Journal of Empirical Finance 16 (2009) 671–685

Contents lists available at ScienceDirect

Journal of Empirical Finance

journal homepage: www.elsevier.com/locate/jempfin

L-performance with an application to hedge funds ☆

Serge Darolles [a,b], Christian Gourieroux [b,c,d], Joann Jasiak [e,*]

[a] SGAM AI, France
[b] CREST, France
[c] CEPREMAP, France
[d] University of Toronto, Canada
[e] York University, Canada

ARTICLE INFO

Article history:
Received 9 July 2008
Received in revised form 13 February 2009
Accepted 19 May 2009
Available online 23 June 2009

JEL classification:
C 14
G32
G23

Keywords:
Hedge fund
Sharpe performance
L-moment
Distortion risk Measure
Ranking
Bias ratio
Manipulation

ABSTRACT

This paper introduces a new parametric fund performance measure, called the L-performance. The L-performance is an alternative to the Sharpe performance, which is commonly used in practice despite its inability to account for skewness and heavy tails of unconditional return distributions. The L-performance improves upon the Sharpe measure in this respect. Technically, it resembles the Sharpe measure in that it is defined as a ratio of the first- and second-order moments, which are the trimmed L-moments instead of the conventional (power) moments. The trimming parameters allow for focusing the L-performance on specific risk levels of interest, according to financial risk criteria. For illustration, a set of L-performances is computed for a variety of hedge funds. The empirical study shows the use of L-performance for fund ranking and return smoothing (manipulation) control.

1. Introduction

Sharpe performance is the basic measure of performance used for fund comparison, ranking and segmentation [see e.g. Sharpe (1966), Lo (2002), Darolles and Gourieroux (2008)]. By definition, the Sharpe performance is a ratio of the sample mean and standard deviation of excess returns. These two statistics do not always provide adequate characterizations of return distributions, especially in the presence of skewness and/or heavy tails. In particular, the sample mean is not a robust location measure, and the standard error may not even exist in heavy tailed distributions. This is of special importance in the analysis of the (unconditional) distributions of hedge fund returns, whose departures from normality are evidenced and related to some liquidity and management effects [see e.g. Getmansky et al. (2003)].

The aim of this paper is to introduce a new parametric performance measure, called the L-performance, which accommodates skewed and heavy tailed distributions better than the Sharpe ratio. By analogy to the Sharpe performance, the L-performance is defined as a location measure divided by a dispersion measure, which are the first- and the second-order trimmed L-moments. Like conventional moments, the (trimmed) L-moments summarize various characteristics of a distribution, such as the location,

☆ This study benefited from the financial support of the Natural Sciences and Engineering Council of Canada (NSERC), the Chair AXA "Large Risks in Insurance" and the Chair GROUPAMA "Behavioral Finance".
* Corresponding author.
E-mail addresses: serge.darolles@sgam.com (S. Darolles), gouriero@ensae.fr (C. Gourieroux), jasiakj@yorku.ca (J. Jasiak).

0927-5398/$ – see front matter
doi:10.1016/j.jempfin.2009.05.003

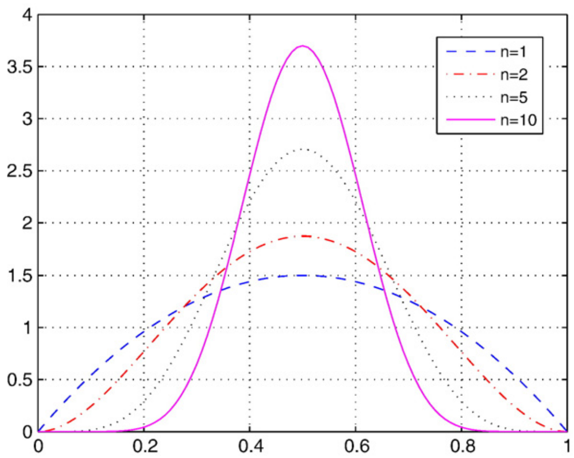

Fig. 1. Weighting polynomials $P_{1,n}$.

dispersion, skewness and kurtosis [see, Elamir and Seheult (2003), Hosking (1990)]. The main area of application of trimmed L-moments is the analysis of catastrophic events such as extreme floods or low flows [see e.g. Hosking and Wallis (1987), Wang (1997), Bayazit and Onoz (2002), Moisello (2007)], rainfall extremes [see e.g. Guttman et al. (1993), Lee and Maeng (2003)], extreme wind speed [see e.g. Whalen et al. (2004)], and extreme traffic volumes in computer networks [see e.g. Hosking (2007)]. So far, there exists only a limited number of applications of L-Moments to financial returns [Karvanen (2006), Brazauskas et al. (2007a,b), Gourieroux and Jasiak (2008), Jurczenko, Maillet and Yanou (2007), Yanou (2008)].

Unlike the Sharpe measure, the L-performance can be focused on selected risk levels. More specifically, the trimming parameters allow us to assign relatively more weight to neighbourhoods of some risk levels of interest. In contrast, the Sharpe ratio assigns equal weights to all observations, regardless if they come from the center or from the tails of a distribution. In this respect, the L-performance is in line with other alternative performance measures that were proposed earlier in the literature, such as the Sortino ratio [Sortino and Prince (1994)], the Omega [Shadwick and Keating (2002)], the mean/VaR(α) ratio [see e.g. Dowd (1999), Gregoriou and Gueyie (2003)], the TVaR(α)/VaR (α) ratio, where TVaR denotes the so-called Tail-VaR [see e.g. Gourieroux and Liu (2007)]. Moreover, a set of L-performances at different risks levels, provides a set of criteria according to which funds can be ranked, in contrast to the traditional approach, which is based on a single Sharpe criterion.

The paper is organized as follows. Section 2 provides the definitions of trimmed L-moments of orders 1 to 4, the interpretation of the trimming parameters, and discusses the differences between the trimmed moments introduced in this paper and those that exist in the statistical literature. The L-performance measure is defined in Section 3 as a ratio of trimmed L-moments of orders 1 and 2. It is compared to the Sharpe measure, and related to the VaR. Next, it is shown to be the outcome of a L-moment portfolio return optimizing behavior, which is analogous to the mean-variance optimizing behavior. The last part of Section 3 introduces an estimator of L-performance and describes its implementation. In Section 4, the L-performances are used to rank a set of hedge funds. In this context, return manipulation (smoothing) is also discussed. Section 5 concludes. Proofs are gathered in the Appendices.

2. Trimmed L-moments

2.1. Conceptual random sample

Let us consider random variable X with cumulative distribution function (cdf) F and quantile function[1] $Q=F^{-1}$. A conceptual random sample is a set $\tilde{X}_1,\ldots,\tilde{X}_N$ of independent and F distributed random variables. The conceptual random sample is long established in statistics, as it was used to develop some early dispersion measures. For example, consider a conceptual random sample of size $N=2$. Intuitively, if the difference $|\tilde{X}_1-\tilde{X}_2|$ is large (resp. small), then the dispersion of distribution F is large (resp. small). This idea leads to a family of dispersion measures defined as $\mu_p=[E(|\tilde{X}_1-\tilde{X}_2|^p)]^{1/p}$. It is known that $\mu_2=[E(|\tilde{X}_1-\tilde{X}_2|^2)]^{1/2}=[2\text{var}(X)]^{1/2}$ and that $\mu_1=E(|\tilde{X}_1-\tilde{X}_2|)$ is the absolute Gini index, up to scale factor 2.[2] These conceptual random sample based dispersion measures are alternatives to the conventional standard error and absolute Gini Index definitions. It is interesting to note that $|\tilde{X}_1-\tilde{X}_2|=\tilde{X}_{2:2}-\tilde{X}_{1:2}$, where $\tilde{X}_{1:2}=\min(\tilde{X}_1,\tilde{X}_2)<\tilde{X}_{2:2}=\max(\tilde{X}_1,\tilde{X}_2)$ are the order statistics of the conceptual random sample.

[1] For ease of exposition, we assume that F is invertible. Loosely speaking, X is a continuous random variable with support (a,b), and has a strictly positive density on $]a,b[$.

[2] $\mu_1=E|\tilde{X}_1-\tilde{X}_2|=2\int_0^1 Q(u)(2u-1)\mathrm{d}u$. If $G(u)=\int_0^u Q(x)\mathrm{d}x$ is the cumulated quantile function, that is the absolute Lorenz curve [Gini (1912)], we obtain by integrating by part: $\mu_1/2=\int_0^1(2u-1)\mathrm{d}G(u)=G(1)-2\int_0^1 G(u)\mathrm{d}u=E(X)\,\text{Gini}(X)$, where Gini($X$) is the (relative) Gini index.

