued with a face value of αI and a nominal interest rate of r. Bankruptcy is excluded and the nominal interest is that of a riskless debt. We do not exclude the possibility of illiquidity where the firm does not earn enough money to satisfy its bondholders. In this case the firm can either borrow or reduce its working capital in order to raise the required cash for servicing the debt. Table I describes the cash-flows to stockholders for the two possible states: firm is liquid (state 1) and firm is illiquid (state 2). The current value of these future cash-flows is therefore:

$$p_0^L = (1 - \beta)\Bigg[V_0[p_1](1 - \tau_g) + V_0[p_0]\tau_g + V_0 \begin{Bmatrix} [(X - \alpha rI)(1 - \tau_c) - \alpha I](1 - \tau_d) & \text{if state 1} \\ [(X - \alpha rI)(1 - \tau_c) - \alpha I](1 - \tau_g) & \text{if state 2} \end{Bmatrix} \Bigg]$$

[4] The model can be adjusted for the case where holdings are liquidated in any other future time, even when investors' holding periods are different. Such a case will complicate the analysis without adding much insight.

[5] This can be shown to be equivalent to the assumption that capital markets are competitive.

[6] For individuals who can carry any loss both backward and foreward, this assumption is a better approximation for a real world situation than for corporations which can carry their losses forward only.

or

$$p_0^L = \frac{(1-\beta)}{1-\dfrac{\tau_g}{1+i}}\left[V_0[p_1](1-\tau_g) + V_0[(X-\alpha rI)(1-\tau_c)-\alpha I](1-\tau_d) + V_0\begin{Bmatrix} 0 & \text{if state 1} \\ [(X-\alpha rI)(1-\tau_c)-\alpha I](\tau_d-\tau_g) & \text{if state 2}\end{Bmatrix}\right] \quad (1)$$

where i is the after-tax discount rate for certain cash flows at time 1: $V_0[p_0]\tau_g = \frac{p_0\tau_g}{1+i}$. Recalling, however, that

$$\beta = \frac{(1-\alpha)I}{\dfrac{p_0^L}{1-\beta}}$$

transforms equation (1) into:

$$p_0^L = \frac{1}{1-\dfrac{\tau_g}{1+i}} V_0[p_1(1-\tau_g) + ((X-\alpha rI)(1-\tau_c)-\alpha I)(1-\tau_d)] - (1-\alpha)I + \frac{1}{1-\dfrac{\tau_g}{1+i}} V_0\begin{Bmatrix} 0 & \text{if state 1} \\ [(X-\alpha rI)(1-\tau_c)-\alpha I](\tau_d-\tau_g) & \text{if state 2}\end{Bmatrix} \quad (2)$$

The net tax advantage of debt financing can be derived by deducting from p_0^L the value of the shares of an unlevered firm ($\alpha = 0$):

$$p_0^U = \frac{1}{1-\dfrac{\tau_g}{1+i}} V_0[p_1(1-\tau_g) + X(1-\tau_c)(1-\tau_d)] - I + \frac{1}{1-\dfrac{\tau_g}{1+i}} V_0\begin{Bmatrix} 0 & X>0 \\ X(1-\tau_c)(\tau_d-\tau_g) & X \le 0\end{Bmatrix} \quad (3)$$

and:

$$\Delta_1 = p_0^L - p_0^U = \alpha I - \frac{1}{1-\dfrac{\tau_g}{1+i}} V_0[-\alpha I(1+r(1-\tau_c))](1-\tau_d) + \frac{1}{1-\dfrac{\tau_g}{1+i}}\left[V_0\begin{Bmatrix} 0 & \text{if state 1} \\ [(X-\alpha rI)(1-\tau_c)-\alpha I](\tau_d-\tau_g) & \text{if state 2}\end{Bmatrix} - V_0\begin{Bmatrix} 0 & \text{if } X>0 \\ X(1-\tau_c)(\tau_d-\tau_g) & \text{if } X\le 0\end{Bmatrix}\right] \quad (4)$$

