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Optimal futures hedging under jump switching dynamics

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ABSTRACT

The article develops a Markov regime switching Generalized Orthogonal GARCH model with conditional jump dynamics (JSGO) for optimal futures hedging. To the author's knowledge, there is no existing study on dynamic futures hedging investigating both the effects of regime switching and conditional jumps. This might be the fact that there is no existing hedging model encompassing both of these features. The JSGO solves this problem by introducing a jump switching filtering algorithm to infer ex post both the distributions of jumps and state variables and a recombining procedure to solve the path-dependency problem. To justify the usefulness of the JSGO on dynamic futures hedging, hedging exercises are performed using FTSE 100 futures data traded in the London International Financial Futures and Options Exchange (LIFFE). JSGO exhibits good out-of-sample performance compared to its jump-free and state-independent counterparts in terms of both criteria of variance reductions and utility improvements.

1. Introduction

It is widely known that when hedging a spot position with a position in the futures market, the minimum variance hedge ratio (MVHR) is equal to the ratio of the covariance of spot and futures returns to the variance of futures returns. A common approach to conditional minimum-variance hedging is to model the time-varying conditional variance–covariance matrix of returns using a multivariate GARCH model, and use forecasts from this model to construct a forecast of the conditional MVHR. Various GARCH models have been applied for estimating the time-varying MVHR. Baillie and Myers (1991) estimate the time-varying MVHR by using a GARCH model with diagonal vech specification. Kroner and Sultan (1993) and Park and Switzer (1995) apply the constant correlation GARCH (CC-GARCH) for estimating the optimal hedge ratios. Gagnon and Lypny (1995), and Brooks et al. (2002) apply BEKK (Baba–Engle–Kraft–Kroner) GARCH, and Bystrm (2003) applies orthogonal GARCH for estimating the time-varying MVHR.

Recent studies recognize that the relationship between spot and futures returns may be characterized by regime shifts (Sarno and Valente, 2000, 2005a, b). The implication is that to improve the futures hedging effectiveness, the state-dependent property between spot and futures series should also be taken account in developing dynamic hedging strategies. Alizadeh and Nomikos (2004), Lee et al. (2006) and Lee and Yoder (2007), respectively, propose regime switching least square model, regime switching state space model, and regime switching GARCH model for futures hedging and find that the hedging effectiveness are improved compared to state-independent hedging strategies.

Although these studies have captured much of the observed behavior in the spot and futures return series, it does not incorporate the jumps which capture the effects of unanticipated news events in the determination of optimal hedge ratio. Chan and Young (2005) develop a jump-BEKK GARCH for estimating the optimal hedge ratio. This jump-BEKK GARCH, however, is state-independent.

Most of the studies in this line of research discuss the effects of time-varying, regime switching and conditional jumps separately. This might be the fact that there is no existing hedging model that captures all these properties. To capture all these

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properties in developing optimal futures hedging strategies, this article develops a Markov regime switching Generalized Orthogonal GARCH model with conditional jump dynamics (Jump Switching GO; JSGO) for estimating the optimal hedge ratio. To the author's knowledge, JSGO is the first multivariate GARCH model that possesses both properties of regime-switching and jump dynamics with time-varying jump intensity. JSGO nests within it the Generalized Orthogonal GARCH (GO) (Van Der Weide, 2002) which is a state-independent and jump-free version of JSGO. A jump switching filtering algorithm is proposed to infer ex post the distributions of jumps and state variables and a recombining procedure is proposed to solve the path-dependency problem caused by the regime shifts and the recursive natural of the GARCH variance processes.

The remainder of the article is organized as follows. The Generalized Orthogonal GARCH (GO) model is introduced next. The proposed Jump Switching Generalized Orthogonal GARCH Model (JSGO) is presented in Section 3. Section 4 gives the jump switching filtering algorithm and recombining procedure. This is followed by measuring hedging performance, data description and empirical results. A conclusion ends the article.

2. Generalized Orthogonal GARCH Model (GO)

The Generalized Orthogonal GARCH model (GO) is a generalization of the Orthogonal GARCH. The Orthogonal GARCH model assumes that the observed data can be linearly transformed into a set of uncorrelated components by means of an orthogonal matrix. This assumption, however, is too restrictive and the model is suffered from identification problems. To solve this problem, Van Der Weide (2002) proposes the GO described below:

Suppose that the observed m-dimensioned economic process $\{R_t\}$ is given by

$$\mathrm{R}_t = \boldsymbol{\mu} + \mathrm{e}_t, \tag{1}$$

where $\boldsymbol{\mu} = [\mu_1 \cdots \mu_m]^T$ is an $m \times 1$ vector of conditional means, "T" stands for transpose, $\mathbf{e}_\mathrm{t} = [e_{1,t} \cdots e_{m,t}]^T = \mathbf{Z}\boldsymbol{\varepsilon}_t$, $\mathbf{Z}$ is the $m \times m$-dimensioned linear map that links the unobserved components with the observed variables, and $\boldsymbol{\varepsilon}_t = [\varepsilon_{1,t} \cdots \varepsilon_{m,t}]^T$ is an m-dimensioned vector and is normally distributed

$$\boldsymbol{\varepsilon}_t | \psi_{t-1} \sim N(\mathbf{0}, \mathbf{H}_t), \tag{2}$$

where N stands for normal distribution, ψ_{t-1} is the information set up to time $t-1$, and each component is described by a GARCH(1,1) model shown as

$$\mathrm{H}_t = \mathrm{diag}\left(h_{1,t}, \dots, h_{m,t}\right), \tag{3}$$

$$h_{i,t} = (1 - \alpha_i - \beta_i) + \alpha_i e_{i,t-1}^2 + \beta_i h_{i,t-1}, \quad i = 1, \dots, m. \tag{4}$$