The definition of trimmed L-moments relies on order statistics $\tilde{X}_{1:N}<\cdots<\tilde{X}_{N:N}$ of a conceptual random sample of size N. The trimmed L-moments are expectations of linear functions of the conceptual order statistics.[3] More specifically, the trimmed L-moments of orders 1,2,3, etc. define families of location, dispersion, skewness, etc. parameters, respectively. The following sections discuss in detail the trimmed L-moments of order 1 and order 2, which correspond to the families of location and dispersion parameters.

2.2. Trimmed L-moments of order 1

The n-trimmed L-moments of order one are defined from a conceptual random sample of size $2n+1$, $n\in\mathcal{N}$. The order statistics of the conceptual random sample are denoted by $\tilde{X}_{i:2n+1}$, $i=1,\ldots,2n+1$, and increase in rank i.

The n-trimmed L-moment of order 1:

$$\lambda_{1,n} = E\left(\tilde{X}_{n+1:2n+1}\right), n\geq 0, \tag{2.1}$$

is equal to the expectation of the median of a conceptual random sample. The n-trimmed L-moment of order 1 of X has the following expressions involving the cdf F and the quantile function Q, respectively [Elamir and Seheult (2003), Hosking (2007)]:

$$\begin{aligned}\lambda_{1,n} &= \frac{(2n+1)!}{(n!)^2}\int xF(x)^n[1-F(x)]^n\mathrm{d}F(x) \\ &= \frac{(2n+1)!}{(n!)^2}\int_0^1 Q(u)u^n(1-u)^n\mathrm{d}u \end{aligned}\tag{2.2}$$

$$= \int_0^1 Q(u)P_{1,n}(u)\mathrm{d}u, \tag{2.3}$$

where

$$P_{1,n}(u) = \frac{(2n+1)!}{(n!)^2}u^n(1-u)^n, \tag{2.4}$$

are nonnegative polynomials with unit mass. The n-trimmed L-moment of order 1, $\lambda_{1,n}$ exists, if $E(|X|^{1/(n+1)})<\infty$ [see Hosking (2007), Theorem 1]. Thus, the trimmed L-moments for $n\geq 1$ can be defined even when the expectation of X does not exist, as for example when X is Student distributed.[4]

Let us now discuss the role of polynomials $P_{1,n}$, $n\in\mathcal{N}$, in the definition of $\lambda_{1,n}$. The patterns of polynomials $P_{1,n}$ depend on n, as displayed in Fig. 1 below.

Polynomial $P_{1,n}$ is the probability density function of a beta distribution with parameters $n+1$, $n+1$. In this context, $\lambda_{1,n}$ is a distortion risk measure (DRM) associated with a beta distortion measure $P_{1,n}$ [Wang (2000a,b), Gourieroux and Liu (2007)]. A distortion measure assigns weights to risk levels $u\in[0,1]$. More specifically, the beta distortion measure is symmetric and centered at 0.5. This measure becomes more peaked when n increases, and tends to a point mass at 0.5 when n tends to infinity. In contrast, it becomes more flat when n decreases and tends to a uniform distribution when n tends to 0. Since less weight is assigned to the extreme risk levels u when n increases, parameter n can be considered as a (quantile) trimming parameter [see Greenwood et al. (1979) for this interpretation].[5] For a sequence of n between 0 and ∞, the trimmed L-moments of order 1 define a family of location parameters, which bridges the gap between the expectation: $\lambda_{1,0} = \int_0^1 Q(u)\mathrm{d}u = E(X)$, and median: $\lambda_{1,n}=\lim_{n\to\infty}\lambda_{1,n}=Q(0.5)$. Let us comment on the informational content of trimmed L-moments of order 1. Polynomials $P_{1,n}$, $n\in\mathcal{N}$, form a (nonorthogonal) basis of the space of even functions (with respect to $u=0.5$). In this context, the trimmed L-moments of order 1 are the inner products of quantile function Q and polynomials $P_{1,n}$, $n\in\mathcal{N}$, and characterize the projection $0.5(Q(u)+Q(1-u))$ of quantile function Q on the space of even functions.[6] Given that distribution F is symmetric iff $Q(u)+Q(1-u)=constant$, we get the following result:

Proposition 1. *The n-trimmed L-moments of order 1 are equal for any $n\in\mathcal{N}$, if and only if the distribution is symmetric.*

Proof. See Appendix A.

Thus, a sequence of n-trimmed L-moments of order 1 allows us to examine the symmetry of a distribution. This approach is much more reliable than direct comparison of the mean and median, because there exist asymmetric distributions whose means and medians are equal. For example, let us consider two gamma distributions $\gamma(\alpha\nu_1,\nu_1)$ and $\gamma(\alpha\nu_2,\nu_2)$ on $(0,\infty)$, where $\nu_1\neq\nu_2$). Suppose X is a continuous random variable that takes positive and negative values with equal probability 0.5, and that the distribution of $X|X\geq 0$ is $\gamma(\alpha\nu_1,\nu_1)$ and that of $-X|X<0$ is $\gamma(\alpha\nu_2,\nu_2)$. The mean and median of X are both equal to 0, while the distribution of X is asymmetric. In this case, an analysis of L-moments $\lambda_{1,n}$ for large n detects asymmetries in a neighbourhood of the median [see Section 4.2].

[3] “L” stands for "linear" (function of the expected conceptual order statistics). Due to linearity, the absolute Gini index $E(X)\mathrm{Gini}(X)$ is a L-moment. The variance of X and the standard error of X are not L-moments, because they can be written only as nonlinear functions of conceptual order statistics.

[4] This is an advantage in the analysis of heavy tailed distributions of financial returns.

[5] This explains the terminology “trimmed L-moment” introduced in Elamir and Seheult (2003).

[6] Any function Q can be written in a unique way as the sum of an even and odd function. The formula is: $Q(u)=0.5(Q(u)+Q(1-u))+0.5(Q(u)-Q(1-u))$.

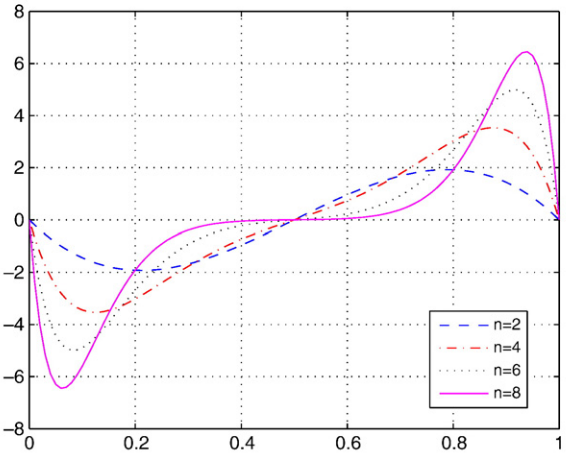

Fig. 2. Polynomials $P_{2,r,n}$ for $r=1$ and $n=2,4,6,8$.

2.3. Trimmed L-moments of order 2

The (r,n)-trimmed L-moments of order 2 are defined from a conceptual sample of size $2n+1$, $n\in\mathcal{N}$, with order statistics $\tilde{X}_{i:2n+1}$, $i=1,\ldots,2n+1$, which increase in rank i. The L-moments of order 2 measure the expected slope of conceptual order statistics, as a function of rank i, as follows: Let us consider ranks $r+1$ and $2n-r+1$, equally distant by $|n-r|$ from the middle rank $n+1$ of the conceptual sample. The (r,n)-trimmed L-moment of order 2 defined by

$$\lambda_{2,r,n}=E\left(\tilde{X}_{2n-r+1:2n+1}-\tilde{X}_{r+1:2n+1}\right), n\in\mathcal{N}, 0\le r\le n-1, \tag{2.5}$$

measures the expected range of a conceptual sample of size $2n+1$, after deleting the r smallest and r largest conceptual order statistics. $\lambda_{2,r,n}$ is an increasing function of r. The trimmed L-moment of order 2 exists when $E[|X|^{1/(r+1)}]<\infty$ [see Hosking (1990)]. For example, $\lambda_{2,0,n}$ requires the existence of the mean, while for $r>0$, $\lambda_{2,r,n}$ exists under milder conditions on the tail index.

The (r,n)-trimmed L-moment of order 2 has the following expressions in terms of the distribution and quantile functions:

$$\begin{aligned}\lambda_{2,r,n}&=\frac{(2n+1)!}{r!(2n-r)!}\int x\left\{F(x)^{2n-r}[1-F(x)]^{r}-F(x)^{r}(1-F(x)]^{2n-r}\right\}\mathrm{d}F(x)\\&=\frac{(2n+1)!}{r!(2n-r)!}\int_0^1 Q(u)\left[u^{2n-r}(1-u)^{r}-u^{r}(1-u)^{2n-r}\right]\mathrm{d}u.\end{aligned} \tag{2.6}$$

The trimmed L-moments of order 2, $\lambda_{2,r,n}$, $n\in\mathcal{N}$, $0\le r\le n-1$ define a family of dispersion parameters. Like the family of $\lambda_{1,n}$, family of $\lambda_{2,r,n}$ contains considerable information about the distribution of X. In particular, the trimmed L-moments of order 2 characterize the odd component $0.5(Q(u)-Q(1-u))$ of the quantile function. Since the family of trimmed L-moments of order 1 characterizes the even component of the quantile function and the family of trimmed L-moments of order 2 characterizes the odd component of the quantile function, the families of trimmed L-moments of orders 1 and 2 characterize the distribution of X.