From equation (4) it appears that the net tax advantage of borrowing consists of three terms: 1. the advantage of collecting the proceeds from debt issuance minus 2. the disadvantage in the form of the current after-tax value of the future debt servicing outflows and 3. the differential effect of capital gains-regular income tax asymmetry. This asymmetry stems from the fact that whereas in case of liquidity (or positive profit for the unlevered firm) dividends are taxed at a rate τ_d, the loss in value due to illiquidity (or negative profit for the unlevered firm) entitles stockholders to a refund at a rate of τ_g. Note that whereas the difference between the first two terms in equation (4) is linear in α, no such statement can be made regarding the difference between the remaining two terms except that it is negative. This difference must be convex in respect to α independently of the option valuation formula used to evaluate these last two terms.[7] As a result, an interior solution to a maximum net tax advantage ($0 < \alpha < 1$) cannot be ruled out.

It is interesting to interpret the role of capital gains taxes on the optimal capital structure. To abstract from other factors assume that the firm is always liquid. In this case the advantage of debt financing consists of the first two terms in equation (4). In this case it is obvious that because of capital gains taxes the advantage declines or may even become negative.

We now examine the conditions under which equation (4) collapses into Miller's condition for debt financing:

$$\frac{(1-\tau_c)(1-\tau_d)}{(1-\tau_b)} < 1 \tag{5}$$

with τ_b as the rate of taxation of interest income. When the distribution of earnings is such that no illiquidity may arise, the tax disparity terms in equation (4) disappear. Where the holding period is very long[8] we have: $\frac{\tau_g}{1+i} \approx 0$. Also, if the bond is a consol, αI is eliminated from the second term and the tax advantage becomes:

$$\Delta_1 = \alpha I - V_0[-\alpha r I(1-\tau_c)(1-\tau_c)].$$

When the consol valuation formula is applied to the perpetual certain debt service outflows we have:

$$\Delta_1 = \alpha I - \frac{\alpha I r(1-\tau_c)(1-\tau_d)}{r(1-\tau_b)} \tag{6}$$

which is identical to Miller's condition (5). It can easily be shown that this condition holds also for a nonconsol debt. In essense, what equation (6) implies is that debt should be used as long as the present value of the liability to bondholders is lower than the proceeds received when the bonds are issued. The

[7] Note that rationality arguments do not provide any assistance in analyzing the nature of this difference as a function of α. Specific valuation formulas (Rubinstein [10], Brennan [2]), are inapplicable because they assume lognormality of the underlying distribution. This assumption cannot be applied to both $X(1-\tau_c)$ and $(X - r\alpha I_0)(1-\tau_c) - I$.

[8] Note, however, that if a long holding period is assumed, arbitrage is impermissable: capital gains will have to be paid when arbitrage is executed. Probably a better assumption is that taxes are not levied on capital gains: $T = 0$.

same interpretation can be given to equation (4) and any other debt advantage formula to be derived later in this paper.

III. Debt Financing by Earning-Retaining Firms: The No-Default Case

The capital structure decision is again that of selecting the proportions α and $(1 - \alpha)$ in which an investment of I is to be financed by debt and equity, respectively. Because no dividends are being paid, stockholders in this company stand to collect from selling their shares, an amount:

$$(1 - \beta)[p_1 + (X - r\alpha I)(1 - \tau_c) - \alpha I]$$

from which

$$[(1 - \beta)(p_1 + (X - r\alpha I)(1 - \tau_c) - \alpha I) - p_0]\tau_g$$

will be paid in capital gains taxes. The current value of the outstanding shares is the value of the net proceeds:

$$p_0^L = \frac{(1 - \beta)(1 - \tau_g)}{\left(1 - \dfrac{\tau_g}{1 + i}\right)} V_0[p_1 + (X - r\alpha I)(1 - \tau_c) - \alpha I] \tag{7}$$

where β is substituted into (7) the resulting value of the earning-retaining firm is:[9]

$$p_0^L = \frac{(1 - \tau_g)}{\left(1 - \dfrac{\tau_g}{1 + i}\right)} V_0[p_1 + (X - r\alpha I)(1 - \tau_c) - \alpha I] - (1 - \alpha)I \tag{8}$$