This implies that the observed m-dimensioned economic process $\{\boldsymbol{R}_t\}$ is also normally distributed

$$\mathrm{R}_t | \psi_{t-1} \sim N(\boldsymbol{\mu}, \mathbf{V}_t), \tag{5}$$

where the conditional covariance matrix $\mathbf{V}_t$ is given by

$$\mathrm{V}_t = \mathbf{Z}\mathbf{H}_t\mathbf{Z}^\mathrm{T}. \tag{6}$$

The conditional correlations $\{\mathbf{C}_t\}$ of the observed process $\{\mathbf{R}_t\}$ can be computed as:

$$\mathrm{C}_t = \mathrm{D}_t^{-1} V_t \mathrm{D}_t^{-1}, \tag{7}$$

where $\mathbf{D}_t = (\mathbf{V}_t \circ \mathbf{I})^{1/2}$, $\mathbf{I}$ is an $m \times m$-dimensioned identity matrix, and $\circ$ denotes the Hadamard product. For a bivariate case, the 2×2 mapping matrix $\mathbf{Z}$ can be specified as

$$\mathrm{Z}(\theta) = \begin{pmatrix} 1 & 0 \\ \cos\theta & \sin\theta \end{pmatrix}, \tag{8}$$

where θ measures the extent to which the uncorrelated components are mapped in the same direction. For $\theta = \frac{\pi}{2}$ the observed variables are completely uncorrelated and for $\theta = 0$ the observed variables are perfectly correlated. The conditional correlation between the observed variables, denoted by ρ_t can be verified as

$$\rho_t = \frac{h_{1t} \cos\theta}{\sqrt{h_{1t}}\sqrt{h_{1t} \cos^2\theta + h_{2t} \sin^2\theta}}, \tag{9}$$

If we define $z_t = \frac{h_{2t}}{h_{1t}}$, then ρ_t can be expressed as:

$$\rho_t = \frac{1}{\sqrt{1 + z_t \tan^2\theta.}} \tag{10}$$

The parameters to be estimated are θ of rotation coefficient and the parameters (α, β) for the univariate GARCH(1,1) specifications and can be estimated by maximizing the following log likelihood function.

$$\begin{aligned} LL &= -\frac{1}{2}\sum_t m\log(2\pi) + \log|\mathrm{V}_t| + \mathrm{R}_t^T \mathrm{V}_t^{-1}\mathrm{R}_t \\ &= -\frac{1}{2}\sum_t m\log(2\pi) + \log|\mathrm{Z}(\theta)\mathrm{H}_t\mathrm{Z}^T(\theta)| + \boldsymbol{\varepsilon}_t^T\mathrm{Z}^T(\theta)\left(\mathrm{Z}(\theta)\mathrm{H}_t\mathrm{Z}^T(\theta)\right)^{-1}\mathrm{Z}(\theta)\boldsymbol{\varepsilon}_t \\ &= -\frac{1}{2}\sum_t m\log(2\pi) + \log|\mathrm{Z}(\theta)\mathrm{Z}^T(\theta)| + \log|\mathrm{H}_t| + \boldsymbol{\varepsilon}_t^T\mathrm{H}_t^{-1}\boldsymbol{\varepsilon}_t \end{aligned} \tag{11}$$

3. Jump Switching Generalized Orthogonal GARCH Model (JSGO)

The proposed Jump Switching Generalized Orthogonal GARCH model (JSGO) nests within it the GO model. Compared to GO, the system parameters in JSGO are state-dependent with state variable governed by a Markov process and the model permits discrete time jumps with a time-varying conditional jump intensity as in Chan and Maheu (2002). The JSGO's state-independent and jump-free counterparts are called Jump GO (JGO) and Switching GO (SGO), respectively in this study. The applied bivariate JSGO is specified as follows:

$$\mathrm{R}_t = \boldsymbol{\mu}_{s_t} + \mathbf{e}_{t,s_t} + \mathrm{J}_t \tag{12}$$

$$= \boldsymbol{\mu}_{s_t} + \mathrm{Z}\left(\theta_{s_t}\right)\boldsymbol{\varepsilon}_{t,s_t} + \mathrm{J}_t, \tag{13}$$

where $\boldsymbol{\mu}_{s_t} = [\mu_{c,s_t}\ \mu_{f,s_t}]^T$ is the state-dependent conditional mean vector and $\mathbf{e}_{t,s_t} = [e_{c,s_t}\ e_{f,s_t}]^T = \mathbf{Z}(\theta_{s_t})\boldsymbol{\varepsilon}_{t,s_t}$ is the state-dependent residual vector, and $\mathbf{R}_t = [r_{c,t}\ r_{f,t}]^T$. $r_{c,t}$ and $r_{f,t}$ are returns on the spot and futures series, respectively and is governed by a linear combination of uncorrelated but state-dependent components $\boldsymbol{\varepsilon}_{t,s_t}$ and a state-independent jump component $\mathbf{J}_t$. The state variable $s_t = \{1, 2\}$ is assumed to follow a first-order, two-state Markov process and the state transition probabilities are assumed to follow a logistic function such that

$$Pr(s_t = 1|s_{t-1} = 1) = \frac{\exp(p_0)}{1 + \exp(p_0)}, \tag{14 – 1}$$

$$Pr(s_t = 2|s_{t-1} = 2) = \frac{\exp(q_0)}{1 + \exp(q_0)}, \tag{14 – 2}$$

where p_0 and q_0 are unconstrained parameters.

The state-dependent linear mapping matrix $\mathbf{Z}(\theta_{s_t})$ is given by

$$\mathrm{Z}\left(\theta_{s_t}\right) = \begin{pmatrix} 1 & 0 \\ \cos\theta_{s_t} & \sin\theta_{s_t} \end{pmatrix}, \tag{15}$$

where θ_{s_t} is a state-dependent rotation angle such that the mapping matrix and as a consequence the correlation between the observed variables is subject to regime shifts.

The state-dependent residual vector $\boldsymbol{\varepsilon}_{t,s_t}$ is a 2-dimensioned vector and is normally distributed

$$\boldsymbol{\varepsilon}_{t,s_t}|\psi_{t-1} \sim N\left(0, \mathrm{H}_{t,s_t}\right), \tag{16}$$

where each component is described by a switching GARCH(1,1) model shown as

$$\mathrm{H}_{t,s_t} = \mathrm{diag}\left(h_{c,t,s_t}, h_{f,t,s_t}\right), \tag{17}$$

$$h_{i,t,s_t} = \gamma_{i,s_t} + \alpha_{i,s_t} e_{i,t-1}^2 + \beta_{i,s_t} h_{i,t-1}, \quad i \in \{c, f\}. \text{[1]} \tag{18}$$

The jump component $\mathbf{J}_t = [J_t\ J_t]^T$ is a 2-dimensioned vector with the common jump component of spot and futures return series J_t defined as

$$J_t = \sum_{k=1}^{n_t} Y_k, \tag{19}$$

[1] This specification contains the expected jump component into the variance process and this allows it to propagate and affect future volatility through the GARCH variance factor. Chan and Maheu (2002) found that this specification gives a substantially better log-likelihood value.