The polynomials:

$$P_{2,r,n}(u)=\frac{(2n+1)!}{r!(2n-r)!}\left[u^{2n-r}(1-u)^{r}-u^{r}(1-u)^{2n-r}\right], n\in\mathcal{N},\ 0\le r\le n-1, \tag{2.7}$$

generate the space of odd functions (with respect to $u=0.5$). The patterns of polynomials vary with n and r, as illustrated in Figs. 2 and 3 below.

Polynomial $P_{2,r,n}$ takes value 0 for risk levels $u=0$, 0.5, 1, and attains its minimum (resp. maximum) for risk level $\alpha_{r,n}\in[0, 0.5]$ (resp $[1-\alpha_{r,n}]\in[0.5,1]$).

When $n\to\infty$, $r\to\infty$, and $r/(2n+1)\to\alpha$, then $\alpha_{r,n}$ tends to α, measure $P_{2,r,n}(u)\mathrm{d}u$ tends to $-\varepsilon_{1-\alpha}+\varepsilon_{\alpha}$, where $\varepsilon_{1-\alpha}$ and ε_{α} denote a point mass at $1-\alpha$ and α, respectively, and the trimmed L-moment of order 2 tends to $Q(1-\alpha)-Q(\alpha)$, which is the difference between an upper and a lower tail α-quantiles.

Let us now interpret the definition of $\lambda_{2,r,n}$ from the quantile function in expression (2.6). Parameters r and n determine two (quantile) trimming parameters n and $r/(2n+1)$, the latter one being close to $\alpha_{r,n}$ for large r and n. Their effect on the quantile function is as follows:

1) When n increases and $r/(2n+1)$ is fixed, $P_{2,r,n}$ assigns more weight to neighbourhoods of risk levels $\alpha_{r,n}$ and $1-\alpha_{r,n}$, and less weight to neighbourhoods of risk levels 0, 0.5, 1.

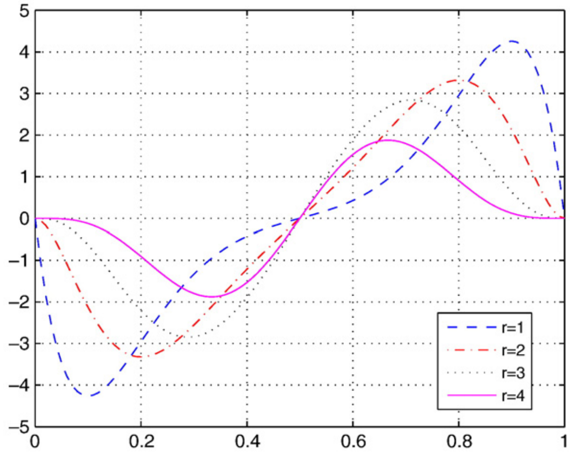

Fig. 3. Polynomials $P_{2,r,n}$ for $n=5$ and $r=1,2,3,4$.

2) The second (quantile) trimming parameter $r/(2n+1)$, for n fixed, is used to change the risk levels $\alpha_{r,n}$ and $1-\alpha_{r,n}$. For instance by decreasing ratio $r/(2n+1)$, we move $\alpha_{r,n}$ and $1-\alpha_{r,n}$ towards the left or right tails, respectively.

Note that when the ratio $r/(2n+1)$ is close to zero, some extremes are amplified rather than attenuated (trimmed). However, this amplifying effect is still compatible with the existence of $E\left(|X|^{\frac{1}{1+r}}\right)$.

2.4. Extensions and comparison with the literature

The definitions given in the previous two sections can be extended to obtain trimmed L-moments of higher orders. The (r,n) trimmed L-moments of order 3 measure the expected curvature of the conceptual order statistics as a function of rank i, and is defined as:

$$\lambda_{3,r,n} = E\left[\tilde{X}_{2n-r+1:2n+1} - 2\tilde{X}_{n+1:2n+1} + \tilde{X}_{r+1:2n+1}\right],\ n\in\mathcal{N},\ 0\le r\le n-1.$$

$\lambda_{3,r,n}$ can be written as:

$$\lambda_{3,r,n} = \int_0^1 Q(u)P_{3,r,n}(u)\mathrm{d}u,$$

where $P_{3,r,n}$ is an even function of u, with respect to $u=0.5$. Since polynomials $P_{1,n}$, $n\in\mathcal{N}$, generate the space of even functions, any trimmed L-moment of order 3 can be written as:

$$\lambda_{3,r,n} = \sum_{k=1}^{\infty}\beta_{r,n,k}\lambda_{1,k},$$

where coefficients $\beta_{r,n,k}$ do not depend of the distribution of X.

The (r^*,r,n) trimmed L-moments of order 4 are defined by:

$$\lambda_{4,r^*,r,n} = E\left[\tilde{X}_{2n-r^*+1:2n+1} - \frac{n-r^*}{n-r}3\tilde{X}_{2n-r+1:2n+1} + \frac{n-r^*}{n-r}\tilde{X}_{r+1:2n+1} - \tilde{X}_{r^*+1:2n+1}\right],$$
$$n\in\mathcal{N},\ 0\le r^*<r\le n-1.$$

The trimmed L-moments of order 4 allows us to explore the tails in neighbourhoods of two different risk levels in each tail. For a given triplet r^*,r,n, the trimmed L-moment of order 4 can take positive or negative values, but is always lower bounded [see Hosking (1990), Fig. 3, for $r^*=0$, $r=1$]. We find that:

$$\lambda_{4,r^*,r,n} = \lambda_{2,r^*,n} - \frac{n-r^*}{n-r}\lambda_{2,r,n}.$$

The trimmed L-moments of order 3 and 4 are, like all trimmed L-moments introduced in this paper, defined from a conceptual random sample of the same size $2n+1$.

Let us now compare our approach with the existing literature on L-moments and trimmed L-moments. The fact that the trimmed L-moments of different orders are all defined from a conceptual random sample of the same *odd* size distinguishes our approach from the literature. The reason is to have all the L-moments, i.e. the dispersion, skewness, etc. defined about the center of conceptual sample $\tilde{X}_{n+1,2n+1}$.

The main difference is, however, in introducing additional trimming parameters for moments of orders higher than 2. For example, $\lambda_{2,r,n}$ extends the definition of n-trimmed L-moment of order 2 in Elamir and Seheult (2003) [see also Hosking (2007)]. The n-trimmed L-moment of order 2, as defined by Elamir and Seheult, is equivalent to $\lambda_{2,r,n}$ for $r = n-1$. This particular definition of the second L-moment renders the examination of the trimming effect on quantile function Q difficult. When n tends to infinity, $\lambda_{2,n-1,n}$ tends to $Q(0.5) - Q(0.5) = 0$, and becomes non-informative.

3. L-performance

The standard Sharpe performance is the ratio of the expected excess return and the standard error of the excess return distribution [Sharpe (1966)]. This section introduces a family of new performance measures defined as ratios of L-location and L-dispersion.[7]

3.1. Definition

The L-performance is a ratio of a trimmed L-moment of order 1 and a trimmed L-moment of order 2:

$$\begin{aligned} L_{r,n} &= \lambda_{1,n} / \lambda_{2,r,n} \\ &= \frac{r!(2n-r)!}{(n!)^2} \int_0^1 Q(u)u^n(1-u)^n \mathrm{d}u \left[\int_0^1 Q(u) \left[u^{2n-r}(1-u)^r - u^r(1-u)^{2n-r} \right] \mathrm{d}u \right]^{-1}, \\ & n \in \mathcal{N},\ 0 \le r \le n-1 \end{aligned} \quad (3.1)$$

For different values of (quantile) trimming parameters n and $r/(n+1)$, we get a battery of L-performances. For example, we have:

$$L_{0,1} = \frac{E\left(\tilde{X}_{2:3}\right)}{E\left(\tilde{X}_{3:3} - \tilde{X}_{1:3}\right)} = 2\int_0^1 Q(u)u(1-u)\mathrm{d}u \left[\int_0^1 Q(u)(2u-1)\mathrm{d}u \right]^{-1}$$

$$\begin{aligned} L_{1,2} &= \frac{E\left(\tilde{X}_{3:5}\right)}{E\left(\tilde{X}_{4:5} - \tilde{X}_{2:5}\right)} = 3/2 \int_0^1 Q(u)u^2(1-u)^2 \mathrm{d}u \left[\int_0^1 Q(u) \left[u^3(1-u) - u(1-u)^3 \right] \mathrm{d}u \right]^{-1} \\ &= 3/2 \int_0^1 Q(u)u^2(1-u)^2 \mathrm{d}u \left[\int_0^1 Q(u)u(1-u)(2u-1)\mathrm{d}u \right]^{-1}. \end{aligned}$$

When $n \to \infty$, $r \to \infty$, and $r/(2n+1) \to \alpha$, say:

$$\lim_{r,n \to \infty} L_{r,n} = \frac{Q(0.5)}{Q(1-\alpha) - Q(\alpha)}. \quad (3.2)$$

The formula above involves the point prediction $Q(0.5)$ of variable X and the range of its prediction interval at level $1–2\alpha$.