The value of the equity of the unlevered firm can be calculated from (8) for $\alpha = 0$. The difference in values between the equities of the levered and the unlevered firms is:

$$\Delta_2 = p_0^L - p_0^U = \frac{1 - \tau_g}{1 - \dfrac{\tau_g}{1 + i}} V_0[-\alpha I(1 + r(1 - \tau_c))] + \alpha I \tag{9}$$

which is nothing but a restatement of the obvious: the firm should issue bonds as long as the proceeds exceed the current value of the future debt service. Recalling that certainty is assumed, this will be true as long as:

$$\frac{(1 - \tau_g)(1 + r(1 - \tau_c))}{\left(1 - \dfrac{\tau_g}{1 + i}\right)(1 + r(1 - \tau_b))} < 1$$

[9] Equation (8) substantiates the claim in the introduction section that even when stock sale is made in the far future capital gains taxes are important. For a long holding period we have $i \to \infty$ and the price of the firm is then penalized by the full $(1 - \tau_g)$ factor. Actually, for short holding periods the effect of capital gains taxes can be ignored for then: $\dfrac{1 - \tau_g}{1 - \dfrac{\tau_g}{1 + i}} \approx 1$. Similar statements can be made regarding the dividend-paying firm (equation (4)).

Substituting $i = r(1 - \tau_b)$ yields the condition:

$$\frac{(1 - \tau_c)(1 - \tau_g)}{(1 - \tau_b)} < 1 \tag{10}$$

The regular income tax rate in equation (5) is now replaced by the capital gains tax rate. In contrast to the dividend-paying firm we are assured that for the earning-retaining firm a corner solution is optimal. Also, by comparing equations (5) and (10) it can be concluded that where τ_d and τ_g (which is the case for most United States taxpayers) the condition for debt financing for the earning-retaining firm is more severe than that of the dividend-paying firm. This result constitutes a testable hypothesis: firms with low pay-out ratio are more likely to be equity financed.

IV. Default Considerations

When default is a possibility, bondholders will no longer be satisfied with a coupon which carries the riskless rate on interest. Rather, they will require a coupon rate of $R = R(\alpha)$ which incorporates bankruptcy risk. In this section we describe how this rate is determined by bondholders and what is the impact of $R(\alpha)$ on the wealth of stockholders. To simplify the presentation some assumptions are being made. The distribution of X is assumed to be discontinuous in the sense that either the full debt service can be paid by the firm (hence: state 1) or none of the debt can be served (state 2). The payoff to bondholders is, therefore, $\alpha I(1 + R(1 - \tau_b))$ if state 1 occurs. If state 2 occurs, bondholders lose their total investment. This loss is assumed to be offset against other capital gains and thus is entitled to a tax refund of $\alpha I \tau_g$. For this prospective cash-flow bondholders require R such that:

$$\alpha I = V_0 \begin{Bmatrix} \alpha I(1 + R(1 - \tau_b)) & \text{if state 1} \\ \alpha I \tau_g & \text{if state 2} \end{Bmatrix} \tag{11}$$

For the sake of the exposition we carry the analysis for the earning-retaining firm only. Similar conclusions can be reached for the dividend-paying firm. For the retaining firm it is easy to show that when X is discontinuous in the above described manner, the advantage of debt is:[10]

$$\Delta = \alpha I = K V_0 \begin{Bmatrix} \alpha I(1 + R(1 - \tau_c)) & \text{if state 1} \\ 0 & \text{if state 2} \end{Bmatrix} \tag{12}$$

where

$$K = \frac{1 - \tau_g}{1 - \dfrac{\tau_g}{1 + i}}.$$

When αI is substituted from (11) into (12) the advantage of debt is:

$$\Delta = V_0 \begin{Bmatrix} \alpha I(1 + R)(1 - K) + \alpha I R(K\tau_c - \tau_b) & \text{if state 1} \\ \alpha I \tau_g & \text{if state 2} \end{Bmatrix} \tag{13}$$

[10] Because of the discontinuity of X equation (9) remains unchanged when bankruptcy risk exists, except for the substitution of R for r.