where the conditional jump size Y_k, given ψ_{t-1}, is presumed to be independent and normally distributed with constant mean τ and constant variance δ^2, and n_t is the discrete counting process governing the number of jumps that arrive between time $t-1$ and time t, which is distributed as a Poisson random variable with the jump intensity parameter $\lambda_t>0$ and density

$$P(n_t = k|\psi_{t-1}) = \frac{\exp(-\lambda_t)\lambda_t^k}{k!}, \quad k = 0, 1, \ldots. \tag{20}$$

The jump intensity $\lambda_t = E[n_t|\psi_{t-1}]$ is allowed to be time-varying and follows

$$\lambda_t = \lambda_0 + a\lambda_{t-1} + b\xi_{t-1}, \tag{21}$$

where ξ_{t-1} is the jump intensity residual and is calculated as

$$\begin{aligned}\xi_{t-1} &= E[n_{t-1}|\psi_{t-1}] - \lambda_{t-1}\\ &= \sum_{k=0}^{\infty} kP(n_{t-1}|\psi_{t-1}) - \lambda_{t-1},\end{aligned} \tag{22}$$

With these definition, the return vector $\mathbf{R}_t$ is normally distributed with covariance matrix $\mathbf{V}_{t,s_t}$, namely,

$$\mathbf{R}_t|\psi_{t-1} \sim N\left(\boldsymbol{\mu}_{s_t}, \mathbf{V}_{t,s_t}\right), \tag{23}$$

where

$$\mathbf{V}_{t,s_t} = \mathbf{Z}\left(\theta_{s_t}\right)\left(\mathbf{H}_{t,s_t}\right)\mathbf{Z}^T\left(\theta_{s_t}\right) + \boldsymbol{\Lambda}_t, \tag{24}$$

and $\boldsymbol{\Lambda}_t$ is the covariance matrix of jump component defined as

$$\boldsymbol{\Lambda}_t = \begin{bmatrix}\kappa_t & \kappa_t\\ \kappa_t & \kappa_t\end{bmatrix}, \quad \kappa_t = \left(\delta^2 + \tau^2\right)\lambda_t. \text{[2]} \tag{25}$$

The state-dependent correlation coefficient of spot and futures return series ρ_{t,s_t} can be shown as

$$\rho_{t,s_t} = \frac{\cos\left(\theta_{s_t}\right)h_{c,t,s_t} + \left(\delta^2 + \tau^2\right)\lambda_t}{\sqrt{\left(h_{c,t,s_t} + (\delta^2 + \tau^2)\lambda_t\right)\left[\cos^2\left(\theta_{s_t}\right)h_{c,t,s_t} + \sin^2\left(\theta_{s_t}\right)h_{f,t,s_t} + (\delta^2 + \tau^2)\lambda_t\right]}}, \tag{26}$$

and the optimal hedge ratio χ_t estimated from JSGO is given by

$$\chi_t = \frac{\mathrm{Cov}\left(r_{c,t}, r_{f,t}|\psi_{t-1}\right)}{\mathrm{Var}\left(r_{f,t}|\psi_{t-1}\right)} = \frac{\cos(\theta)h_{c,t} + \left(\delta^2 + \tau^2\right)\lambda_t}{\cos^2(\theta)h_{c,t} + \sin^2(\theta)h_{f,t} + (\delta^2 + \tau^2)\lambda_t}, \tag{27}$$

where θ, $h_{c,t}$, and $h_{f,t}$ are respectively the recombined values of θ_{s_t}, h_{c,t,s_t}, and h_{f,t,s_t} and are derived with the recombining procedure described in the next section.

4. Jump Switching Filtering Algorithm and Recombining Procedure

To apply maximum likelihood approach for estimating the system parameters, a jump switching algorithm is proposed to infer ex post the distributions of jumps and state variables.[3] The proposed filter is an intervening filter of Hamilton filter (Hamilton, 1989, 1994) and Chan and Maheu's filter (Chan and Maheu, 2002). Let $f(\mathbf{R}_t|n_t=k, s_t=i, \psi_{t-1})$ denote the conditional density of return vector given that k jumps occur and state variable is in the i state. The conditional density of returns given information set ψ_{t-1} is completed by integrating out the state variable s_t and the possible number of jumps n_t.

$$\begin{aligned}f(\mathrm{R}_t|\psi_{t-1}) &= \sum_{i=1}^{2}\sum_{k=0}^{\infty} \mathrm{f}(\mathrm{R}_t, \mathrm{n_t} = \mathrm{k}, \mathrm{s_t} = \mathrm{i}|\psi_{t-1})\\ &= \sum_{i=1}^{2}\sum_{k=0}^{\infty} \mathrm{f}(\mathrm{R}_t|\mathrm{n_t} = \mathrm{k}, \mathrm{s_t} = \mathrm{i}, \psi_{t-1}) \times \mathrm{P}(\mathrm{n_t} = \mathrm{k}|\psi_{t-1}) \times \mathrm{P}(\mathrm{s_t} = \mathrm{i}|\psi_{t-1}).\end{aligned} \tag{28}$$

[2] The expectations of J_t and J_t^2 can be shown to be $\tau\lambda_t$ and $\delta^2\lambda_t + \tau^2(\lambda_t + \lambda_t^2)$, respectively and therefore the variance of J_t is equal to $(\delta^2 + \tau^2)\lambda_t$.

[3] Although the jump-switching version of GO GARCH is illustrated in this article, the jump switching filter proposed in this study is a general algorithm and the application of the algorithm is not limited to the GO GARCH. For example, a jump-switching version of the varying correlation (VC) GARCH (Tse et al., 2002) can also be easily derived.

Here, the possible number of jumps n_t is assumed to be state-independent. Having observed $\mathbf{R}_t$ and using the Bayes rule, we can infer post the probability of the occurrence of k jumps at time t with Chan and Maheu's filter defined as

$$P(n_t = k|\psi_t) = \frac{f(\mathbf{R}_t|n_t = k, \psi_{t-1}) \times P(n_t = k|\psi_{t-1})}{f(\mathbf{R}_t|\psi_{t-1})}, \quad k = 0, 1, 2, \ldots, \tag{29}$$

where $f(\mathbf{R}_t|\psi_{t-1})$ is from Eq. (28), $P(n_t = k|\psi_{t-1})$ is from Eq. (20) and $f(\mathbf{R}_t|n_t = k,\psi_{t-1})$ is given by

$$\begin{aligned} f(\mathbf{R}_t|n_t = k, \psi_{t-1}) &= \sum_{i=1}^{2} \mathrm{f}(\mathbf{R}_t, \mathrm{s}_t|\mathrm{n}_t = \mathrm{k}, \psi_{t-1}) \\ &= \sum_{i=1}^{2} \mathrm{f}(\mathbf{R}_t|\mathrm{n}_t = \mathrm{k}, \mathrm{s}_t = \mathrm{i}, \psi_{t-1}) \times \mathrm{P}(\mathrm{s}_t = \mathrm{i}|\psi_{t-1}). \end{aligned} \tag{30}$$