The limit in Eq. (3.2) highlights the relevance of L-performances in financial applications. Given that the Value-at-Risk (VaR) is a quantile from an excess return distribution, Eq. (3.2) can be written and calculated in terms of the VaR as

$$\lim_{r,n \to \infty} L_{r,n} = \frac{\mathrm{VaR}(0.5)}{\mathrm{VaR}(1-\alpha) - \mathrm{VaR}(\alpha)}.$$

This formula resembles the standard Sharpe performance with the expected return replaced by the median, and the volatility replaced by the difference of two extreme quantiles.

3.2. L-performance versus Sharpe performance

The Sharpe ratio is a widely used measure of performance. From the theoretical point of view, there are two arguments that support that concept: 1) the Sharpe ratio is compatible with the CARA utility optimizing behavior, under the hypothesis of normally distributed returns, and 2) regardless of the return distribution, the Sharpe ratio is an outcome of the mean-variance

[7] The paper is focused on symmetric measures of performance, that treat the left and right parts of a distribution in a similar way. The extension to asymmetric measures is straightforward, in the spirit of the Sortino ratio, the Omega statistic, and the TVaR/VaR ratio.

Table 1
Funds by style.

Ticker	Fund name	Style	Manager	Fund assets
GU EH	Exane Investors Gulliver Fund	Equity Hedge	Exane Structured Asset Management	918,048,000
IC EH	Ibis Capital, LP	Equity Hedge	Ibis Management, LLC	35,000,000
OAM EH	Odey European Inc.	Equity Hedge	Odey Asset Management Limited	637,070,000
PF EH	Platinum Fund Ltd.	Equity Hedge	Optima Fund Management	1,009,357,000
PM EH	PM Capital Australian Opportunities Fund Ltd.	Equity Hedge	PM Capital Limited	19,000,000
RA EH	RAB Europe Fund	Equity Hedge	RAB Capital plc	189,000,000
RB EH	Robeco Boston Partners Long/Short Equity, L.P.	Equity Hedge	Robeco Investment Management	45,990,000
SP EH	Sprott Opportunities Hedge Fund, L.P.	Equity Hedge	Sprott Asset Management Inc.	313,000,000
RC EH	Robbins Capital Partners, L.P.	Equity Hedge	T. Robbins Capital Management, LLC	25,000,000
FS EM	Fairfield Sentry Ltd.	Equity Market Neutral	Fairfield Greenwich Group	6,800,000,000
I EM	Invesco QLS Equity	Equity Market Neutral	Invesco Structured Products Group	806,461,000
LC EN	Large Cap Core Equity	Equity Non-Hedge	First Quadrant L.P.	3,063,300,000
TR EN	Thames River European Fund	Equity Non-Hedge	Thames River Capital LLP	42,888,960
LES SS	Leveraged Short Equity Index Hedge L.P.	Short Selling	Derivative Consulting Group L.L.C.	3,000,000
APM MA	APM Global Fixed Income Composite Fund	Macro	Absolute Plus Management, LLC	177,000,000
AS MA	Aspect Diversified Fund Limited	Macro	Aspect Capital Limited	1,420,000,000
FO MA	FORT Global Contrarian LP	Macro	Fort, Inc.	694,721,000
FX MA	FX Concepts Global Currency Program	Macro	FX Concepts, Inc.	3,600,000,000
HC MA	Haidar Jupiter International Ltd.	Macro	Haidar Capital Management, LLC	80,387,000
ML MA	Maple Leaf Macro Volatility Fund	Macro	Maple Leaf Capital	635,000,000
PE MA	Permal Global Opportunities Ltd. A	Macro	Permal Asset Management, Inc.	770,000,000
WI MA	Winton Futures Fund	Macro	Winton Capital Management	4,440,000,000
DF MF	Discus Fund Limited	Managed Futures	Capital Fund Management	47,989,382
CG RV	Clinton Multistrategy Fund	Relative Value Arbitrage	Clinton Group, Inc.	685,000,000
EF RV	Endeavour Fund L.P.	Relative Value Arbitrage	Endeavour Capital LLP	2,767,000,000
WE RV	Western Investment Institutional Partners, LLC	Relative Value Arbitrage	Western Investment, LLC	62,000,000
AC CA	Advent Convertible Arbitrage Fund, L.P.	Convertible Arbitrage	Advent Capital Management, LLC	40,000,000
AI CA	Aristeia International, Ltd.	Convertible Arbitrage	Aristeia Capital LLC	2,746,479,000
PI MA	Paulson International Ltd.	Merger Arbitrage	Paulson & Co., Inc.	3,373,000,000
CSS ED	Courage Special Situations Offshore Fund, Ltd.	Event-Driven	Courage Capital Management, LLC	350,567,000
RO ED	Rosseau Limited Partnership	Event-Driven	Rosseau Asset Management Ltd.	157,263,000
TT ED	TT Event-Driven Fund	Event-Driven	TT International	990,006,144
BF FI	BlackRock Fixed Income Global Opportunities Fund LLC	Fixed Income Arbitrage	BlackRock Financial Management, Inc.	1,333,000,000
BR FI	Blue River Advantaged Muni Fund	Fixed Income Arbitrage	Blue River Asset Management	1,600,000,000
DM FI	Drake Global Opportunities Fund, Ltd.	Fixed Income Arbitrage	Drake Management LLC	3,896,000,000
RCG DS	RCG Carpathia Overseas Fund, Ltd.	Distressed Securities	Ramius Capital Group, LLC	498,000,000

optimization, along with the optimal mean-variance portfolio, the two-fund separation theorem, and the mean-variance efficiency frontier. In this Section, we show that the L-performance has a similar theoretical background.

1) Let us first assume that returns are such that $x_t = m + \sigma u_t$, where the error terms u_t have a symmetric distribution and m,σ are location and scale parameters, respectively. We have: $Q(u) = m + \sigma H(u)$, where H is the quantile function of the error term u_t. By the symmetry condition, we have: $H(-u) = H(1-u)$. It follows that the L-performances are:

$$L_{r,n} = \frac{r!(2n-r)!}{(n!)^2}\frac{m}{\sigma}\left[\int_0^1 H(u)\left[u^{2n-r}(1-u)^r - u^r(1-u)^{2n-r}\right]\mathrm{d}u\right]^{-1}.$$

The L-performances are proportional to the ratio m/σ, up to a scale factor that depends on r, n, and on the error term distribution. In particular, for Gaussian returns, L-performances are all equal to the standard Sharpe performance measure, up to a scale factor, function of the trimming parameters only. In that case, portfolio rankings based on L-performances are all identical, and are equal to the ranking based on the Sharpe performance.

2) In general, financial returns are not normally distributed. Therefore, let us consider a portfolio with allocation a, and the trimmed L-moments of portfolio returns. Then the L-moment optimization analogue of the mean-variance optimization is:

$$\mathrm{Min}_{a_0,a}\ \lambda_{2,r,n}\left(a_0 r_f + a' r_t\right) \tag{3.3}$$

subject to

$$\lambda_{1,n}\left(a_0 r_f + a' r_t\right) \geq \overline{\lambda}_1,$$
$$a_0 + a' e = w,$$

where $e=(1,\dots1)'$, r_f (resp. r_t) is the riskfree return (resp. risky return), w the wealth, $\bar{\lambda}_1$ a lower bound of the L-moment of order 1 and $\lambda_{1,n}(X)$, $\lambda_{2,r,n}(X)$ denote the L-moments of variable X. Since the quantile function is such that

$$Q(\alpha + a'r_t, u) = \alpha + Q(a'r_t, u),\ \forall\alpha,$$
$$Q\left(\frac{a'r_t}{\mu}, u\right) = \frac{1}{\mu}Q(a'r_t, u), \forall\mu,$$

and the L-moments are linear functions of the quantile function, the above optimization becomes:

$$\mathrm{Min}_a\ \lambda_{2,r,n}\left[a'\left(r_t - r_f\right)\right] \tag{3.4}$$

subject to

$$\lambda_{1,n}\left[a'\left(r_t - r_f\right)\right] \geq \bar{\lambda}_1 - wr_f,$$

by solving the budget constraint. Equivalently, we have

$$\mathrm{Min}_a \lambda_{2,r,n}\left(\frac{a'\left(r_t - r_f\right)}{\bar{\lambda}_1 - wr_f}\right)$$

subject to

$$\lambda_{1,n}\left(\frac{a'\left(r_t - r_f\right)}{\bar{\lambda}_1 - wr_f}\right) \geq 1.$$