If no default occurs the advantage to bond financing consists of two terms: $\alpha I(1 + R)(1 - K)$ and $\alpha IR(K\tau_c - \tau_b)$. The first term is always positive and it stems from the fact that debt servicing reduces the tax base for capital gains tax computation. The second term (which may be either positive or negative) is the result of the tax differential between corporate leverage and personal leverage. Note, however, that because this differential is adjusted for the effect of capital gains taxes, it may become negative even if $\tau_c > \tau_b$. The total tax advantage in state 1 can thus be either positive or negative. In contrast, the tax advantage to debt in state 2 is always positive. If in both states the advantages are positive, then the firm should opt for a full debt financing. If, however, the advantage in state 1 is negative while it is positive in state 2, then an interior solution to the optimal capital structure is a possibility.

V. The Case of Varying Tax Rates

When all individuals are taxed at identical rates and no illiquidity risk exists it was shown above that the optimal capital structure is a corner solution. This conclusion is nothing but a substantiation of Miller's statement as reflected by equation (5). In attempting to explain the fact that firms do no pursue extreme capital structures, Miller [8] (and recently De Angelo and Masulis [4]) examined an economy in which different individuals are taxed at different rates and firms maximize their values. Using this framework, Miller has shown that firms do not have an optimal capital structure although the economy as a whole does. We claim now that if individuals vary in regards to their personal tax rates value maximization is no longer a Pareto-optimum solution. To prove this point we adopt a framework similar to that used by De Angelo and Masulis. Observe a firm with a prospective per share, before-tax cash flow of X_s in state nature s. If the firm is totally equity financed, the owner of one share may expect to obtain $X_s(1 - \tau_c)$ one period from now on which he will be paying a personal tax of τ_d leaving him with: $X_s(1 - \tau_c)(1 - \tau_d)$. This shareholder can borrow against this future cash flow an amount of: $\dfrac{X_s(1 - \tau_c)(1 - \tau_d)}{1 - \tau_b}$. If the current market price of future dollars delivered at state s is P_E and P_B, for equity and debt respectively, the current wealth of this stockholder is:

$$W^u = X_s(1 - \tau_c)(1 - \tau_d)P_E - \frac{X_s(1 - \tau_c)(1 - \tau_d)}{1 - \tau_b} P_B.$$

If, on the other hand, the firm is fully debt financed it commits all of its future cash flows to its bondholders. The proceeds from selling these bonds $P_B X_s$ are now distributed to stockholders who pay taxes and are therefore left with a personal wealth of:

$$W^L = P_B X_s(1 - \tau_d).$$

Nominally these stockholders still own shares but because the firm committed its total future cash flows, these shares have no value. Now, for any given pair of P_B, P_E, it is possible that for one investor $W^u > W^L$ while for another investor in a different tax bracket $W^L > W^u$. The two stockholders will disagree as to what financing scheme should be adopted. Value maximization is therefore a meaningless rule.

VI. Conclusions

This paper examined the impact of taxation on the optimal capital structure of the firm when all investors belong to the same tax brackets. The impact of taxes on capital gains was explicitly considered. It was shown that for the dividend-paying firm, interior solutions for the capital structure decision is possible due to the disparity between the capital gains and dividend income tax rates and the possibility of illiquidity. When the dividend-paying firm is always liquid, the solution to the capital structure problem coincides with that of Miller's [8]. For the earning-retaining firm without bankruptcy the optimal capital structure is always a corner solution. When bond default is allowed, an interior solution is possible. This study also commented on recent attempts to explain the fact that firms do not pursue corner solution by means of differential rates of taxation. It was shown that differential rates are inconsistent with the value maximization rule.

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