The state probability $P(s_t = i|\psi_{t-1})$ can be derived as follows

$$P(s_t = i|\psi_{t-1}) = \sum_{j=1}^{2} \mathrm{P}(\mathrm{s}_t = \mathrm{i}|\mathrm{s}_{t-1} = \mathrm{j}) \times \mathrm{P}(\mathrm{s}_{t-1} = \mathrm{j}|\psi_{t-1}), \tag{31}$$

where $P(s_t = i|s_{t-1} = j)$ are the state transition probabilities defined in Eqs. (14—1) and (14—2), and $P(s_{t-1} = j|\psi_{t-1})$ can be derived recursively via the Hamilton filter which is given by

$$P(s_t = j|\psi_t) = \frac{f(\mathbf{R}_t|s_t = j, \psi_{t-1}) \times P(s_t = j|\psi_{t-1})}{f(\mathbf{R}_t|\psi_{t-1})}. \tag{32}$$

The conditional state probability $P(s_t = j|\psi_{t-1})$ is from Eq. (31), $f(\mathbf{R}_t|\psi_{t-1})$ is from (28), and $f(\mathbf{R}_t|s_t = j,\psi_{t-1})$ can be derived as

$$f(\mathbf{R}_t|s_t = i, \psi_{t-1}) = \frac{\sum_{k=0}^{\infty} f(\mathbf{R}_t, n_t = k, s_t = i|\psi_{t-1})}{P(s_t = i|\psi_{t-1})}. \tag{33}$$

To clearly address this filtering algorithm, a flow chart of the complete filtering algorithm is shown in the Appendix A.

The assumptions in Eqs. (16) and (19) imply that the distribution of return conditional on the most recent information set, state i, and k jumps is normally distributed as

$$\begin{aligned} f(\mathbf{R}_t|n_t = k, s_t = i, \psi_{t-1}) &= \frac{1}{2\pi}|\mathbf{V}_{t,i,k}|^{-\frac{1}{2}}\exp\left\{-\frac{1}{2}\mathbf{u}_{t,k}^T\mathbf{V}_{t,i,k}^{-1}\mathbf{u}_{t,k}\right\} \\ &= \frac{1}{2\pi}\left|\mathbf{Z}(\theta_i)\mathbf{H}_{t,i}\mathbf{Z}(\theta_i)^T + \boldsymbol{\Lambda}_{t,k}\right|^{-\frac{1}{2}} \times \exp\left\{-\frac{1}{2}\mathbf{u}_{t,k}^T\left[\mathbf{Z}(\theta_i)\mathbf{H}_{t,i}\mathbf{Z}(\theta_i)^T + \boldsymbol{\Lambda}_{t,k}\right]^{-1}\mathbf{u}_{t,k}\right\}, \end{aligned} \tag{34}$$

where $\mathbf{V}_{t,i,k}$ stands for the covariance matrix of $\mathbf{R}_t$ given information set ψ_{t-1}, state i, and k jumps, and the state-independent $\mathbf{u}_{t,k}$ and $\boldsymbol{\Lambda}_{t,k}$ are given by

$$\mathbf{u}_{t,k} = \begin{bmatrix} r_{c,t} \\ r_{f,t} \end{bmatrix} - \begin{bmatrix} k\tau \\ k\tau \end{bmatrix}, \tag{35}$$

$$\boldsymbol{\Lambda}_k = \begin{bmatrix} k\delta^2 & k\delta^2 \\ k\delta^2 & k\delta^2 \end{bmatrix}. \tag{36}$$

Eqs. (28) and (33) involve an infinite sum over the possible number of jumps n_t. In practice, the maximum number of jumps η is truncated at a large enough value such that the probability of η or more jumps to the machine precision is 0 and the likelihood and the parameter estimate do not change.

In addition to the filtering algorithm, a recombining procedure has to be proposed to make JSGO tractable. As shown in Eq. (18), the variance process is recursive. When a recursive process is subject to regime switching, the recursive nature of the process makes the model intractable (Cai, 1994; Hamilton and Susmel, 1994; Gray, 1995, 1996; Lee et al., 2006; Lee and Yoder, 2007) which is the well known path-dependency problem in regime switching GARCH literature. To solve this path-dependency problem, one has to recombine all conditional variances at each time step. The conditional variance of the spot return $v_{c,t}$ can be recombined using the following equation.

$$\begin{aligned} v_{c,t} &= E\left[r_{c,t}^2|\psi_{t-1}\right] - E\left[r_{c,t}|\psi_{t-1}\right]^2 \\ &= p_{1t}\left[h_{c,t,1} + \left(\delta^2 + \tau^2\right)\lambda_t\right] + (1 - p_{1t})\left[h_{c,t,2} + \left(\delta^2 + \tau^2\right)\lambda_t\right] - \{p_{1t}(\tau\lambda_t) + (1 - p_{1t})(\tau\lambda_t)\}^2 \\ &= p_{1t}h_{c,t,1} + (1 - p_{1t})h_{c,t,2} + \left(\delta^2 + \tau^2\right)\lambda_t - (\tau\lambda_t)^2, \end{aligned} \tag{37}$$

where p_{1t} is the conditional probability of being in regime 1 at time t defined in Eq. (31).

Since the variance of spot return is equal to $h_{c,t}+(\delta^2+\tau^2)\lambda_t$, it follows that

$$h_{c,t} = v_{c,t} - \left(\delta^2 + \tau^2\right)\lambda_t. \tag{38}$$

The conditional variance of the futures return $v_{f,t}$ can be recombined using the following equation

$$\begin{aligned} v_{f,t} &= E\left[r_{f,t}^2 | \psi_{t-1}\right] - E\left[r_{f,t} | \psi_{t-1}\right]^2 \\ &= p_{1t}\left[\cos^2(\theta_1)h_{c,t,1} + \sin^2(\theta_1)h_{f,t,1} + \left(\delta^2 + \tau^2\right)\lambda_t\right] \\ &\quad + (1-p_{1t})\left[\cos^2(\theta_2)h_{c,t,2} + \sin^2(\theta_2)h_{f,t,2} + \left(\delta^2 + \tau^2\right)\lambda_t\right] - (\tau\lambda_t)^2. \end{aligned} \tag{39}$$

Since the variance of futures return is equal to $\cos^2(\theta)h_{c,t}+\sin^2(\theta)h_{f,t}+(\delta^2+\tau^2)\lambda_t$, it follows that

$$h_{f,t} = \frac{v_{f,t} - \cos^2(\theta)h_{c,t} - \left(\delta^2 + \tau^2\right)\lambda_t}{\sin^2(\theta)}, \tag{40}$$

where the rotation coefficients θ is equal to the weighted average of θ_1 and θ_2 , weighted by the conditional state probability p_{1t}.