Therefore, the optimal risky allocation α^* is given by $a^* = (\bar{\lambda}_1 - wr_f)\alpha_1^*$, where α_1^* is the solution to optimization Eq. (3.4) with $\bar{\lambda}_1 = 1 + wr_f$. Then, the allocation in riskfree asset is obtained by solving the budget constraint. It follows that the L-moment efficient portfolio combines the efficient risky allocation α_1^* with the riskfree asset, which is the two-fund separation theorem. Thus, there exist a two-fund separation theorem and the possibility of defining a L-moment efficiency frontier for any pair of L-location and L-dispersion moments $(\lambda_{1,n},\lambda_{2,r,n})$. The L-performance measures the distance between a portfolio and the L-moment efficiency frontier. In particular, the maximum L-performance is attained by a L-moment efficient portfolio [see Gourieroux and Liu (2007) for the estimation of the L-moment efficiency frontier and test for L-moment portfolio efficiency].

3.3. Estimation of L-performance

Let us now consider a sample $x_t = r_t - r_f$, $t=1,\dots,T$, of excess returns, and focus on the estimation of L-performance from trimmed L-moments of the unconditional excess return distribution. For ease of exposition we assume[8] that excess returns are independent and identically distributed (iid) with the observed order statistics: $x_{1:T} \leq \dots \leq x_{T:T}$. The observed order statistics have to be distinguished from the conceptual order statistics $\tilde{X}_{1:2n+1} < \dots < \tilde{X}_{2n+1;2n+1}$ introduced in Section 2. The latter ones are random, unobserved and used to define the theoretical L-moments. The trimming parameters r,n, which define the theoretical L-moment of interest, need to be fixed according to financial criteria, and have to be applied to all assets in a given category.

The L-performances are easily estimated from their sample counterparts. The estimator defined by:

$$\hat{L}_{r,n,T} = \frac{\sum_{i=1}^{T} x_{i:T} P_{1,n}(i/T)}{\sum_{i=1}^{T} x_{i:T} P_{2,r,n}(i/T)}. \tag{3.5}$$

is a ratio of two linear combinations of order statistics.[9] These estimators are consistent and asymptotically normal, under standard regularity conditions (see Appendix A).

The estimation consists of the following steps: 1. Choose a pair of r and n. 2. Consider the sample $x_1,\dots,x_T$ of size T of excess returns on portfolio A. The sample is first ordered, i.e. the observations are ranked in an ascending order. As a result, we obtain sample order statistics $x_{i:T}$, $i=1,2,\dots,T$ arranged as follows: $x_{1:T} < x_{2:T} < \cdots < x_{T:T}$. 3. Compute the values of polynomials $P_{1,n}$, $P_{2,r,n}$ for arguments $u=i/T$, $i=1,\dots,T$, and the selected values of r and n. 4. Plug in the numerator (denominator) of formula (3.5) the

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[8] The standard Sharpe performance measure can be extended to non-iid returns as follows:

i) First, one can consider the historical performance measures, and derive their asymptotic distributions in a non-iid framework [see e.g. Lo (2002) for this approach applied to Sharpe performance].
ii) Second, one can consider conditional performance measures instead of unconditional measures [see e.g. Darolles and Gourieroux (2008) for conditional Sharpe performances and Gourieroux and Jasiak (2008) for the definition of conditional L-Moments]. These extensions are clearly out of the scope of the present paper.

[9] A linear combination of order statistics is called a L-statistic in the statistical literature.

sample order statistics $x_{i:T}$ multiplied by the associated values of polynomial $P_{1,n}$ (resp. $P_{2,r,n}$), evaluated at rank i/T, and sum up the products across the sample.

L-performances calculated for excess returns of portfolios A,B,C,…etc., can be used to rank these portfolios. In practice, it is insightful to consider several different pairs of parameters r,n and obtain alternative rankings of portfolios with respect to $\hat{L}_{r,n,T}$ [see Section 4 below].

4. Application to hedge funds

In this section we derive and compare the rankings of hedge funds according to the unconditional Sharpe performance and a set of unconditional L-performance measures. The first part describes the set of pure hedge funds used in the application. The second part provides the summary statistics of fund return distributions to highlight the differences in fund behaviors, depending on their style. The third part compares the rankings obtained from the performance measures of interest.

4.1. The selected funds

The Hedge Fund Research, Inc. (HFR) database includes 2294 pure hedge funds and 1926 funds of funds, for which the return data are available for the period July 2004–June 2007, and expressed in US dollars. The assets under management in the database add up to a total value of $ 1400 billion for both types of funds and to $ 860 billion for the pure hedge funds.

At inception, the hedge funds are self-declared in one or several categories, called styles [see e.g. Das and Das (2004) for a description of styles in various databases]. These categories describe either the type of assets in the portfolio (styles “Currencies”, “Distressed Securities”), or the type of portfolio management (styles “Global Macro”, “Merger Arbitrage”), or both (styles “Fixed Income Arbitrage”, “Equity Long/Short”). A majority of pure hedge funds belong in 9 categories, that are “Equity Long/Short”, “Fixed Income”, “Global Macro”, “Currency”, “Futures”, “Equity Long Short Equally Weighted”, “Fixed Income Arbitrage”, “Merger Arbitrage” and “Distressed Securities”.

We select 36 funds that represent various styles and report the information on the management company, the self-declared strategy and the assets under management. This information is displayed in Table 1, where tickers are used to abbreviate the name

Table 2
L-moments of order 1 (in %).

Ticker	$\lambda_{1,n=0}$	$\lambda_{1,n=1}$	$\lambda_{1,n=2}$	$\lambda_{1,n=3}$	$\lambda_{1,n=4}$	$\lambda_{1,n=5}$	$\lambda_{1,n=\infty}$
AC CA	0.056	0.027	0.056	0.071	0.081	0.087	0.202
AI CA	0.371	0.292	0.299	0.305	0.308	0.310	0.325
APM MA	0.546	0.419	0.388	0.359	0.333	0.311	0.276
AS MA	0.814	0.906	0.975	0.986	0.978	0.962	0.601
BF FI	−0.071	−0.065	−0.055	−0.050	−0.048	−0.046	−0.026
BR FI	0.820	0.725	0.759	0.769	0.770	0.767	0.593
CG RV	0.348	0.262	0.267	0.270	0.272	0.275	0.291
CSS ED	0.499	0.436	0.444	0.448	0.450	0.452	0.443
DF MF	2.344	2.138	2.162	2.174	2.182	2.187	2.256
DM FI	1.072	0.770	0.730	0.712	0.704	0.701	0.781
EF RV	0.042	−0.024	−0.017	−0.013	−0.012	−0.013	−0.164
FO MA	0.909	0.838	0.827	0.814	0.805	0.800	0.865
FS EM	0.303	0.260	0.258	0.259	0.262	0.264	0.304
FX MA	0.859	1.016	1.107	1.128	1.127	1.116	0.883
GU EH	0.197	0.215	0.231	0.235	0.236	0.236	0.228
HC MA	0.806	0.873	0.976	1.033	1.069	1.095	1.252
I EM	1.091	1.065	1.126	1.167	1.196	1.217	1.426
IC EH	0.470	0.187	0.215	0.231	0.241	0.246	0.455
LC EN	0.908	0.922	0.999	1.042	1.067	1.083	1.063
LES SS	−1.076	−1.392	−1.477	−1.530	−1.564	−1.586	−1.584
ML MA	0.460	0.373	0.379	0.370	0.359	0.348	0.309
OAM EH	0.688	0.488	0.472	0.452	0.435	0.420	0.483
PE MA	1.277	1.156	1.165	1.152	1.134	1.116	1.099
PF EH	0.636	0.609	0.656	0.683	0.698	0.708	0.680
PI MA	1.222	0.796	0.774	0.772	0.775	0.779	0.852
PM EH	1.133	1.096	1.126	1.139	1.146	1.151	1.357
RA EH	0.839	0.602	0.569	0.538	0.512	0.491	0.305
RCG DS	0.907	0.694	0.684	0.681	0.681	0.682	0.956
RO ED	2.695	2.477	2.630	2.735	2.808	2.860	3.539
RB EH	0.758	0.736	0.755	0.760	0.761	0.761	0.799
SP EH	2.132	1.744	1.714	1.718	1.732	1.749	2.100
RC EH	−0.540	−0.950	−1.005	−1.043	−1.071	−1.093	−1.164
TR EN	1.180	1.199	1.290	1.337	1.365	1.385	1.563
TT ED	1.031	0.986	1.027	1.046	1.058	1.066	1.399
WE RV	0.611	0.461	0.437	0.428	0.425	0.426	0.510
WI MA	1.197	1.241	1.373	1.462	1.528	1.581	2.237

of each fund in the sample. This sample of 36 funds is used as an illustration. The results on other fund performances (funds of funds excluded) are available at: www.crest.fr/content/view/173/299.

4.2. Fund return distributions

The unconditional distributions of monthly hedge fund excess returns are summarized by their power moments and L-moments. Table 2 below shows the estimated L-moments of order 1 for selected values of n.

The first and last columns in Table 2 report the mean and median, respectively. From Proposition 1, it follows that the fund return distribution is symmetric if and only if all L-moments of order 1 are equal. Although a few funds such as CSS ED or DF MF, have almost symmetric distributions, the distribution of fund returns is skewed, in general. We noted earlier in the text that a simple test of symmetry based on a comparison of the mean and median can be misleading. Let us consider funds IC EH and FX MA. We observe that their respective means and medians are quite close. Fund IC EH has the L-moments at order 1 in columns 2 to 6, at $n=1,..,5$, significantly smaller than the mean and median. For fund FX MA, we find that its largest moment at order 1 is not the mean or median, but instead $\lambda_{1,3}$ in column 4, which indicates an asymmetry in the central part of the distribution. This type of asymmetry can originate when some losses are intentionally unreported by a hedge fund manager, in order to attract investors [Burgstahler and Dichev (1997), Asness, Krail and Liew (2001), Weinstein and Abdulali (2002)]. More precisely, the manager arbitrarily increases the number of small positive returns, to make the number of small gains exceed the number of small losses [Bollen and Pool (2007), Fig. 1]. If a portfolio manager uses some return smoothing techniques, the return distribution can become skewed about zero. Such data manipulation can be detected by measuring the departure of fund return distribution from symmetry in the neighbourhood of the median. The difference $Man=\lambda_{1,2}-\lambda_{1,1}$ is a measure of this effect, and can be used as a test statistic in a test of manipulation. In particular, the value of L-moment of order 1 for $n=2$ greater than the mean and the median is a warning that such a manipulation could have occurred. This is observed in our data for fund FX MA. In this respect, the L-moment analysis comes as an alternative to the Bias Ratio, which compares the number of positive returns to the number of negative returns within one standard deviation of 0 [Abdulali (2006)]. Compared to the Bias Ratio, the L-moment approach is more

Table 3
Power moments and L-moments of order 2 (in %).

	Power moments	L-moments				
Ticker	σ	$\lambda_{2,r=0,n=1}$	$\lambda_{2,r=1,n=2}$	$\lambda_{2,r=1,n=3}$	$\lambda_{2,r=1,n=4}$	$\lambda_{2,r=1,n=5}$
AC CA	1.14	1.55	1.01	1.54	1.89	2.15
AI CA	0.95	1.33	0.93	1.40	1.70	1.90
APM MA	1.90	2.92	2.09	3.13	3.76	4.17
AS MA	3.63	5.63	3.93	5.92	7.14	7.98
BF FI	0.30	0.41	0.27	0.41	0.51	0.58
BR FI	2.80	3.97	2.32	3.67	4.69	5.52
CG RV	1.22	1.67	0.98	1.53	1.95	2.28
CSS ED	1.42	2.12	1.39	2.12	2.60	2.96
DF MF	4.88	7.55	5.24	7.94	9.65	10.84
DM FI	3.38	4.92	3.05	4.75	5.97	6.92
EF RV	1.24	1.79	1.12	1.73	2.16	2.49
FO MA	1.87	2.72	1.66	2.59	3.26	3.78
FS EM	0.41	0.58	0.37	0.57	0.71	0.81
FX MA	3.09	4.34	2.51	3.99	5.13	6.06
GU EH	0.71	0.99	0.56	0.89	1.14	1.35
HC MA	2.26	3.19	1.97	3.06	3.84	4.45
I EM	2.24	3.44	2.38	3.62	4.41	4.95
IC EH	3.36	4.33	2.72	4.22	5.28	6.10
LC EN	2.35	3.56	2.43	3.73	4.60	5.20
LES SS	3.23	4.90	3.40	5.17	6.28	7.05
ML MA	1.67	2.13	1.27	2.00	2.53	2.96
OAM EH	2.66	3.76	2.44	3.75	4.63	5.30
PE MA	3.03	4.47	2.98	4.53	5.54	6.27
PF EH	2.04	3.09	1.98	3.07	3.83	4.40
PI MA	2.53	2.50	1.56	2.42	3.02	3.49
PM EH	1.69	2.49	1.65	2.52	3.10	3.53
RA EH	2.65	3.46	2.20	3.39	4.20	4.81
RCG DS	2.11	2.91	1.90	2.90	3.57	4.07
RO ED	5.93	8.75	5.89	8.95	10.93	12.35
RB EH	1.67	2.42	1.43	2.25	2.84	3.33
SP EH	3.31	4.73	3.31	4.96	5.97	6.66
RC EH	4.88	6.91	4.36	6.70	8.30	9.49
TR EN	2.74	3.91	2.51	3.87	4.80	5.51
TT ED	2.20	3.18	1.95	3.04	3.82	4.45
WE RV	1.05	1.43	0.88	1.38	1.74	2.01
WI MA	4.21	6.58	4.79	7.15	8.54	9.44

Table 4
Power moments and L-moments of order 3 and 4.

	Power moments		L moments	
Ticker	Skewness	Kurtosis	L-skewness	L-kurtosis
AC CA	0.12	1.49	0.02	0.06
AI CA	0.61	0.80	0.06	−0.12
APM MA	0.14	−1.12	0.04	−0.17
AS MA	−0.55	−0.50	−0.02	−0.11
BF FI	−0.49	1.43	−0.01	0.05
BR FI	−0.14	0.81	0.02	0.35
CG RV	0.34	1.45	0.05	0.35
CSS ED	−0.04	−0.14	0.03	0.04
DF MF	−0.04	−0.89	0.03	−0.10
DM FI	0.38	0.17	0.06	0.19
EF RV	0.11	0.41	0.04	0.18
FO MA	−0.40	0.90	0.03	0.23
FS EM	0.62	0.47	0.07	0.09
FX MA	−1.23	2.03	−0.04	0.38
GU EH	−1.16	2.68	−0.02	0.44
HC MA	−0.62	0.90	−0.02	0.20
I EM	−0.24	−0.81	0.01	−0.09
IC EH	0.92	2.82	0.07	0.15
LC EN	−0.39	−0.64	−0.00	−0.06
LES SS	0.36	−0.87	0.06	−0.10
ML MA	0.30	2.83	0.04	0.29
OAM EH	0.33	0.58	0.05	0.07
PE MA	−0.08	−0.06	0.03	−0.00
PF EH	−0.26	−0.24	0.01	0.09
PI MA	2.78	11.03	0.17	0.17
PM EH	−0.18	−0.03	0.01	0.02
RA EH	0.62	2.65	0.07	0.11
RCG DS	0.72	1.14	0.07	0.05
RO ED	0.06	−0.25	0.02	−0.02
RB EH	−0.51	1.07	0.01	0.31
SP EH	0.66	0.07	0.08	−0.12
RC EH	0.24	0.98	0.06	0.14
TR EN	−0.38	0.63	0.00	0.10
TT ED	−0.28	0.53	0.01	0.22
WE RV	1.02	1.06	0.10	0.21
WI MA	−0.36	−1.16	−0.01	−0.21

flexible, since it does not assume the existence of standard error and allows for using different values of n. While a scalar performance measure can be manipulated, the set of first- and second-order L-moments cannot, since it characterizes the entire distribution.

Table 3 shows the L-moments of order 2, for $n=1,\ldots,5$ (columns 2 to 6).[10] Column 1 reports the (power) standard error. We observe that fund RO ED has the largest standard error and the largest L-moments of order 2, for any value of trimming parameters. However, it is not always the case that the standard error and the L-moments of order 2 lead to the same ranking with respect to risk. For example, funds with the next largest standard errors, such as funds DF MF, RC EH or WI MA, have different ranks with respect to their L-moments of order 2. In particular, fund RC EH is more risky than WI MA in standard error, but less risky in L-moments of order 2, for $n=2,3,4$.

Table 4 reports the power skewness and excess kurtosis, and the L-skewness and L-kurtosis. Parameters r and n in this example are chosen so that the resulting ratios are directly compatible with the L-skewness and L-kurtosis defined by Sillitto (1951), Hosking (1990). They are given by $\lambda_{3,0,1}/\lambda_{2,0,1}$, and $\lambda_{4,0,1,2}/\lambda_{2,0,2}$, respectively. As expected, the power and L-skewness of funds CSS ED and DF MF are close to zero (see discussion of Table 2). Interestingly, some funds, such as WI MA and DF MF appear very risky in Table 3, with respect to their moments of order 2, but have rather small power excess kurtosis and L-kurtosis (L-kurtosis of the standard Normal is 0.149).