The unknown parameters in JSGO are $\Theta=\{p_0, q_0, \alpha_{c,s_t}, \alpha_{f,s_t}, \beta_{c,s_t}, \beta_{f,s_t}, \theta_{s_t}, \tau, \delta^2, \lambda_0, a, b\}$ for $s_t=\{1,2\}$, which can be estimated by maximizing the following log-likelihood function

$$LL = \sum_{t=1}^{T} \log[p_{1t}f(\mathbf{R}_t | s_t = 1, \psi_{t-1}) + (1-p_{1t})f(R_t | \mathbf{s}_t = 2, \psi_{t-1})], \tag{41}$$

where $f(\mathbf{R}_t|s_t=i,\psi_{t-1})$, $i\in\{1,2\}$ is defined in Eq. (33).

5. Measuring hedging performance, data description and empirical results

Hedging performance is evaluated from both a risk-minimization and a utility standpoint. Let χ_t be the estimated optimal hedge ratios derived from various hedging strategies, the return from the hedged portfolio can be expressed as $r_{p,t+1}=r_{c,t+1}-\chi_t r_{f,t+1}$. From a risk-minimization standpoint, a hedger chooses a hedging strategy to minimize the variance of the hedged portfolio return or equivalently to maximize the variance reduction of a hedging strategy compared to the unhedged position. Due to the fact that dynamic hedging strategies[4] require frequent rebalancing and are more costly to implement, the economic significance of these dynamic hedging methods is also investigated. Consider a hedger with a mean-variance expected utility function:

$$E\left[U\left(r_{p,t}\right) | \psi_{t-1}\right] = E\left[r_{p,t} | \psi_{t-1}\right] - \vartheta Var\left(r_{p,t} | \psi_{t-1}\right), \tag{42}$$

where ϑ is the degree of risk aversion. A dynamic hedging strategy is considered to be superior to a static ordinary least square (OLS) method if it has higher expected utility net of transaction costs.

FTSE 100 nearby futures contract traded in the London International Financial Futures and Options Exchange (LIFFE) is examined in this study. Both spot and futures rates were collected from Datastream from January 1985 to December 2007. Estimation of all models was conducted using daily data from January 1985 to December 2000; the remaining data are used for out-of-sample analysis. Summary statistics for spot and futures returns are shown in Table 1 and parameter estimates of alternative models are presented in Table 2. The parameters are estimated by maximizing the log-likelihood functions using GAUSS. The unconditional covariance matrix $\mathbf{H}_0=\mathbf{I}$ and $\lambda_0/(1-a)$, the unconditional value of λ_t are used as initial values for the jump switching filter, where $\mathbf{I}$ is a 2 by 2 identity matrix. For JGO and JSGO, the maximum number of jumps $\eta=20$ was selected as the truncation point based on the selection method discussed in section 4.

Table 3 presents summary statistics regarding the effects of static and dynamic hedging strategies on in- and out-of-sample hedging effectiveness. The performance of JSGO is compared with performances of OLS, the ordinary least square, GO GARCH, JGO GARCH, the state-independent jump GO GARCH, and SGO the jump-free regime switching GO GARCH. Results show that JSGO hedging method exhibits good hedging performance in terms of variance reduction in sample. SGO has the lowest variance which is equal to 10.95%. JSGO is only second to the SGO with a variance of 11.04%. The variance of the hedged portfolio with JSGO hedging is reduced by 0.08%, 0.84%, and 1.04% compared to OLS, GO, and JGO hedging.

[4] Although GARCH hedging strategies rebalanced on a daily or weekly basis are commonly called dynamic hedging strategies in the futures hedging literature, they are really static one-period hedge because the hedging rule is not developed from a true dynamic optimization problem. In this article JSGO hedging is still called a dynamic hedging strategy in accordance with the convention of this line of research even if it is actually a static one-period hedge.

Table 1
Summary statistics for spot and futures returns of daily FTSE 100 index data.

	FTSE 100 daily return data			
	In-sample		Out-of-sample	
	Spot	Futures	Spot	Futures
Max	0.0760	0.0809	0.0590	0.0595
Min	−0.1303	−0.1675	−0.0589	−0.0606
Mean	0.0004	0.0004	0.0000	0.0000
SD	0.0097	0.0112	0.0111	0.0110
Skewness	−1.0574	−1.0629	−0.2441	−0.1887
Kurtosis	15.2813	16.2357	3.8572	4.0835

Note: Returns are calculated as the differences in the logarithm of price multiplied by 100. The in-sample data period is from January 1985 to December 2000 and the out-of-sample data period is from January 2001 to December 2007.

Out-of-sample, the proposed JSGO has the best hedging performance. The variance of the hedged portfolio with JSGO hedging is reduced by 2.96%, 0.2%, 0.14% and 0.10% compared to OLS, GO, JGO and SGO hedging. All time-varying hedging methods considered in this study improve the hedging effectiveness compared to static OLS hedging out-of-sample. Allowing the model to be either

Table 2
Estimates of unknown parameters of alternative models.

	FTSE 100 daily data			
	GO	JGO	SGO	JSGO
p_0			2.648	0.115
			(0.158)**	(0.129)
q_0			−0.025	2.574
			(0.036)	(0.146)**
μ_{c1}	0.063	0.070	0.058	0.028
	(0.012)**	(0.014)**	(0.014)**	(0.036)
μ_{c2}			0.049	0.064
			(0.023)*	(0.015)**
μ_{f1}	0.062	0.069	0.048	0.125
	(0.013)**	(0.014)**	(0.015)**	(0.019)**
μ_{f2}			0.156	0.054
			(0.042)**	(0.015)**
α_{c1}	0.075	0.078	0.104	0.000
	(0.009)**	(0.009)**	(0.018)**	(0.018)
α_{c2}			0.000	0.091
			(0.010)	(0.010)**
α_{f1}	0.034	0.037	0.240	0.001
	(0.006)**	(0.006)**	(0.213)	(0.010)
α_{f2}			0.000	0.226
			(0.004)	(0.056)**
β_{c1}	0.905	0.916	0.896	0.933
	(0.013)**	(0.010)**	(0.054)**	(0.041)**
β_{c2}			0.749	0.909
			(0.273)**	(0.011)**
β_{f1}	0.949	0.946	0.760	0.883
	(0.010)**	(0.010)**	(0.026)**	(0.052)**
β_{f2}			0.749	0.774
			(0.049)**	(0.028)**
θ_1	0.369	0.370	0.060	0.214
	(0.004)**	(0.004)**	(0.039)	(0.036)**
θ_2			0.168	0.077
			(0.112)	(0.013)**
τ		−0.538		−1.968
		(0.234)		(0.470)**
δ^2		2.709		0.978
		(1.028)**		(0.236)**
λ_0		0.034		0.008
		(0.016)**		(0.007)
a		0.050		0.188
		(0.067)		(0.388)
b		0.033		0.167
		(0.024)		(0.058)**
LL	−7306.05	−6525.25	−7143.46	−6433.02

Note. Figures in parentheses are standard errors and LL stands for log-likelihood.
*Significant at the 5% level; **Significant at the 1% level.

Table 3
In- and out-of-sample hedging effectiveness.

	FTSE 100 daily data					
	In sample		Out-of-sample			
	Variance of Hedged Portfolio Return	Improvement of JSGO Over Other Hedging Strategies[1]	Variance of Hedged Portfolio Return	Improvement of JSGO Over Other Hedging Strategies	Expected Daily Utility[2]	Utility Gains of JSGO Over Other Hedging Strategies[3]
Unhedged	93.93%		123.74%			
OLS	11.12%	0.08%	8.12%	2.96%	−32.46%	14.54%
GO	11.82%	0.84%	4.71%	0.20%	−18.90%	0.98%
SGO	10.95%	−0.10%	4.59%	0.10%	−18.43%	0.50%
JGO	12.01%	1.04%	4.63%	0.14%	−18.55%	0.63%
JSGO	11.04%		4.46%		−17.92%	

Note. 1. Improvement of JSGO over other hedging strategies is defined as the difference of the percentage variance reduction of JSGO and the percentage variance reduction of alternative hedging strategies.
2. Expected daily utility is calculated based on Eq. (42).
3. Utility gains of JSGO over other hedging strategies are defined as the differences of the expected utility of JSGO and the expected utilities of alternative models.

state-dependent or with conditional jumps improves the hedging performance and the out-of-sample hedging effectiveness is improved most when both properties are taken into consideration. In general, allowing the model to be state-dependent and with conditional jumps exhibits good hedging performance both in- and out-of-sample.

To better understand the economic significance of portfolio variance reduction, the utility based criterion is also used in comparing the performances of alternative models. Table 3 gives the utility gains of JSGO over other hedging strategies. Following other empirical studies in the literature (Park and Switzer, 1995; Alizadeh and Nomikos, 2004; Lee et al., 2006), the hedger is assumed to have an expected utility function given by Eq. (42) with a degree of risk aversion equal to 4. As shown in Table 3, JSGO has the best hedging performance in terms of utility gain. JSGO provides utility gains of 14.54%, 0.98%, 0.63%, and 0.5% compared to OLS, GO, JGO and SGO hedging. The hedger's net benefit from using JSGO hedging over OLS hedging is 14.54% net of transaction cost from dynamic rebalancing. Since the typical round trip transaction cost is around 0.02% to 0.04%, a mean-variance expected utility-maximizing hedger will benefit from hedging with JSGO even after taking account of these transaction costs.

Figs. 1 to 5 display the hedge ratios, state-dependent conditional correlations, regime probabilities, and jump intensities for the FTSE 100 futures data. Only out-of-sample data are plotted to make the figure clearer. Fig. 1 compares the hedge ratios of GO and SGO and Fig. 2 compares the hedge ratios of JGO and JSGO. All hedge ratios are time-varying and fluctuate within the range of 0.758 and 0.996. The small fluctuation range simply reflects the fact that spot and futures returns on daily FTSE 100 data are highly correlated. Fig. 3 shows the JSGO estimates of the state-dependent time-varying correlations. The spot and futures return series has an average correlation equals to 0.986 in the higher correlation state and an average correlation

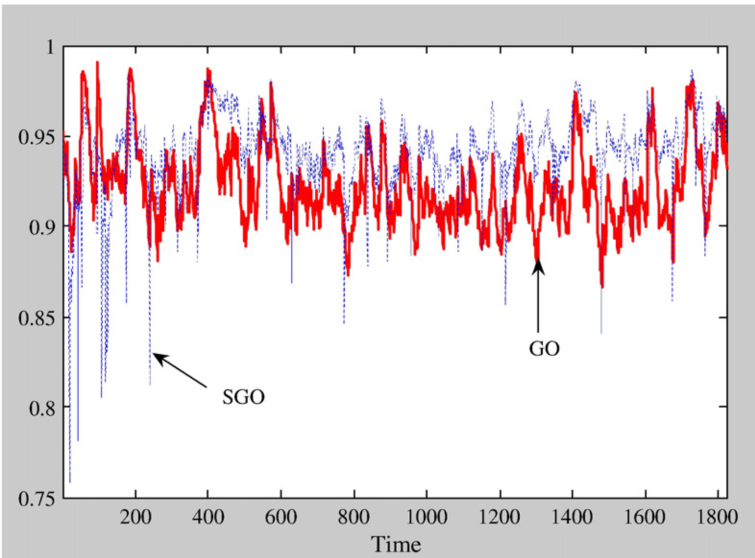

Fig. 1. GO and SGO hedge ratios.