4.3. Comparison of estimated performances

In the next step, we compute the estimated Sharpe performance $\hat{S}_T$ and the estimated L-performances $\hat{L}_{0,1,T}$, $\hat{L}_{1,2,T}$, $\hat{L}_{1,3,T}$, $\hat{L}_{1,4,T}$, $\hat{L}_{1,5,T}$. The (quantile) trimming parameter $r/(2n+1)$ is equal to 0%, 20%, 14%, 11%, 9%, respectively. The estimated performances and the associated rankings are given in Table 5.

[10] For comparison, the values of trimmed L-moments of order 2 for $N(0,1)$ are: $\lambda_{2,0,1}=1.69$, $\lambda_{2,0,2}=2.33$, $\lambda_{2,1,2}=0.99$, $\lambda_{2,1,3}=1.51$, $\lambda_{2,1,4}=1.87$, $\lambda_{2,1,5}=2.12$.

Table 5
Sharpe and L-performances.

	Sharpe performance		L-performances									
Ticker	SP	Rank	$L_{0,1}$	Rank	$L_{1,2}$	Rank	$L_{1,3}$	Rank	$L_{1,4}$	Rank	$L_{1,5}$	Rank
AC CA	0.05	32	0.02	32	0.06	32	0.05	32	0.04	32	0.04	31
AI CA	0.39	15	0.22	18	0.32	21	0.22	20	0.18	20	0.16	20
APM MA	0.29	23	0.14	29	0.19	30	0.11	30	0.09	30	0.07	30
AS MA	0.22	30	0.16	26	0.25	27	0.17	26	0.14	26	0.12	24
BF FI	−0.24	35	−0.16	35	−0.20	34	−0.12	34	−0.09	34	−0.08	34
BR FI	0.29	22	0.18	23	0.33	20	0.21	22	0.16	23	0.14	23
CG RV	0.28	25	0.16	27	0.27	25	0.18	25	0.14	25	0.12	25
CSS ED	0.35	18	0.21	20	0.32	22	0.21	21	0.17	22	0.15	22
DF MF	0.48	8	0.28	11	0.41	14	0.27	15	0.23	14	0.20	14
DM FI	0.32	19	0.16	28	0.24	28	0.15	28	0.12	28	0.10	28
EF RV	0.03	33	−0.01	33	−0.01	33	−0.01	33	−0.01	33	−0.01	33
FO MA	0.49	5	0.31	8	0.50	7	0.31	10	0.25	11	0.21	11
FS EM	0.74	1	0.45	1	0.69	1	0.45	1	0.37	1	0.33	1
FX MA	0.28	26	0.23	17	0.44	13	0.28	13	0.22	15	0.18	15
GU EH	0.28	27	0.22	19	0.41	15	0.26	16	0.21	16	0.17	17
HC MA	0.36	17	0.27	13	0.50	8	0.34	7	0.28	5	0.25	5
I EM	0.49	6	0.31	6	0.47	11	0.32	8	0.27	7	0.25	6
IC EH	0.14	31	0.04	31	0.08	31	0.05	31	0.05	31	0.04	32
LC EN	0.39	16	0.26	14	0.41	16	0.28	14	0.23	13	0.21	13
LES SS	−0.33	36	−0.28	36	−0.43	36	−0.30	36	−0.25	36	−0.22	36
ML MA	0.28	28	0.18	24	0.30	23	0.19	24	0.14	24	0.12	26
OAM EH	0.26	29	0.13	30	0.19	29	0.12	29	0.09	29	0.08	29
PE MA	0.42	14	0.26	15	0.39	17	0.25	17	0.20	17	0.18	16
PF EH	0.31	21	0.20	21	0.33	19	0.22	19	0.18	19	0.16	21
PI MA	0.48	7	0.32	5	0.50	9	0.32	9	0.26	10	0.22	10
PM EH	0.67	2	0.44	2	0.68	2	0.45	2	0.37	2	0.33	2
RA EH	0.32	20	0.17	25	0.26	26	0.16	27	0.12	27	0.10	27
RCG DS	0.43	13	0.24	16	0.36	18	0.23	18	0.19	18	0.17	18
RO ED	0.45	10	0.28	12	0.45	12	0.31	12	0.26	9	0.23	8
RB EH	0.45	11	0.30	10	0.53	3	0.34	6	0.27	8	0.23	9
SP EH	0.64	3	0.37	3	0.52	5	0.35	3	0.29	3	0.26	3
RC EH	−0.11	34	−0.14	34	−0.23	35	−0.16	35	−0.13	35	−0.12	35
TR EN	0.43	12	0.31	9	0.51	6	0.35	4	0.28	4	0.25	4
TT ED	0.47	9	0.31	7	0.53	4	0.34	5	0.28	6	0.24	7
WE RV	0.58	4	0.32	4	0.50	10	0.31	11	0.24	12	0.21	12
WI MA	0.28	24	0.19	22	0.29	24	0.20	23	0.18	21	0.17	19

The values of different performance measures cannot be compared directly, as they do not have the same interpretation. In contrast, the rankings are comparable. We observe that rankings based on the Sharpe ratio and the different L-performances are quite similar. For example, LES SS is always the worst performing fund, while FS EM is always the best performer. In financial analysis, it is important to detect hedge funds whose rankings vary significantly with respect to the performance measure. For example, the ranking of WE RV fluctuates between 4 and 12, which can be due to the presence of extreme returns, and is compatible with the self-declared style “Relative Value Arbitrage”. The rankings of the self-declared “Equity Hedge” funds are stable with respect to all L-performances, except for funds GU EH and RB EH, which have few extreme returns. Finally, note that fund FS EM (Fairfield Sentry) ranked top one according to all performance measures was invested with the Madoff Investment Securities. The small values of its power and L-moments of order 2 are coherent with the reported remarkably consistent returns of some 1% per month.

5. Concluding remarks

The Sharpe performance is commonly used in the hedge fund industry. While it is a convenient measure for retail investors, it can be insufficient for banks or regulators. The Sharpe performance is very sensitive to outliers and unable to correctly assess the skewness and tails of unconditional return distributions. The L-performance improves upon the Sharpe performance in this respect. The L-performance is based on the trimmed L-moments of order 1 and 2, which are more informative than the power moments, and exist even when the power moments do not. From an economic point of view, a L-performance is an outcome of a portfolio optimizing behavior, based on alternative definitions of L-efficient portfolios and L-efficiency frontiers, in the spirit of Roy [Roy (1952), see also Bawa (1978), Haley and McGee (2006)]. The L-performance is parametrized by two (quantile) trimming parameters which allow for fine-tuning of L-performance according to financial risk criteria. Therefore, for a comprehensive fund performance analysis, it is possible to calculate a variety of performance measures. A comparison of fund rankings with respect to various L-performances can reveal the risk effect on fund performance, and detect return smoothing. Finally, note that it is very unlikely that the same fund remains ranked top one according to all performance measures, for many consecutive periods of time. Therefore, if that happens, the management of such a fund should be audited.

Appendix A

Proof of Proposition 1

The set of polynomials $P_{1,n}$, $n=1,2,\ldots$ defines a basis of the space of functions g on [0,1], which depends of u by means of $u(1-u)$. This space is the space of even functions with respect to 0.5: $g(u)=g(1-u), \forall u$.

The L-moments of order 1 are equal if and only if

$$\begin{aligned}
&\lambda_{1,n} = \lambda_{1,0}, \forall n,\\
&\Leftrightarrow \int_0^1 Q(u)P_{1,n}(u)\mathrm{d}u = \int_0^1 Q(u)\mathrm{d}u = m,\ \forall n,\\
&\Leftrightarrow \int_0^1 [Q(u)-m]P_{1,n}(u)\mathrm{d}u = 0,\ \forall n,\\
&\Leftrightarrow Q-m \text{ belongs in the space of functions on } [0,1]
\end{aligned}$$

which are orthogonal to the even functions,

$$\begin{aligned}
&\Leftrightarrow Q-m \ \text{ is an odd function}\\
&\Leftrightarrow Q(u)-m = -[Q(1-u)-m],\ \forall u,\\
&\Leftrightarrow [Q(u)+Q(1-u)]/2 = m,\ \forall u.
\end{aligned}$$

This is the condition of symmetry of the distribution of X. □

Appendix B

Asymptotic properties of the L-performance estimator

The methodology used in this Appendix is based on the empirical process and the so-called Bahadur' expansion. The advantage of this approach is that it is straightforward and easy to understand. It requires, however, a rather strong assumption on the distribution of variable X. Alternatively, we could follow an approach based on the theory of L-statistics that requires weaker assumptions [see e.g. Chernoff et al. (1967), Serfling (1980), Mason (1984)].

1. Asymptotic properties of L-moments

A trimmed L-moment is a linear function of quantile function Q. It can be written as

$$\lambda(Q,P) = \int_0^1 Q(u)P(u)\mathrm{d}u,$$

where P is a polynomial. It can consistently be estimated by the sample counterpart.

$$\hat{\lambda}_T(Q,P) = \lambda\left(\hat{Q}_T,P\right) = \int_0^1 \hat{Q}_T(u)P(u)\mathrm{d}u,$$

where $\hat{Q}_T(u) = \inf\left\{x : \frac{1}{T}\sum_{t=1}^T \mathbb{1}_{(X_t \le x)} \ge u\right\}$ is the sample quantile function. This estimator is also equal to

$$\hat{\lambda}_T(Q,P) = \frac{1}{T}\sum_{t=1}^T x_{t:T}\int_{(t-1)/T}^{t/T} P(u)\mathrm{d}u = \frac{1}{T}\sum_{t=1}^T x_{t:T}P(t/T) + o_p(1).$$

The asymptotic properties of the estimated L-moment follow from the asymptotic properties of the quantile function estimators. The quantile function estimator is related to the sample cdf by the Bahadur representation [see e.g. Koenker (2005), Section 4.3]:

$$\sqrt{T}\left[\hat{Q}_T(u) - Q(u)\right] = -\frac{1}{f[Q(u)]}\sqrt{T}\left[\hat{F}_T(Q(u)) - u\right] + o_P(1;u),$$

where $\hat{F}_T(x) = \frac{1}{T}\sum_{t=1}^T \mathbb{1}_{X_t \le x}$ is the sample cdf and $o_P(1;u)$ denotes a negligible term in probability, which depends on u. Moreover, it is known that $\sqrt{T}\left[\hat{F}_T(Q(u)) - u\right]$ weakly converges to a Brownian bridge B defined on [0,1]. Recall that a Brownian bridge is a Gaussian process on [0,1], with zero-mean and covariance $\mathrm{Cov}[B(u_1),B(u_2)]=\min(u_1,u_2)-u_1u_2$. Heuristically, we deduce the (functional) limit theorem for the sample quantile:

$$\sqrt{T}\left[\hat{Q}_T(u)-Q(u)\right]\Rightarrow -\frac{1}{f[Q(u)]}B(u), \tag{A.1}$$

where $\Rightarrow$ denotes the weak convergence (convergence in distribution), and the asymptotic behavior of a trimmed L-moment.[11]

$$\begin{aligned}\sqrt{T}\left[\lambda\left(\hat{Q}_T,P\right)-\lambda(Q,P)\right] &= \textstyle\int_0^1 \sqrt{T}\left[\hat{Q}_T(u)-Q(u)\right]P(u)\mathrm{d}u\\ \xrightarrow{d} -&\textstyle\int_0^1 \frac{1}{f[Q(u)]}B(u)\mathrm{d}u.\end{aligned} \tag{A.2}$$

In particular, estimator $\lambda(\hat{Q}_T,P)$ is consistent, asymptotically Gaussian with zero-mean and a variance given by:

$$V_{as}\left(\sqrt{T}\left[\lambda\left(\hat{Q}_T,P\right)-\lambda(Q,P)\right]\right)=\int_0^1\int_0^1[\min(u_1,u_2)-u_1u_2]P(u_1)P(u_2)\mathrm{d}Q(u_1)\mathrm{d}Q(u_2). \tag{A.3}$$

2. L-performance estimator

We have:

$$\hat{L}_{r,n,T}=\hat{\lambda}_{1,n,T}/\hat{\lambda}_{2,r,n,T},$$

where $\hat{\lambda}$ denotes the estimated L-moment. Since $\hat{\lambda}_{1,n,T}$ (resp. $\hat{\lambda}_{2,r,n,T}$) converges to $\lambda_{1,n}$ (resp. $\lambda_{2,r,n}$, and since $\lambda_{2,r,n}$ is strictly positive, it follows that $\hat{L}_{r,n,T}$ tends to $L_{r,n}$.

Moreover, we have:

$$\begin{aligned}&\sqrt{T}\left(\hat{L}_{r,n,T}-L_{r,n}\right)\\ &=\sqrt{T}\left(\frac{\hat{\lambda}_{1,n,T}}{\hat{\lambda}_{2,r,n,T}}-\frac{\lambda_{1,n}}{\lambda_{2,r,n}}\right)\\ &=\frac{1}{\lambda_{2,r,n}}\sqrt{T}\left(\hat{\lambda}_{1,n,T}-\lambda_{1,n}\right)-\frac{\lambda_{1,n}}{\lambda_{2,r,n}^2}\sqrt{T}\left(\hat{\lambda}_{2,r,n,T}-\lambda_{2,r,n}\right)+o_P(1),\end{aligned}$$

(by the delta method)

$$=\frac{1}{\lambda_{2,r,n}}\int_0^1\sqrt{T}\left[\hat{Q}_T(u)-Q(u)\right]\left[P_{1,n}(u)-L_{r,n}P_{2,n}(u)\right]\mathrm{d}u.$$

The expression of the asymptotic variance of the estimated L-performance follows directly from Eq. (A.1) applied to polynomial $P=\frac{1}{\lambda_{2,r,n}}(P_{1,n}-L_{r,n}P_{2,n})$.

Proposition 2. *Let us suppose that excess returns are independent and identically distributed. We have:*

$$\sqrt{T}\left(\hat{L}_{r,n,T}-L_{r,n}\right)\xrightarrow{d}N\left(0,\eta_{r,n}^2\right),$$

where the asymptotic variance is given by:

$$\eta_{r,n}^2=\lambda_{2,r,n}^{-2}\textstyle\int_0^1\int_0^1[\min(u_1,u_2)-u_1u_2]\left[P_{1,n}(u_1)-L_{r,n}P_{2,r,n}(u_1)\right]\left[P_{1,n}(u_2)-L_{r,n}P_{2,r,n}(u_2)\right]\mathrm{d}Q(u_1)\mathrm{d}Q(u_2).$$

[11] If the integral of the negligible term $o_P(1{:}u)$ with respect to $P(u)\mathrm{d}u$ is negligible. This assumption of uniformity of the negligible term is rather strong and can be avoided in an alternative proof of weak convergence based on the theory of L-statistics. Loosely speaking, it is difficult to estimate the tails of a distribution. Therefore, it is easy to derive the functional limit theorem Eq. (A.1) on any compact strictly included in (0,1). In contrast, it is difficult to do that on the entire interval (0,1) that includes the extreme risk levels, close to 0 and 1, without requiring that $f[Q(u)]$ is lower bounded by a strictly positive scalar in neighbourhoods of 0 and 1. However, the convergence in Eq. (A.2) remains valid if the weighting function P vanishes sufficiently fast at 0 and 1 [see e.g. Mason (1984), Shorack and Wellner (1986), Norvaisa (1993)]. Such a conditions is, for example, given in Jones and Zitikis (2003), and is clearly satisfied by polynomials $P_{1,n}$ and $P_{1,r,n}$.

3. Estimation of the asymptotic variance of $\hat{L}_{r,n,T}$

The asymptotic variance can be alternatively written as:

$$\eta_{r,n}^2 = \lambda_{2,r,n}^{-2} \int\int min\,(F(x_1),F(x_2)) - F(x_1)F(x_2)\Big[P_{1,n}(F(x_1)) - L_{r,n}P_{2,r,n}(F(x_1))\Big]\Big[P_{1,n}(F(x_2)) - L_{r,n}P_{2,r,n}(F(x_2))\Big]\mathrm{d}x_1\mathrm{d}x_2,$$

by applying the change of variable $x = Q(u)$. From this expression we infer a consistent estimator of the asymptotic variance [see e.g. Jones and Zitikis (2003, 2005)]:

$$\hat{\eta}_{r,n,T}^2 = \hat{\lambda}_{2,r,n,T}^{-2}\sum_{i=1}^{T-1}\sum_{j=1}^{T-1}\Big\{(min(i/T,j/T) - i/Tj/T)\Big[P_{1,n}(i/T) - \hat{L}_{r,n,T}P_{2,r,n}(i/T)\Big]\Big[P_{1,n}(j/T) - \hat{L}_{r,n,T}P_{2,r,n}(j/T)\Big](x_{i+1:T} - x_{i:T})\Big(x_{j+1:T} - x_{j:T}\Big)\Big\},$$

where:

$$\hat{\lambda}_{2,r,n,T} = T^{-1}\sum_{t=1}^{T}\Big[x_{t:T}P_{2,r,n}(t/T)\Big].$$

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