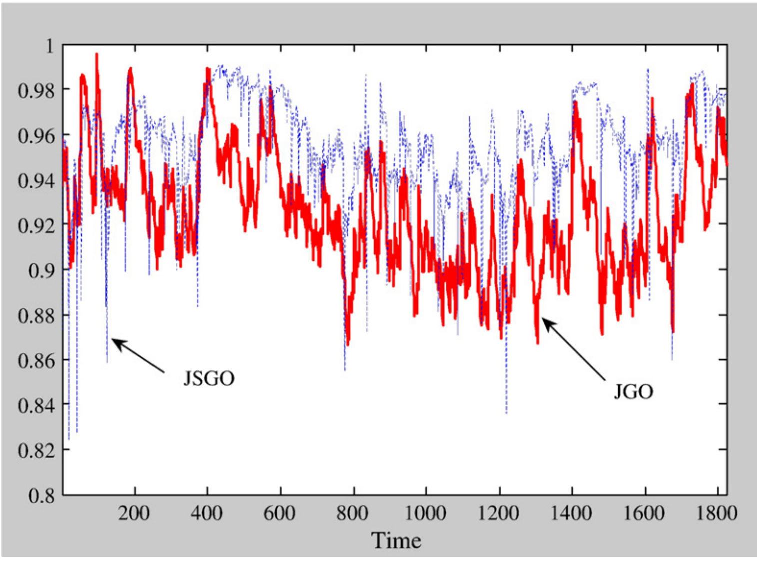

Fig. 2. JGO and JSGO hedge ratios.

equals to 0.901 in the lower correlation state. Even the lower correlation state exhibits high correlation between spot and futures returns. The state probabilities of being in the lower correlation state estimated from JSGO are shown in Fig. 4. It can be seen that the higher correlation state dominates the regime. About 95.6% of the time in the out-of-sample period, market is more likely to stay in the higher correlation state. Fig. 5 shows the time-varying jump intensities estimated from JSGO. The average jump intensity is 0.125. In Table 2, the estimated coefficient of λ_{t-1} is not significant, but the coefficient of ξ_{t-1} is significant at 1% level. This implies that the time-varying property of jump intensities is mainly contributed by the jump intensity residual term.

6. Conclusions

The focus of this article has been proposing a general model for developing dynamic futures hedging strategies. The proposed Markov regime switching Generalized Orthogonal GARCH model with conditional jump dynamics (JSGO) captures

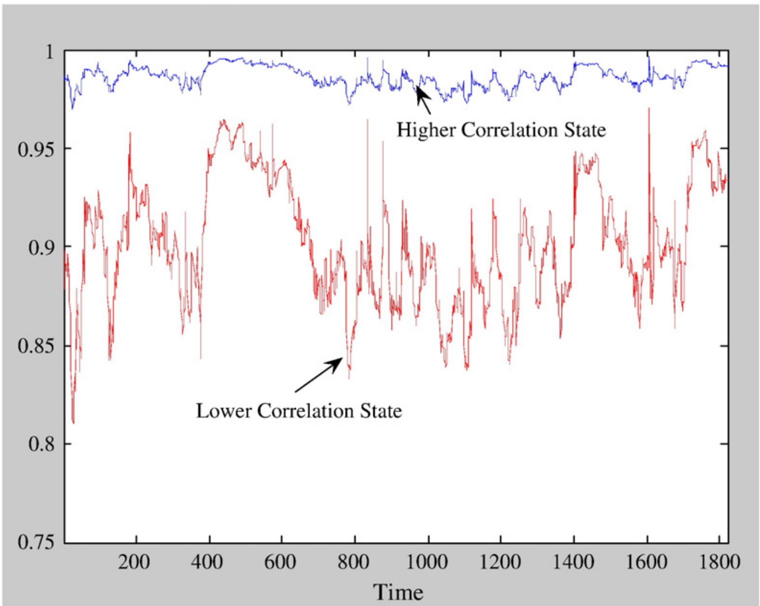

Fig. 3. Time varying correlations estimated from JSGO.

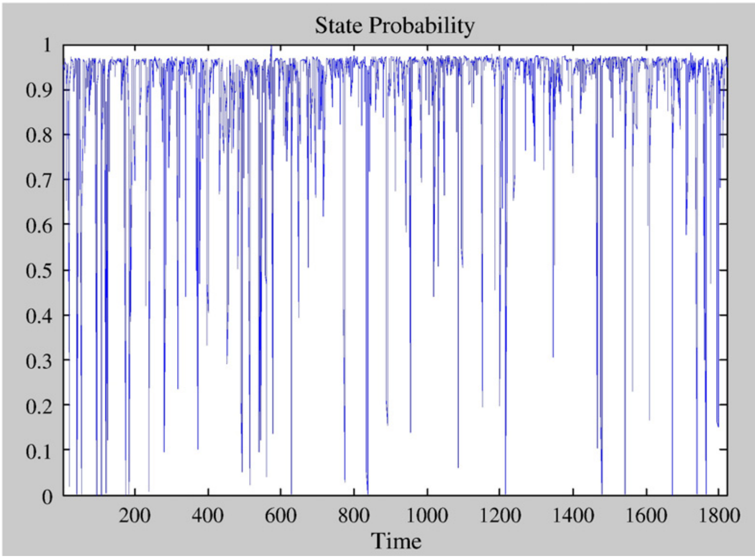

Fig. 4. Regime probability of higher correlation state estimated from JSGO.

all properties of time-varying, regime-switching, and conditional jumps which are considered as important properties in studying the risk structure of the spot and futures markets in the time-varying minimum variance hedging literature. The proposed jump switching filtering algorithm and recombining procedure allows us to simultaneously investigating all these effects on the futures hedging effectiveness that have been discussed separately in the previous studies.

FTSE 100 futures data traded in the London International Financial Futures and Options Exchange (LIFFE) are used for the hedging exercise to justify the usefulness of JSGO. Empirical results suggest that allowing a hedging strategy to possess all properties of time-varying, regime-switching, and conditional jumps improves the out-of-sample hedging effectiveness in terms of both criteria of variance reduction and utility maximization. In sample, JSGO is only second to SGO, the jump-free counterpart of JSGO. Overall, JSGO provides a general framework for dynamic futures hedging and exhibits good hedging performance.

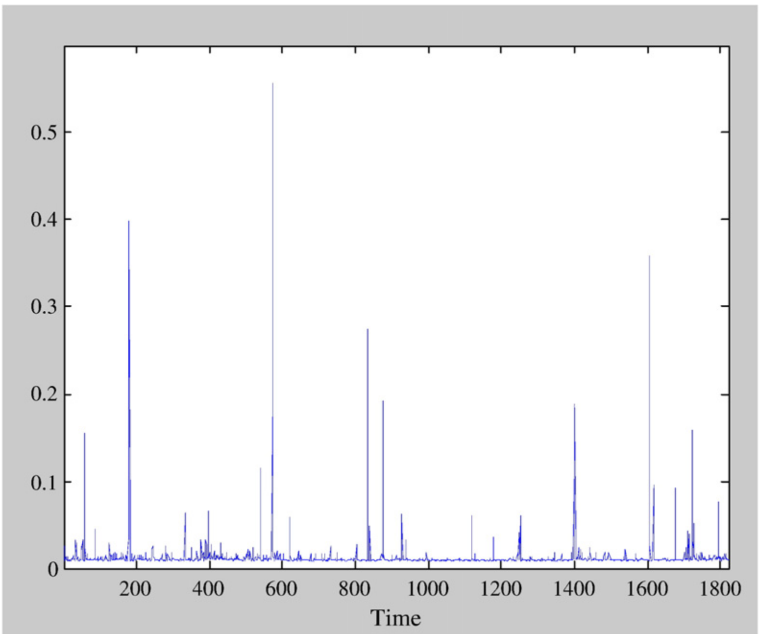

Fig. 5. Time-varying jump intensity estimated from JSGO.

Acknowledgement

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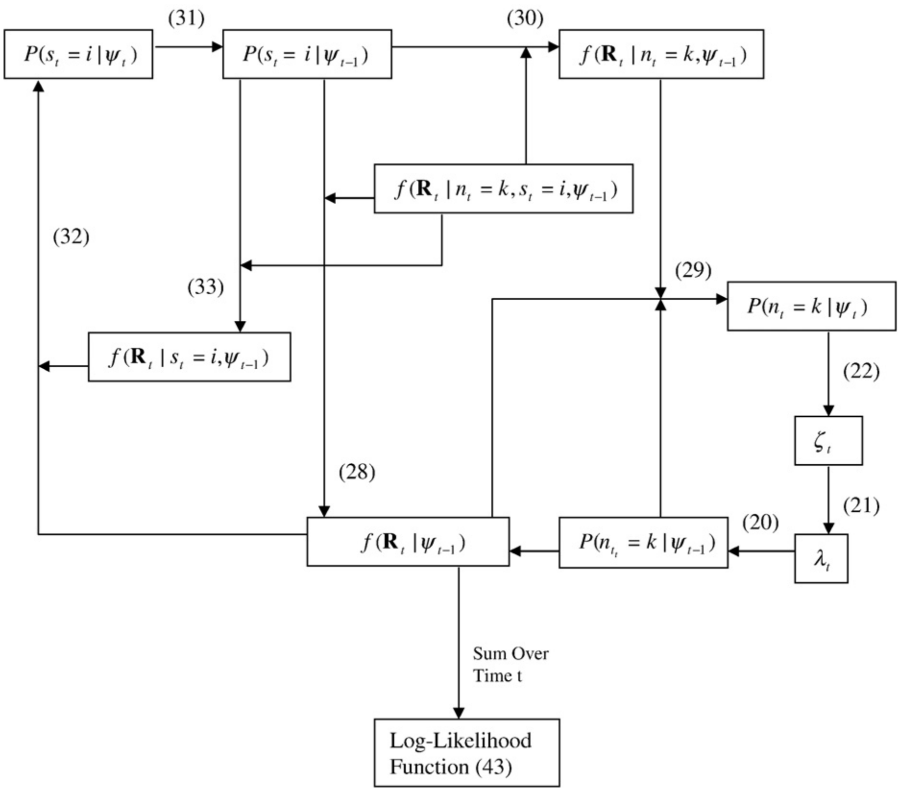
Appendix A. Flow chart of the proposed jump switching filtering algorithm



Note:figures in the parenthesis stand for the equation numbers

References

Alizadeh, Amir, Nomikos, Nikos, 2004. A Markov regime switching approach for hedging stock indices. J. Futures Mark. 24, 649–674.
Baillie, Richard T., Myers, Robert J., 1991. Bivariate GARCH estimation of the optimal commodity futures hedge. J. Appl. Econ. 6, 109–124.
Brooks, Chris, Henry, ólan T., Persand, Gita, 2002. The effect of asymmetries on optimal hedge ratios. J. Bus. 75, 333–352.
Byström, H.N.E., 2003. The hedging performance of electricity futures on the Nordic power exchange. Appl. Econ. 35, 1–11.
Cai, Jun, 1994. A Markov model of switching-regime ARCH. J. Bus. Econ. Stat. 12, 309–316.
Chan, Wing H., Maheu, John M., 2002. Conditional jump dynamics in stock market returns. J. Bus. Econ. Stat. 20, 377–389.
Chan, Wing H., Young, Denise, 2005. Jumping hedges: an examination of movements in copper spot and futures markets. J. Futures Mark. 26, 169–188.
Gagnon, Louis, Lypny, Greg, 1995. Hedging short-term interest risk under time-varying distribution. J. Futures Mark. 15, 767–783.
G ray, Stephen F., 1995. An analysis of conditional regime-switching models. Working Paper. Duke University.
G ray, Stephen. F., 1996. Modeling the conditional distribution of interest rates as a regime-switching process. J. Financ. Econ. 42, 27–62.
Hamilton, J.D., 1989. A new approach to the economic analysis of nonstationary time series and the business cycle. Econometrica 57 (2), 357.
Hamilton, J.D., 1994. Time series analysis. Princeton University Press.
Hamilton, James D., Susmel, Raul, 1994. Autoregressive conditional heteroscedasticity and changes in regime. J. Econ. 64, 307–333.
Kroner, Kenneth F., Sultan, Jahangir, 1993. Time-varying distribution and dynamic hedging with foreign currency futures. J. Financ. Quant. Anal. 28, 535–551.
Lee, Hsiang-Tai, Yoder, Jonathan K., 2007. Optimal hedging with a regime-switching time-varying correlation GARCH model. J. Futures Mark. 27, 495–516.
Lee, Hsiang-Tai, Yoder, Jonathan K., Mittelhammer, Ron C., McCluskey, Jill J., 2006. A random coefficient autoregressive Markov regime switching model for dynamic futures hedging. J. Futures Mark. 26, 103–129.
Park, Tae H., Switzer, Lorne N., 1995. Bivariate GARCH estimation of the optimal hedge ratios for stock index futures: a note. J. Futures Mark. 15, 61–67.
Sarno, Lucio, Valente, Giorgio, 2000. The cost of carry model and regime shifts in stock index futures markets: an empirical investigation. J. Futures Mark. 20, 603–624.
Sarno, Lucio, Valente, Giorgio, 2005a. Empirical exchange rate models and currency risk: some evidence from density forecasts. J. Int. Money Financ. 24, 363–385.
Sarno, Lucio, Valente, Giorgio, 2005b. Modelling and forecasting stock returns: exploiting the futures market, regime shifts, and international spillovers. J. Appl. Econ. 20, 345–376.
Tse, Y.K., Tsui, Albert, K.C., 2002. A multivariate generalized autoregressive conditional heteroscedasticity model with time-varying correlations. J. Bus. Econ. Stat. 20, 351–362.
Van Der Weide, Roy, 2002. GO-GARCH: a multivariate generalized orthogonal GARCH model. J. Appl. Econ. 17, 549–564.